CENCO Physics
Selective Experiments In Physics

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THERMAL CONDUCTIVITY
(ADVANCED THEORY)

OBJECT: To determine the thermal conductivity of a thin slab of material of low conductivity.

METHOD: A thin slab of a material whose thermal conductivity is to be measured is placed between an upper vessel kept at constant temperature and a lower insulated block of copper of known thermal properties. The heat conducted through the material raises the temperature of the copper block by a measured amount. Thermocouples and a galvanometer are used to indicate temperature differences. From the rate at which heat is conducted through the material, and the area, thickness and temperature difference of the faces of the specimen of material, its thermal conductivity is calculated.

THEORY: Consider a uniform slab of material of thickness *l* and cross-sectional area *A* whose faces are at temperature θ_1 and θ_0 degrees where θ_1 is greater than θ_0. Heat is conducted through the slab from the face at the higher temperature to the face at the lower temperature. If the slab is thermally insulated so that no heat escapes from the sides, then the lines of heat flow are perpendicular to the faces and the rate of conduction is the same for all equal cross-sections.
The quantity of heat *Q* conducted through the slab in time *t* is proportional to the time *t*, to the cross-sectional area *A*, to

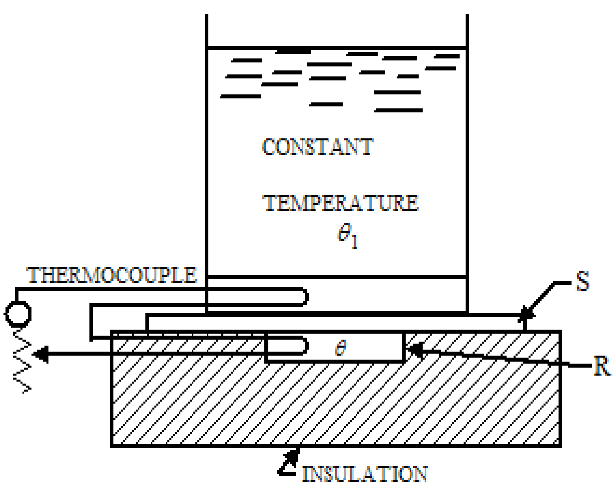

Fig. 1. Diagrammatic sketch of apparatus showing slab S of specimen whose thermal conductivity is to be measured, and copper heat receiver R.

the temperature difference (θ_1 - θ) and inversely proportional to the thickness *l*, so that

$$Q = KA(\theta_1 - \theta)t/l \tag{1}$$

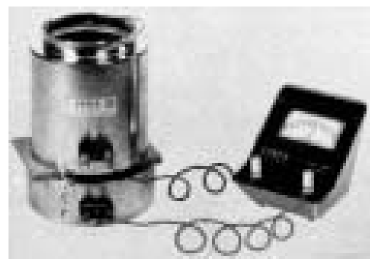
Fig. 2. Thermal Conductivity Apparatus, with constant temperature source above the slab and receiver, and with connections to the galvanometer.

where *K*, the constant of proportionality, is called the *thermal conductivity* of the material of the slab. In the cgs system of units *Q* is measured in calories, *A* in square centimeters, (θ_1 - θ) in centigrade degrees, *t* in seconds and *l* in centimeters. In this experiment, see Fig. 1, a thin slab *S* of material of low conductivity is placed between a hot source at constant temperature θ_1 and a heat receiver *R* consisting of a thermally insulated block of copper of known mass whose initial temperature θ slowly changes. Suppose that in a small interval of time *dt*, the temperature of the copper block changes by an amount $d\theta$ so that $d\theta/dt$ is its rate of increase of temperature. The heat received by the copper block per unit time *Q/t* is

$$Q/t = Mc\,d\theta/dt \tag{2}$$

where *M* is the mass and *c* is the specific heat of the copper block.
Let us first assume that no heat escapes from the copper block. Then the heat conducted through the slab, given by Eq. (1), is equal to the heat received by the copper block, Eq. (2), in a given time interval. Thus

$$KA(\theta_1 - \theta)/l = Mc\,d\theta/dt \tag{3}$$

The temperature difference between the source and the receiver is measured by a copper-constantan thermocouple and a microammeter or galvanometer, Fig. 2. If the temperature difference is small the current *i* produced by the thermocouple is proportional to the temperature difference, so that

$$i = C(\theta_1 - \theta) \tag{4}$$

where C is a constant. From Eq. (4) it follows that

$$di/dt = -C\,d\theta/dt \tag{5}$$

since θ_1 is constant. By substitution of Eqs. (4) and (5) in Eq. (3) it follows that

$$dt = -\frac{lMc}{KA}\frac{di}{i} \tag{6}$$

Integration of Eq. (6) gives

$$t = -\frac{lMc}{KA}\ln i + K_1 \tag{7}$$

where *ln i* is the natural logarithm or the logarithm of *i* to base *e* and K_1 is the constant of integration. If the initial conditions are $i = i_o$ at time $t = 0$ then Eq. (7) gives

$$K_1 = \frac{lMc}{KA}\ln i_o \tag{8}$$

and Eq. (7) becomes

$$t = -\frac{lMc}{KA}(\ln i - \ln i_o) \tag{9}$$

If logarithms to base 10 are used then, since *ln* 10 = 2.303 approximately, it follows that Eq. (9) can be written as

$$t = -2.303\frac{lMc}{KA}(\log i - \log i_o) \tag{10}$$

where it is understood that "*log*" without subscript signifies that base 10 is used.
From Eq. (10) it follows that the graph of *log i* plotted against t should be a straight line. Such a graph may be plotted on ordinary linear graph paper using the logarithms of *i* corresponding to different values of *t* Fig. 3, or on semi-log graph paper using the values of *i* and *t*, Fig. 4. The slope m_1 of the graph of Fig. 3 is

$$m_1 = \frac{\log i}{t} = -\frac{KA}{2.303 lMc} \tag{11}$$

From the experimental data it is seen that the graph of *log i* plotted against *t* is approximately a straight line for a time interval of about 10 minutes, Figs. 3 and 4, but for longer times the current *i* or *log i* deviates from a straight line and becomes constant as shown in Fig. 5a. This indicates that the assumption made in deriving Eq. (10), namely that the copper receiver does not lose any heat, is not valid. The copper block does lose heat despite its being surrounded by very low conducting material. Thus the copper block will reach a constant temperature when the rate at which it receives heat from the hot source is equal to the rate at which it loses heat to the outside air.

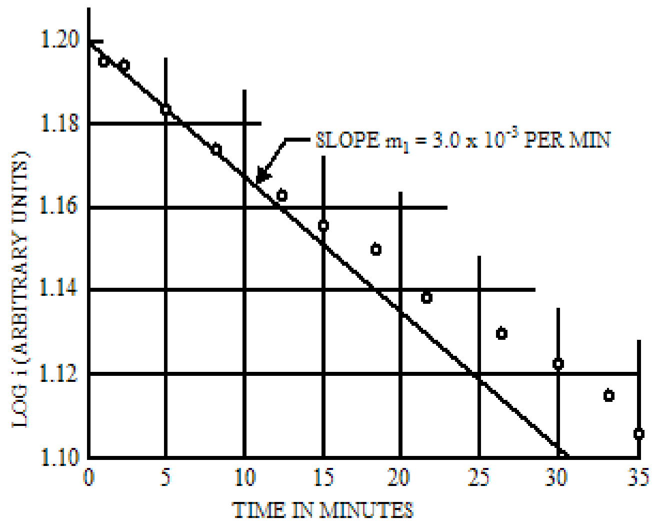

Fig. 3. Graph of logarithm of current i against time.

The rate of increase of temperature θ with time *t*, when the copper block is both receiving and losing heat, can be written as

$$d\theta/dt = m_1(\theta_1 - \theta) - m_2(\theta - \theta_a) \tag{12}$$

where θ_a is the temperature of the air, assumed constant; m_l is given by Eq. (11) and m_2 is another constant. Equation (12) may be written as

$$\begin{aligned} d\theta/dt &= m_1(\theta_1 - \theta) + m_2(\theta_1 - \theta) - m_2(\theta_1 - \theta_a) \\ &= (m_1 + m_2)(\theta_1 - \theta) - m_2(\theta_1 - \theta_a) \end{aligned} \tag{13}$$

Substituting Eqs. (4) and (5) in Eq. (13) gives

$$-d\theta/dt = (m_1 + m_2)i - m_2 i_o \tag{14}$$

where $i_o = C(\theta_1 - \theta_a)$ and is the same i_o as used in Eq. (8) for at time zero, the temperature of the copper block is equal to the air temperature θ_a. For convenience of integration Eq. (14) can be written as

$$\frac{di}{\left(i - \dfrac{m_2 i_o}{m_1 + m_2}\right)} = -(m_1 + m_2)dt \tag{15}$$

Integration of this equation gives

$$\ln\left(i - \frac{m_2 i_o}{m_1 + m_2}\right) = -(m_1 + m_2)t + K_2 \tag{16}$$

where K_2 is the constant of integration and is obtained from the same initial condition used for K_1, namely $i = i_o$ when $t = 0$.

Hence

$$K_2 = \ln\left(i_o - \frac{m_2 i_o}{m_1 + m_2} \right) = \ln\left(\frac{m_1 i_o}{m_1 + m_2} \right) \qquad (17)$$

and

$$\ln\left(i - \frac{m_2 i_o}{m_1 + m_2} \right) - \ln\left(\frac{m_1 i_o}{m_1 + m_2} \right) = -(m_1 + m_2)t \qquad (18)$$

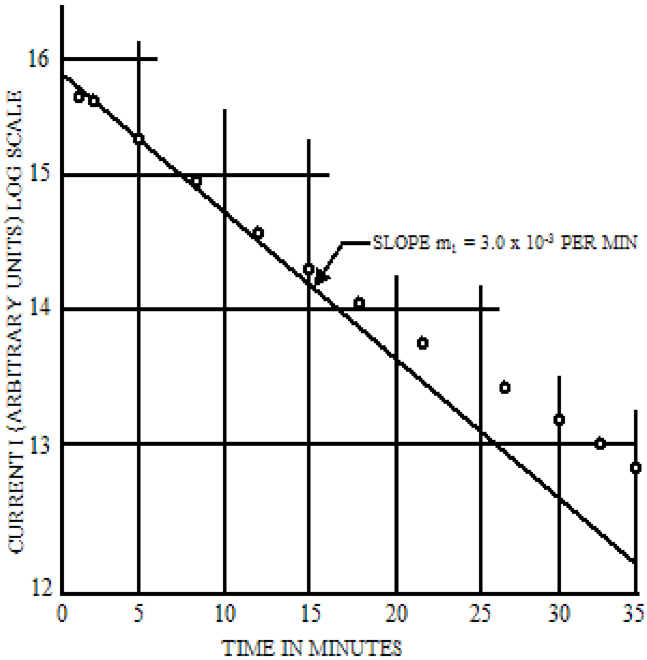

Fig. 4. Graph of current i against time on semilog paper.

Equation (18) can be written as

$$i - \frac{m_2 i_o}{m_1 + m_2} = \frac{m_1 i_o}{m_1 + m_2} e^{-(m_1 + m_2)t} \qquad (19)$$

As the time t becomes very large the negative exponential approaches zero and the current attains the steady value i_s given by

$$i_s = m_2 i_o / (m_1 + m_2) \qquad (20)$$

This is the steady value attained when the copper block is losing heat to the air at the same rate that it is receiving heat from the constant temperature source. Substituting Eq. (20) in Eq. (18) gives

$$\ln(i - i_s) - \ln[m_1 i_o / (m_1 + m_2)] = -(m_1 + m_2)t \qquad (21)$$

or

$$2.303 \log(i - i_s) - 2.303 \log[m_1 i_o / (m_1 + m_2)] = -(m_1 + m_2)t \qquad (22)$$

If *log* $(i - i_s)$ is plotted against t and the assumed conditions hold then a straight line should result. Such a straight line is shown in Fig. 5b, whereas Fig. 5a shows *log i* plotted against t. The slope of the straight line shown in Fig. 5b has the value of $(m_1 + m_2)$. From Eq. (20) it follows that

$$m_1 = (1 - i_s / i_o)(m_1 + m_2) \qquad (23)$$

Thus if $(m_1 + m_2)$ is measured then m_1 can be obtained from Eq. (23) and the values of is and i_o which can be obtained from Fig. 5a. The value of m_1 obtained from Eq. (23) should be more accurate than that obtained from Eq. (10) since the latter is only approximately correct. Having obtained the value of m_1 the thermal conductivity K can be calculated from Eq. (11) using other known or measured quantities.

APPARATUS: The apparatus essentially consists of two parts, the "source" or vessel which holds the liquid at constant temperature, and the "receiver" or the receptacle containing a heat-insulated copper plug. The source is essentially a copper vessel, heat insulated on the sides, with an extra-heavy base which is carefully ground and nickel-plated. One junction of a copper-constantan thermal junction is embedded in the heavy copper base of this source and leads are brought to the binding posts on the sides of the vessel (Fig. 2). The constantan terminal is provided with a constantan lock-washer (gray) while the copper terminal is provided with a copper lock-washer.
The receiver consists of a copper plug, face-ground and nickel-plated, carefully secured in a heat-insulated vessel. A second copper-constantan thermal junction is embedded in the copper plug and terminals brought out to binding posts in a manner similar to those used on the source. The mass M of the copper plug is stamped on the apparatus. A piece of constantan wire is provided for joining the constantan junctions on the source and receiver. A galvanometer having a linear scale is also necessary. Figure 2 shows the apparatus and galvanometer. Depending on the type of galvanometer used, series and shunt resistors may be necessary to keep the needle on the scale. A thin piece (less than one centimeter thick) of the material whose thermal conductivity is to be measured is required. The specimen should be uniform in thickness and may be glass, cork, blotting paper, wall board, etc. A micrometer caliper is needed for determining the thickness of the specimen and also a pair of calipers for determining the area A of the copper plug in the receiver. An immersion heater keeps the water boiling and thus establishes a constant source temperature. A large mass (about 5kg) is used on top of the source to keep the specimen in intimate contact with the source and receiver.

PROCEDURE:

Experimental: Connect the constantan wire to the constantan junction binding posts on the source and the receiver. Connect the copper binding posts with pieces of copper wire to the galvanometer terminals. The material to be tested is placed beneath the source vessel, but not on the receiver. The source vessel is filled with hot water and the immersion heater placed in it to maintain the water at a constant temperature. The receiver should be at approximately room temperature. If the galvanometer is too sensitive introduce a suitable series resistance to bring the deflection onto the scale. When the galvanometer deflection is steady, the test sample and source are placed on the receiver. A heavy mass (about 5kg) should be placed on the top of the source vessel so as to prevent any air from forming between the sample and the source or receiver. The galvanometer deflection is then taken at regular intervals of two or three minutes for about 15 minutes and then every 5 or 10 minutes for a total of about 2 hours. This long value is needed to obtain is with reasonable accuracy. Measure the thickness of the sample by means of the micrometer caliper, applying about the same pressure to the sample as used in the experiment. Measure the diameter of the copper plug on top of the receiver.

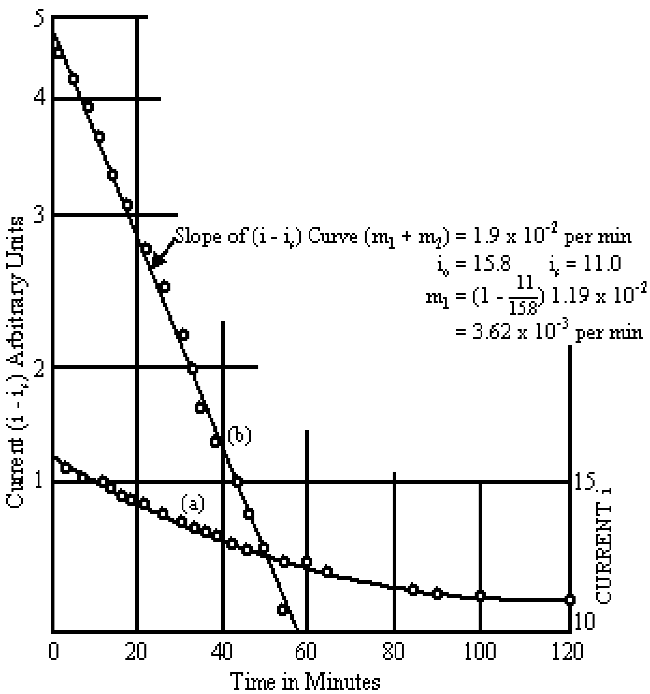

Fig. 5 a, b. a. Graph of current i against time, b. Graph of (i - i$_s$) against time on semilog paper.

Analysis and Calculations. On semilog paper plot the graph of *i* against *t*, Fig. 5a, and from this obtain the steady value i_s. Use this value to construct a table of (i - i_s) values and plot this against ton semilog paper as shown in Fig. 5b. Obtain the slope (m_1 + m_2). Notice that in these graphs the values of *i* may be multiplied by any constant factor so that the *i* values fit on the graph paper, for this does not alter the slope of the line.

The slope of the straight line on semilog paper can be obtained by choosing two values on the line and reading off the *i* and *t* values, then finding the *log i* or *ln i* values from a table of logarithms. The slope is then ($\log i_2 - \log i_1$)/($t_2 - t_1$). Since distances on semilog scales are proportional to the logarithms of the numbers it follows that the slope of the straight line can be obtained by measurement. Measure the distance in centimeters between i_2 and i_1 corresponding to the times t_2 and t_1. The ratio of this distance to the length of one cycle measured in centimeters gives the ($\log i_2$ - $\log i_1$) and this divided by the time interval t_2 - t_1 gives the slope m_1. Notice that the time must be given in seconds in calculating the thermal conductivity. From the value of (m_1 + m_2) obtained from the graph of *log* (i - i_s) calculate the value of m_1, using the necessary constants and Eq. (24).

Use the value of m_1, the mass *M*, area *A* and specific heat *c* of the copper receiver, and the measured thickness of the slab of material used, calculate the thermal conductivity *K*.

Optional Calculation: Plot the graph of log *i* against *t* on semilog paper for the first twenty minutes in which readings were taken. Find the slope m_1 of the best straight line and calculate the thermal conductivity *K* from this value.

QUESTIONS: 1. Would this type of apparatus be suitable for measuring the thermal conductivity of a good conductor such as copper? Explain.

2. Would an air film between the sample and the source or receiver appreciably affect the value obtained for the thermal conductivity of the sample? Would the value obtained for the thermal conductivity with an air film present be higher or lower than the true value for the material of the sample?

3. Could the thermal conductivity of a sample be determined by filling the source vessel with ice and water in place of boiling water?

4. Is it necessary that the two constantan junctions be connected together with constantan wire and constantan washers? Explain.

5. State the physical quantities which determine the value of m_2 used in Eq. (12).

6. Convert your value of *K* to *mks* units.