THERMAL CONDUCTIVITY

OBJECT: To determine the thermal conductivity of a thin piece of poorly conducting material.

METHOD: A thin piece of the material whose thermal conductivity is to be measured is placed between an upper vessel kept at constant temperature and a lower insulated block of copper of known thermal capacity. The heat conducted through the material raises the temperature of the copper block by a measured amount. Thermocouples and a galvanometer are used to indicate temperature differences. From the rate at which heat is conducted through the material, and the area, thickness and temperature difference of the faces of the specimen of material, its thermal conductivity is calculated.

THEORY: Consider a uniform slab of material of thickness l and cross-sectional area A whose faces are kept at constant temperatures T_1 and T respectively, where T_1 is greater than T, Fig. 1. Heat is conducted through the slab from the face at the higher temperature to the face at the lower temperature. If the slab is thermally insulated so that no heat escapes from the sides, then the lines of heat flow are perpendicular to the faces and the amount of heat conducted across any cross section of the slab is constant.

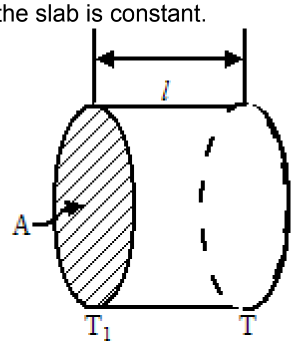

Fig. 1. Transmission of heat through slab of substance having faces at temperatures T_1 and T respectively.

The rate of flow of heat R through the bar- that is, the quantity of heat flowing through the bar per unit time-is (a) directly proportional to the temperature difference (T_1 - T), (b) directly proportional to the cross-sectional area A, (c) inversely proportional to the thickness l, or

$$R = KA(T_1 - T)/l \qquad (1)$$

where K, the factor of proportionality, is called the thermal conductivity of the material of the slab. The thermal conductivity may be defined as numerically equal to the quantity of heat which is conducted per unit time through unit area of a slab of unit thickness having unit temperature difference between its faces. The c.g.s. system of units is usually used for measuring K, so that R is measured in calories per second, A in square centimeters, l in centimeters and (T_1 - T) in degrees centigrade.

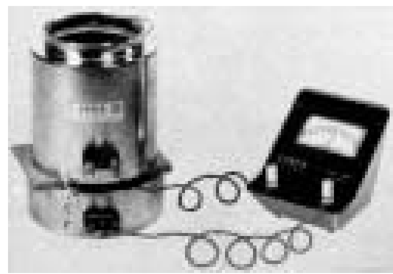

Fig. 2. Thermal Conductivity Apparatus, with constant temperature source above the slab and receiver, and with connections to the galvanometer.

In this particular experiment a thin slab of material of poor conductivity is placed between a constant temperature source and a receiver consisting of a thermally insulated cylindrical block of copper of known thermal capacity (Fig. 2). Thus the upper face of the slab is at a constant temperature T_1, while the temperature T of the lower face is slowly changing. Suppose that in a small interval of time dt the temperature of the copper block and the lower face change by a small amount dT where T is the instantaneous temperature of the copper block. Then dT/dt is the rate of increase in temperature of the copper block. The number of calories per second R received by the copper block is

$$R = McdT/dt \qquad (2)$$

where M is the mass and c the specific heat of the copper block. If no heat escapes from the copper block, the amount of heat conducted through the specimen per unit time is equal to the number of calories received by the copper block per second or

$$KA(T_1 - T)/l = McdT/dt \qquad (3)$$

where K is the thermal conductivity, l the thickness of the specimen whose upper face has a constant temperature T_1 and whose lower face a temperature T, and where A is the surface area of the copper block.

The temperature differences in this experiment are measured by two copper-constantan thermocouples placed respectively in the source and receiver as shown in Figs. 2 and 3. When the temperature difference is small the current produced by such thermocouples is proportional to the temperature difference.

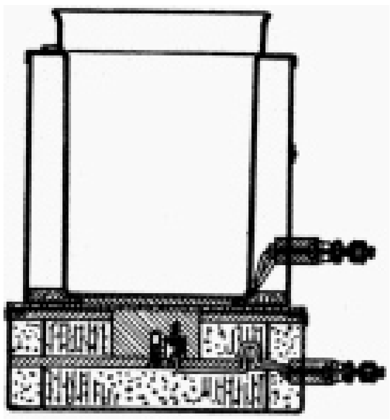

Fig. 3. Diagrammatic sketch of Thermal Conductivity Apparatus showing position of thermocouples.

produced by such thermocouples is proportional to the temperature difference. Thus the current *i* produced by the thermocouple having junctions at the temperature T_1 and T is given by

$$i = C(T_1 - T) \tag{4}$$

where C is the constant of proportionality or the current produced when the junctions are at unit temperature difference. In practice the current is measured by a galvanometer which has a linear scale so that the current is proportional to the deflection of the galvanometer. When such a galvanometer is used, deflections may be substituted for currents in Eq. (4). Now, if T_1 remains constant and T changes, any change in T produces a corresponding change in the current *i*. Thus, if the instantaneous rate of change of temperature of the copper block is dT/dt, the instantaneous rate of change of the thermoelectric current is given by differentiation of Eq. (4)

$$\frac{di}{dt} = -C\frac{dT}{dt} \tag{5}$$

By substitution of Eqs. (4) and (5) in Eq. (3) it follows that

$$\frac{KAi}{l} = -Mc\frac{di}{dt} \tag{6}$$

or

$$dt = -\frac{lMc}{KA}\frac{di}{i} \tag{7}$$

By integration

$$i = -\frac{lMc}{KA}\log_e i + K' \tag{8}$$

where e is the base of natural logarithms and K' the constant of integration. To evaluate K', substitute the initial conditions $i = i_o$ at time $t = 0$ whence

$$K' = -\frac{lMc}{KA}\log_e i_o \tag{9}$$

Thus the Eq. (8) becomes

$$t = -\frac{lMc}{KA}\left(\log_e i - \log_e i_o\right) \tag{10}$$

If the logarithms to base 10 are used, * in order to simplify the analysis of the experimental data, Eq. (10) may be reduced to

$$t = -2.303\frac{lMc}{KA}\left(\log i - \log i_o\right) \tag{11}$$

* The symbol for logarithms to base 10 is “log” without any subscript.

From Eq. (11) it follows that the graph of t plotted against log i should be a straight line since all the other quantities in the equation, including log i_o, are constant. Such a graph of experimental data is shown in Fig. 4a. The plotting of the graph may be considerably simplified by using semi-logarithmic graph paper, Fig. 4b. In such a case the values of i are plotted directly without the necessity of finding their logarithms. The slope m of the graph of t plotted against log i is given by Eq. (11) as

$$m = -2.303lMc/KA \tag{12}$$

APPARATUS: The apparatus essentially consists of two parts, the "source" or vessel which holds the liquid at constant temperature, and the "receiver" or the receptacle containing a heat-insulated copper plug. The source is essentially a copper vessel, heat-insulated on the sides, with an extra-heavy base which is carefully ground and nickel-plated. One junction of a copper-constantan thermal junction is embedded in the heavy copper base of this source and leads are brought to the binding posts on the sides of the vessel (Fig. 3). The constantan terminal is provided with a constantan lock-washer (gray) while the copper terminal is provided with a copper lock-washer.

The receiver consists of a copper plug, face-ground and nickel-plated, carefully secured in a heat-insulated vessel. A second copper-constantan thermal junction is embedded in the copper plug and terminals brought out to binding posts in a manner similar to those used on the source. The mass M of the copper plug is stamped on the apparatus. A piece of constantan wire is provided for joining the constantan junctions on the source and receiver. A galvanometer having a linear scale is also necessary. Fig. 2 shows the apparatus and galvanometer. Depending on the type of galvanometer used, series and shunt resistances may be necessary to keep the needle on the scale. A thin piece (less than one centimeter thick) of the material whose thermal conductivity is to be measured is required. The specimen should be uniform in thickness and may be glass, cork, blotting paper, wall board, etc. A pair of micrometer calipers is needed for determining the thickness of the specimen and also a pair of calipers for determining the area A of the copper plug in the receiver. In order to keep the temperature of the water in the source constant, an immersion heater (Fig. 5) is recommended. A large weight (about 5kg) is used on top of the source to keep the specimen in intimate contact with the source and receiver.

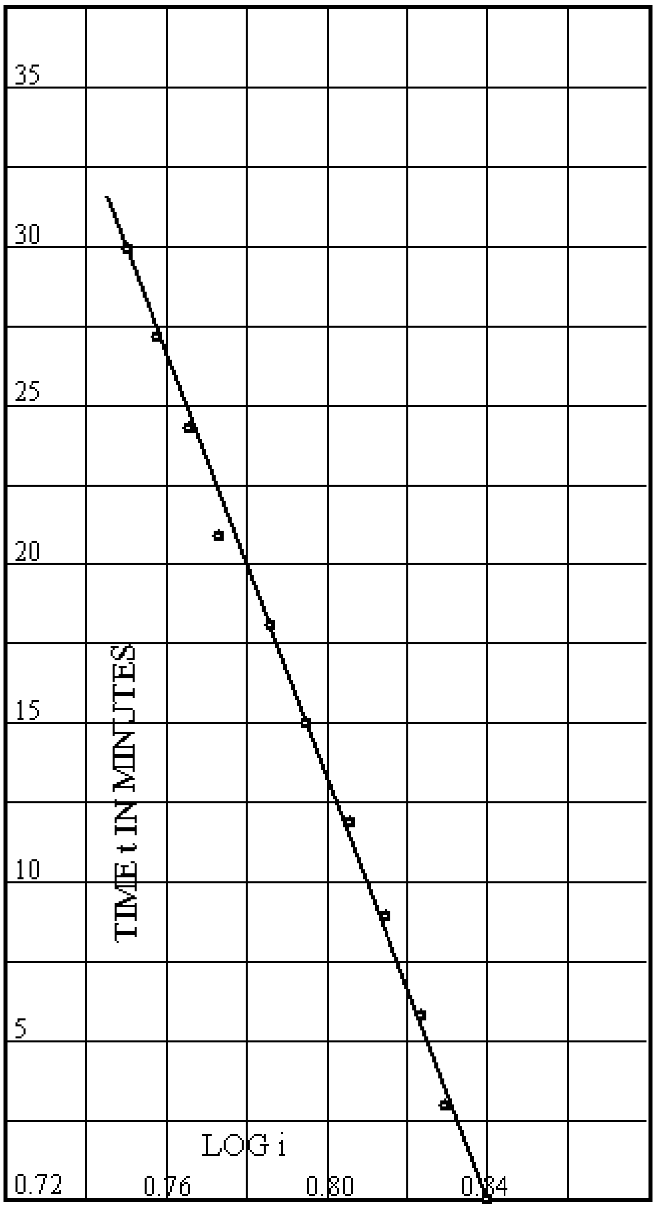

Fig. 4a. Graph of logarithm of galvanometer deflection or current plotted against time.

PROCEDURE:

Experimental: Connect the constantan wire to the constantan junction binding posts on the source and receives. Connect the copper binding posts with pieces of copper wire to the galvanometer terminals. The material to be tested is placed beneath the source vessel, but not on the receiver. The source vessel is filled with hot water and the immersion heater placed in it to maintain the water at a constant temperature. The receiver should be at

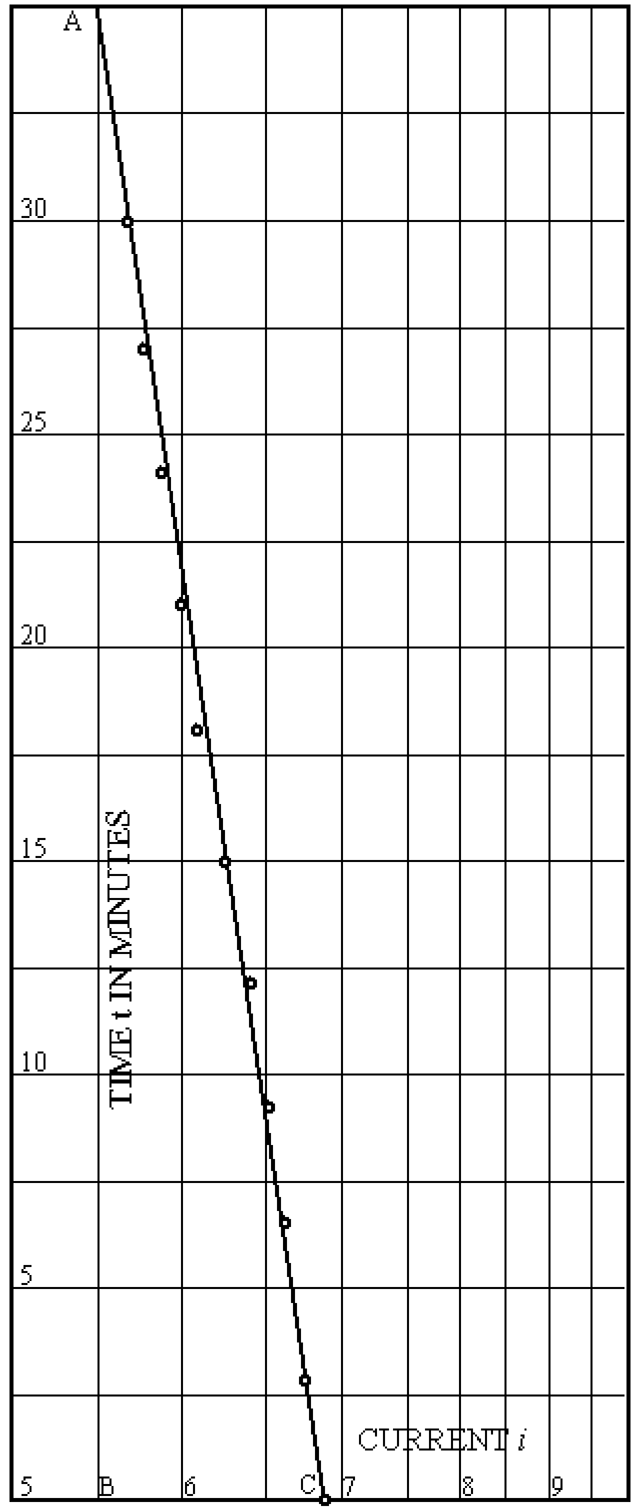

Fig. 4b. Graph of galvanometer deflection or current plotted against time on semi-logarithmic graph paper.

approximately room temperature. If the galvanometer is too sensitive, introduce a suitable series or shunt resistance to bring the deflection onto the scale. When the galvanometer deflection is steady, the test sample and source are placed on the receiver. A heavy weight (about 5kg) should be placed on top of the source vessel so as to prevent any air film from forming between the sample and the source or receiver. The galvanometer deflection is then taken at regular intervals of one, two or each three minutes according to the rate of change of the deflection. A set of about ten readings should be taken. Measure the thickness of the sample by means of the micrometer, applying about the same pressure to the sample as was used in the experiment. Measure the diameter of the copper plug on the top of the receiver.

Fig. 5. Immersion Heater

Analysis and Calculations: Plot a graph of the data using the time *t* as ordinate and log *i* as abscissa. Determine the slope of the graph. If semi-logarithmic graph paper is used, the slope is given by the change in the ordinate corresponding to a change of one cycle in the abscissa which is in logarithmic units. This method of finding the slope when using semi-logarithmic plots is discussed in the instructions on the plotting of curves given on page 2. The slope of the graph shown in Fig. 4b is AB/BC times the length in centimeters of one cycle of the logarithmic scale where AB is the change in the time equal to 35 x 60 seconds corresponding to a change BC in the abscissa measured in centimeters.

Using the value of the slope from the graph, the mass *M* and the area *A* of the copper plug, the specific heat *c* of copper and the thickness *l* of the sample, calculate the thermal conductivity *K* of the sample using Eq. (12).

QUESTIONS: 1. If the copper plug in the receiver is not sufficiently well heat-insulated, would the graph of *t* plotted against log *i* be a straight line?

2. Would this type of apparatus be suitable for measuring the thermal conductivity of a good conductor such as copper? Explain.

3. Are the statements numbered (a), (b) and (c) of the Theory Section (1) pure assumptions or (2) justified by experiment? Explain.

4. Would an air film between the sample and the source or receiver appreciably affect the value obtained for the thermal conductivity of the sample? Would the value obtained for the thermal conductivity with an air film present be higher or lower than the true value for the material of the sample?

5. Could the thermal conductivity of a sample be determined by filling the source vessel with ice and water in place of boiling water?

6. Is it necessary that the two constantan junctions be connected together with constantan wire and constantan washers? Explain.