CENCO Physics — Selective Experiments In Physics

• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

BOYLE'S LAW

OBJECT: To study Boyle's law, at moderate pressures above and below atmospheric, by both analytical and graphical methods.

METHOD: A fixed mass of air confined in a glass tube is kept at room temperature and subjected to various pressures, ranging from half to double atmospheric pressure. A series of corresponding pressures and volumes are observed and Boyle's law is checked by noting the constancy of their products. The data are plotted in several graphical forms the interpretation of which also indicates the validity of Boyle's law.

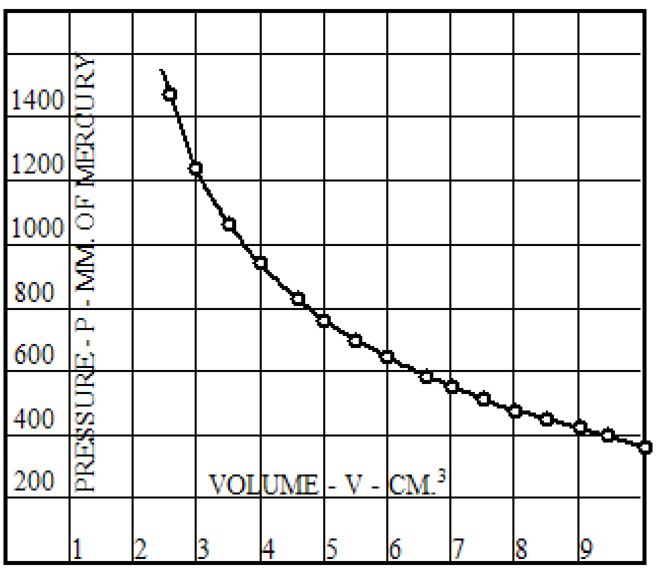

Fig. 1. Curve showing the variation of the volume of a gas at constant temperature as a function of the pressure.

THEORY: The relation existing between the pressure exerted by a confined gas and its volume is given by what is usually known as Boyle's law, namely: *The temperature remaining constant, the volume V occupied by a given mass of gas is inversely proportional to the pressure P to which it is subjected*. In symbols

$$V \propto \frac{1}{P}$$

or

$$V = k \times \frac{1}{P}$$

whence

$$PV = k \qquad (1)$$

where k is (numerically) a constant under given conditions.
It is apparent that Eq. (1) is an equation of the second degree, since the left side is the product of two variable quantities. When the pressure is plotted as a function of the volume, an equilateral hyperbola, as shown in Fig. 1, is obtained.
The actual pressure P may be thought of as consisting of the *atmospheric or barometric pressure B plus an added pressure p*, the algebraic sign of the added pressure depending upon whether the actual pressure is above or below atmospheric pressure. Eq. (1) may therefore be written

$$(B + p)V = k \qquad (2)$$

or

$$B + p = k \times \frac{1}{V} \qquad (2a)$$

By placing $1/V = x$, Eq. (2) becomes

$$B + p = kx \qquad (3)$$

or

$$p = kx - B \qquad (4)$$

Since B is numerically constant (for any given case), it will be seen that Eq. (4), being of the first degree, is the equation of a straight line. If, then, not the *actual* pressure but merely the *added* pressure p may be plotted as ordinates and the reciprocal of volume, $1/V$ or x, be plotted as abscissas, the resultant curve should be a straight line (Fig. 2). If the curve is produced downward until it intersects the pressure axis, i.e., when x or $1/V$ equals zero, the intercept on the p-axis gives immediately the negative of the barometric pressure at the time of the experiment. This is obvious from Eq. (4), for if x is placed equal to 0, then

$$p = -B \qquad (4a)$$

While the present experiment is designed ostensibly to furnish a check upon the approximate validity of Boyle's law, it also furnishes a splendid example of a study of both graphical and analytical representation and interpretation of experimental data. In addition it offers an excellent illustration of the processes used in logical scientific reasoning. When this experiment is performed with this point of view in mind and the differences in the apparatus, procedure and precision are considered, it will be seen that the present experiment is far from a mere repetition of one of the commonest experiments of elementary-school physics.

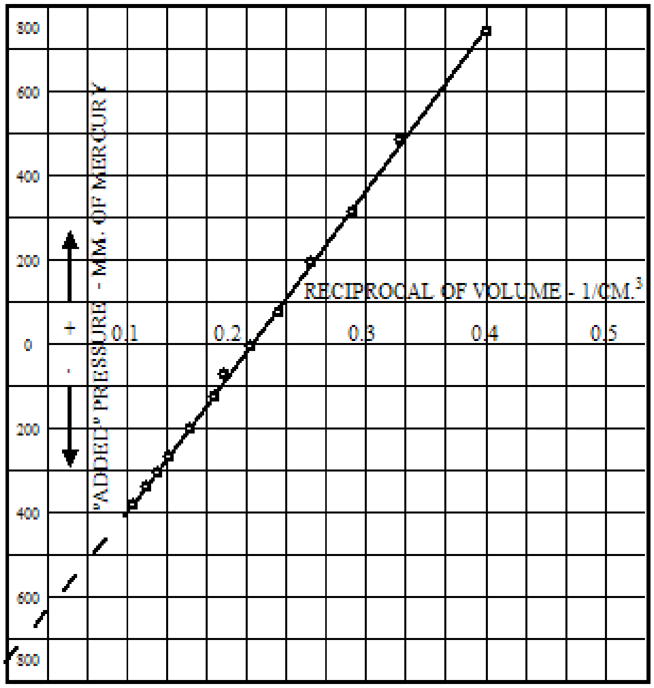

Fig. 2. Curve showing the variations of the reciprocal of the volume of a fixed mass of gas at constant temperature, as a function of the pressure above and below the atmospheric pressure.

APPARATUS:

The Barometer: The pressure of the atmosphere may be most accurately determined by the mercurial barometer illustrated in Figs. 3 and 4. The glass tube G, closed at its upper end, projects into the mercury cistern w. The metric scale S and the British scale S' have their zeros at the lower end of the ivory tip I. Consequently, it is necessary that the upper level of the mercury in the cistern always be brought to this point before a reading is taken. This is done by adjusting a screw at the bottom of the barometer. A vernier reading device V makes possible accurate settings and readings at the top of the mercury column.

The following procedure is used in making a measurement of atmospheric pressure by means of a mercurial barometer:

1. Adjust the mercury level to the tip of the ivory point by turning the well upward with great care until the tip of the ivory point just touches the mercury surface. When the mercury is clean this contact may be judged by noting apparent contact between the tip of the point and its image in the mercury. When the mercury has become contaminated, the mercury level is adjusted upward until the point has made the slightest depression discernible in the mercury surface. Another way of making the setting is to hold apiece of white paper behind the well as a background, keeping the eye at the level of the ivory point and adjusting the mercury surface until the light between the point and the mercury is just cut off.

2. Move the vernier up until the top of the mercury column appears below its lower edge. Tap the barometer lightly to permit the mercury to form a free meniscus. Then move the vernier downward until the sighting edges A and A' are in line with the uppermost point of the meniscus.

3. Read the scale in millimeters and determine tenths with the aid of the vernier.

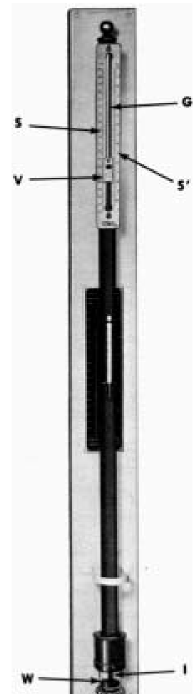

Fig. 3. Mercury Barometer

The Boyle's Law Apparatus: The apparatus used in studying Boyle's law is shown in Fig. 5. The mass of air on which the measurements are made is confined in the calibrated glass tube T which has a stopcock K at its upper end. The stopcock tube T and another glass tube T' form the opposite ends of what might be designated an adjustable U-tube The glass tubes are connected through suitable metal couplings by means of heavy, flexible tubing and this adjustable U-tube is filled with the proper amount of mercury. By means of the metal couplings C and C' the glass tubes are supported in clamps which can be moved vertically on the support rods G and held in any desired position. Midway between the two support rods is the vertical millimeter scale B, graduated on metal. By means of this scale and a suitable reading device R, the height of mercury column in either of the two glass tubes may be ascertained within 0.1mm.

The reading device consists of a sleeve which slides freely along the graduated square tube and to which is attached the mirror M and the vernier V. A fine horizontal line etched on the mirror permits setting without parallax on the mercury columns, and after the setting has been made the reading is taken by means of the vernier. The reading device is held in any desired position on the graduated tube by means of spring friction. The clamp which holds the open tube is provided with a micrometer adjusting screw S by means of which a setting of the mercury level in either tube may be made with precision. The entire apparatus is supported on a massive tripod base A provided with leveling screws.

When the tubes are turned in toward the mirror, great care must be observed not to strike them against the top of the support.

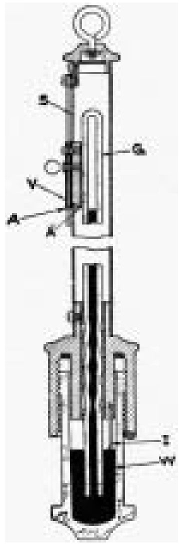

Fig. 4. Detail of Mercury Barometer

PROCEDURE:

Experimental: Place the apparatus in good light where the scale may easily be read. With the stopcock open, adjust the levels of mercury until a volume of air is enclosed which occupies about half the total volume of the closed tube. Level the apparatus by adjusting the leveling screws in the base until the mercury levels in the two glass tubes coincide with the horizontal line on the mirror. Then close the stopcock tightly and keep it closed throughout the progress of the experiment. (While adjusting the stopcock, steady the top of the closed tube with the hand – otherwise the glass tube may be snapped off. Fasten the stopcock with a rubber band. Keep it well lubricated with stopcock grease.)

Test the system by lowering the open tube or raising the closed tube to decrease the pressure as far as the apparatus will conveniently permit. Allow it to remain in this condition for a few minutes and note whether there is any change in the mercury levels. The barometer may be read during this interval.

Take a series of ten or twelve readings of volumes at various pressures ranging from the lowest to the highest attainable. The volumes are read directly on the graduated glass scale of the closed tube. The pressure is determined by reading the height of the mercury columns on the open and closed tubes and subtracting these values to obtain the “added” pressure p. The actual pressure P is $B + p$ when the level in the open tube is above that in the closed tube.

In tabulating the date the following should be recorded: (a) the reading of the mercury level in the open tube; (b) the reading of the level in the closed tube; (c) the “added” pressure p; (d) the actual pressure $B + p$; (e) the volume V; (f) the product PV; (g) percentage difference between the observed PV and the means of all the values; (h) $1/V$.

In varying the pressure it is convenient to set the mirror index on the closed tube at 1/2cc reduction in volume each time and to raise the open tube until the pressure is properly adjusted to give the desired volume. Final adjustment of the levels is made by the use of the slow-motion screw on the open tube. When the open tube is near the top of the frame, the closed tube may be lowered to produce the same effect as further raising of the open tube. Make all changes slowly, to avoid changing the temperature, and wait a few minutes before taking the readings. Do not handle the closed tube after the preliminary adjustments are completed. (Why?) While changing the positions of the tubes be careful to avoid spilling the mercury out of the open tube.

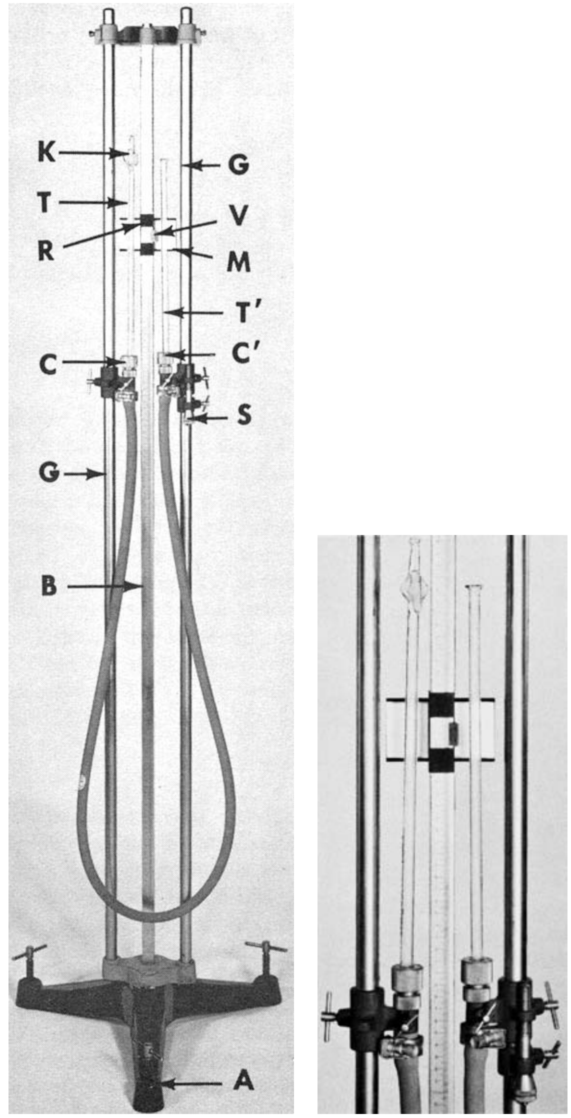

Fig. 5. Boyle’s Law Apparatus. The insert shows a “close-up” of the upper end of the tubes.

Interpretation of Data: Calculate the various PV products and take their average. Determine the percentage variation between the individual values of PV and their mean. What is the physical significance of the constancy of the various values of PV?

Curves: Plot the following curves: (1) P vs. V (begin both axes at zero); (2) "added" pressure p vs. reciprocal of volume $1/V$. Choose the axis of p near the center of the page and be sure that $-p$ extends as far as 770mm below the axis. In laying off the scale for $1/V$, begin with $1/V = 0$ at the origin. Carefully interpret in the report the significance of these curves. From curve (2) determine the barometric pressure by extrapolating the observed portion of the curve (use a dotted line for extrapolated portion) back to the intercept where $1/V = 0$. Compare this pressure-intercept value of *B* with that observed by the barometer.

QUESTIONS: 1. Show by dimensional reasoning that the constant *k* in the equation $PV = k$ is *not* a mere proportionality constant, i.e., one having no unit, but that it has the dimensions of work. On what does the value of *k* depend?

2. Considering the accuracy of the observations of the volume in the present experiment, was it necessary to measure the pressures to tenths of millimeters in order to secure accurate values of *PV*? Would more precise readings of the mercury levels have given more accurate values of *PV*?

3. A certain automobile tire is labeled, “Inflate to 35lb. or 2.5kg." What does this mean? Is the statement clear? Is 35lb. equal to 2.5kg?

4. The moving coil of an ammeter weighs 1/2 gram and is supported in a jewel bearing by a needle-point pivot which is rounded off to have a radius at the tip of 75/100,000 inch. Calculate the pressure, in pounds per square inch, which the pivot exerts on the bearing.

5. The mercury stands at a height of 74cm in a barometric tube. The top of the tube is 6cm above the top of the mercury column and the cross-sectional area of the tube is $0.6 cm^2$. A quantity of air is introduced above the mercury which then falls to 64cm. What was the volume of the air before it was introduced into the tube?

6. The closed end of uniform U-tube containing mercury has an air space 20cm long. The mercury in the open arm stands 5cm lower than in the closed arm. If more mercury is poured into the open arm until its level in this arm is 10cm higher than in the closed arm, how long will the air space be? The barometric height is 75cm of mercury.

7. A barometer having a little air in the top of the tube has the mercury 70cm above that in the cistern. If the level of the mercury in the cistern is raised so that the volume of the air space at the top is half as great as before, the mercury column stands 67cm above the level of that in the reservoir. What is the barometric pressure at the time?