CENCO Physics Selective Experiments In Physics

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YOUNG'S MODULUS OF ELASTICITY
MEASUREMENT OF STRETCH WITH THE OPTICAL LEVER

OBJECT: To determine Young's modulus of elasticity of a wire by stretching.

METHOD: A long vertical wire, securely fastened at its upper end, is stretched by attaching known weights. The extension, or stretch, for a given applied force as well as the cross section and the original length are measured. From these quantities and the applied force, Young's modulus of elasticity is calculated.

THEORY: Over a considerable range it is found by experiment that the deformation of a body produced by forces is proportional to the magnitude of these forces. This proportionality between deformation and applied forces is called Hooke's law and is the basis of the theory of elasticity. The value of the stress for which Hooke's law just ceases to hold is called the elastic limit of the substance. The elastic limit may also be defined as the magnitude of the applied stress which produces the maximum amount of recoverable deformation.

The fractional change in the dimensions of a body produced by a system of forces in equilibrium is called a strain.

Even within the limits of perfect elasticity, different bodies show distinct differences in their behavior. Some recover their form immediately after the removal of the force, while others, though they recover it ultimately, take considerable time to do so. This delay in recovering the original condition of the substance is called the elastic *after-effect*, or elastic lag.

When a wire is stretched beyond its elastic limit and its cross section is reduced, as in drawing through a die, its structure is broken down and the surface appears to be an amorphous layer of flowed material. This flowed layer becomes proportionately thicker with repeated drawings and the density, hardness and elasticity of the material are profoundly changed. For instance, when a wire of Swedish iron has its diameter reduced from 0.75mm to 0.10mm by repeated drawings, the breaking strength measured in force per unit area is doubled. Keeping in mind these changes, the student will not expect an exact check of his results for the modulus of elasticity with accepted values given in tables.

When a wire is stretched, there is not only a change in its length but also a much smaller change in its diameter. Young's modulus of elasticity Y takes into account only the change in length- the longitudinal strain which occurs. This strain is the change in length per unit length.

The longitudinal stress produced by the applied forces is measured in terms of force per unit area. Young's modulus is defined as the ratio of the longitudinal stress to the longitudinal strain.

If L represents the initial length of the wire (Fig. 1), r its average radius, e the stretch produced by the weight of a mass M, then

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{\text{force/area}}{\text{elongation/length}}$$

$$Y = \frac{Mg/\pi r^2}{e/L} = \frac{MgL}{\pi r^2 e} \qquad (1)$$

The Optical Lever: The optical lever furnishes an interesting application of the law of reflection of light. If a beam of light TO (Fig. 2) originating at a point T on the scale S meets the mirror perpendicularly, it is reflected back on its path; but if the mirror is turned through an angle θ (NOT), an incident beam PO originating at a point P on the scale Swill on reflection take the direction OT where the angle of incidence θ (NOP) is equal to the angle of reflection, ON being the normal to the mirror in the position OM'. By the motion of the mirror through the angle θ, the reflected beam has therefore been turned through an angle 2θ. If a telescope is mounted adjacent to the scale at T and if the mirror is properly adjusted, both reflected beams will enter the telescope and the angle 2θ may easily be determined from the distance D and the scale distance R.

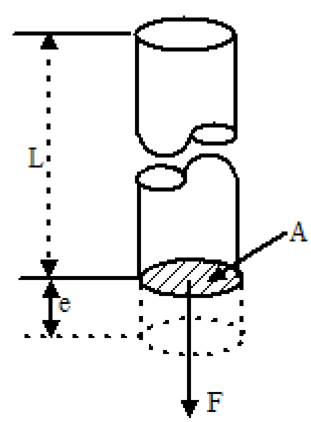

Fig. 1. Stress and strain involved in the stretching of a wire. The stress, which is the force per unit area, is F/A; the strain, which is the elongation per unit length, is e/L.

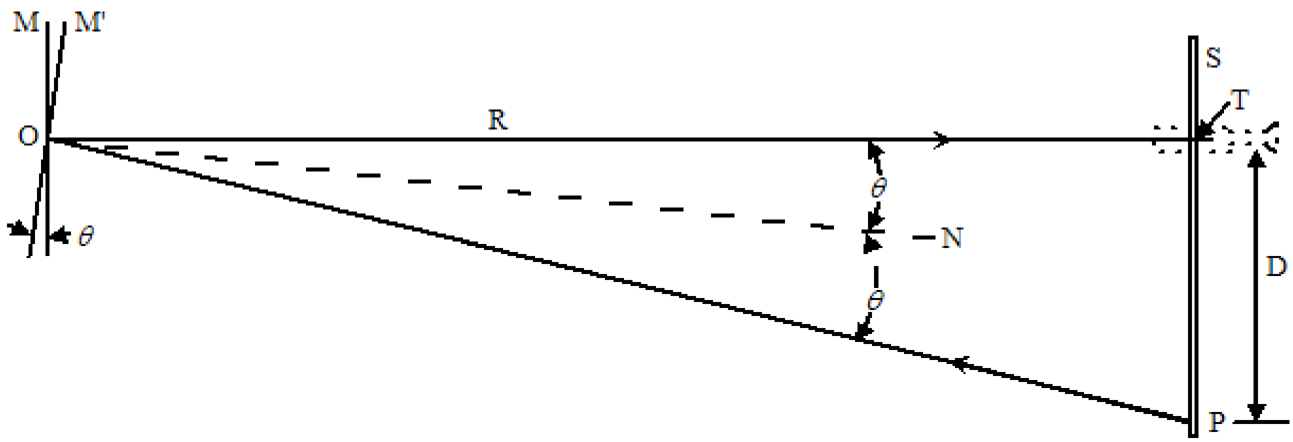

Fig. 2. Rotation of a mirror through an angle θ turns the reflected ray through an angle 2θ.

and the angle 2θ may easily be determined from the distance *D* and the scale distance *R*.

In use the optical lever stands upon three hardened steel points, A B C (Fig. 3). Two of these, A and B, resting in a groove, form the fulcrum of the lever, while the third, C, rests on the chuck H which grips the lower end of the wire W under test. The lever (Fig. 4) carries a mirror which is adjustable about a horizontal axis. From Fig. 3 it is seen that the elongation e is given by the relation

$$e = d\sin\theta \qquad (2)$$

where *d* is the perpendicular distance from the line AB to the point C.

The double angle 2θ may be determined from the distance *R* from the mirror to the scale and the average distance *D* between readings of the telescope produced by adding masses *M* of 1000 grams each to the wire by means of the relation

$$\tan 2\theta = D/R \qquad (3)$$

Since the angle θ is small, 1/2 tan 2θ = tan θ = sinθ. In this case

$$e = \frac{dD}{2R} \qquad (4)$$

APPARATUS: The apparatus consists of a heavy tripod base with leveling screws (Fig. 5). On the base are mounted two nickel- plated steel support rods 140cm long, with a yoke clamped to their upper ends. This yoke carries a chuck for supporting the wire to be tested. A weight hanger supports the load by means of which the tension is applied to the wire. Near its lower end the wire passes through a hole in an adjustable bridge or platform which - is clamped to the support rods. The measuring apparatus consists of an optical lever (Fig. 4) which rests in the groove on the bridge and on the face of the chuck which is tightly clamped to the wire. The chuck rides easily in the hole in the bridge. As weights are applied to load the wire, the extension tilts the optical lever so that a different part of the scale is seen under the cross hair of the telescope.

As auxiliary apparatus ten- slotted kilogram weights, a telescope and scale, a micrometer caliper and a meter stick are needed.

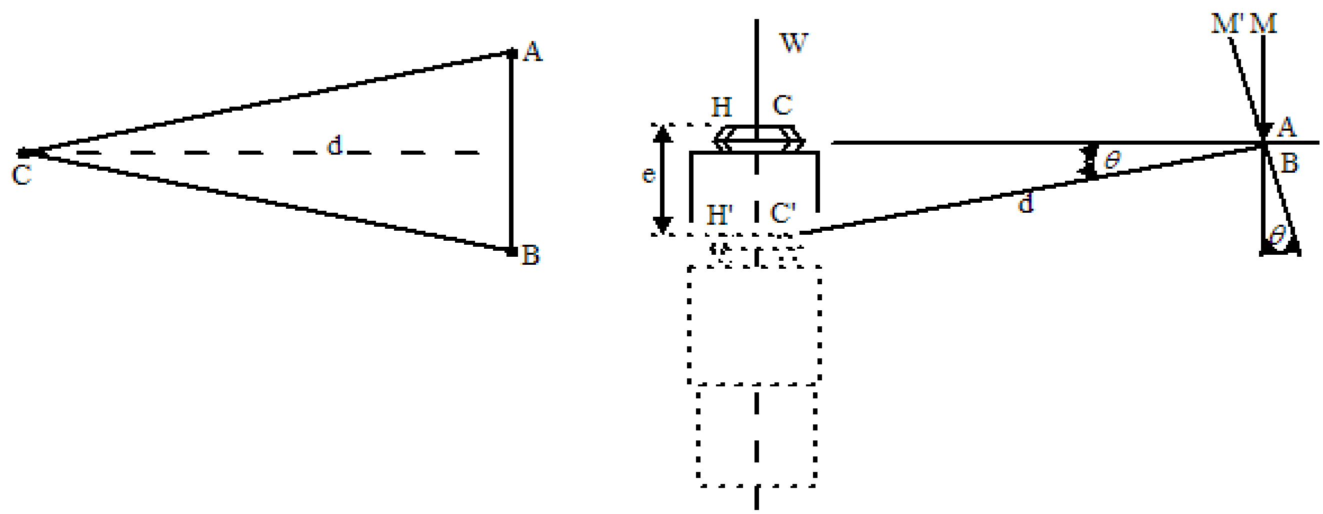

Fig. 3. Schematic diagram of the Optical Lever.

PROCEDURE:

Experimental: In case it is necessary to install a wire in the apparatus, select a sample about two meters long as free from kinks and bends as possible. To ensure a fairly straight wire, clamp one end in a vise and, wrapping the other end about a stick of wood, give the wire one or two sharp jerks. Then place the wire between two flat sticks and, while pressing them tightly together, pull the wire through two or three times, giving the sticks a slight tilt while pulling. After the straightening process, cut off four or five inches from each end of the wire. Pass the wire through the chuck in the adjustable bridge and upward through the chuck in the yoke, fastening it securely. Adjust the bridge so that there is at least a meter of wire between the two chucks, then close the

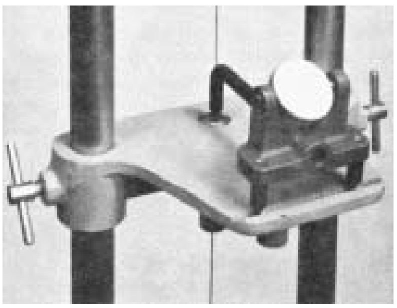

Fig. 4. The Optical Lever as used to means the stretch of a wire.

lower chuck. Pass the bottom end of the wire three times around the hook of the weight hanger, bringing the end up under the turns of wire, and adjust the length so that the bottom of the hanger is about an inch above the base. Then hold the two wires firmly with the pliers about an inch above the top of the hanger and twist the hanger until the wires are tightly wrapped around each other. Cut off the loose end. Level the apparatus so that the lower chuck moves freely in the bridge.

Fig. 5. Apparatus for determining Young's modulus. In this experiment the micrometer screw and level shown mounted on the bridge are replaced by the optical level shown in Fig. 4.

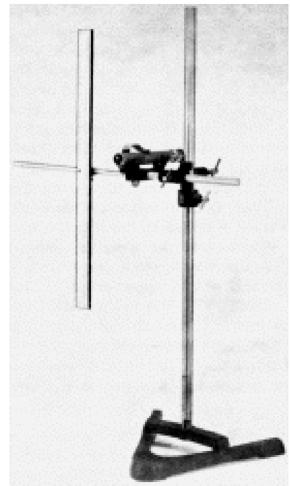

Fig. 6. Telescope and Scale as used with the Optical Lever.

Place the optical lever on the apparatus with the mirror vertical. Set the telescope and scale (Fig. 6) at least a meter away from the mirror and at about the same height. Adjust the eyepiece of the telescope so that the cross hairs are clearly seen as soon as the eye looks into the instrument. The main fact to be kept in mind in endeavoring to locate the image of the scale in the telescope is that it cannot be seen in the instrument unless it is possible to see it *with the unaided eye* by sighting along and over the telescope. To do this, get near the mirror and move the head until the image of the eye is seen in the mirror. Keeping the image of the eye in sight, move away from the mirror at least one meter. Still keeping the image of the eye in sight, move the telescope until it is directly under the line of sight. Then move it slightly until the scale can be seen in the mirror while looking over the telescope along the barrel. Next, without touching the eyepiece, pull out the drawtube carrying the eyepiece nearly to its full extent. Place the fingers of the two hands against the sides of the telescope and, *while looking through it*, slowly push in the drawtube with the thumbs. The first thing to come in focus should be the mirror; further motion should bring the scale in sight. Center the scale in the field of view. Tilt the mirror until the part of the scale seen is that on the same horizontal line as the center of the telescope. To complete the adjustment, note whether there is any relative motion of the scale image with reference to the cross hairs when the head and eye are moved from side to side. If the image appears to move *with the eye*, and the cross hairs move in the opposite direction, then the image is in front of the plane of the cross hairs and it will be necessary to push the tube containing the cross hairs *in* in order to bring them into a common plane. If, however, the image moves in the opposite direction to the eye and the cross hairs appear to move with it, then the drawtube should be pulled *out*.

Carefully place 10kg on the weight hanger, keeping one hand under the weight hanger as each weight is added and allowing the wire to take up each additional load slowly. Read the scale position in line with the horizontal cross hair. Wait a full minute and see whether this reading slowly changes. If it does, remove 1kg and observe again. In this way, find the maximum load within the elastic limit. Remove this load and then carefully replace it twice, reading the scale each time.

Remove all the weights and take a reading with the hanger alone. Then take a series of readings, increasing the load in steps of 1kg up to the maximum. Take a second series of readings as the weights are removed. Measure with the meter stick the length of the wire from the under side of the upper chuck to the upper side of the lower chuck. With the micrometer calipers, make ten determinations of the diameter of the wire, two measurements at right angles to each other at five different points along the wire. Record these measurements in centimeters. Measure the distance *R* from the mirror to the scale.

Interpretation of Data: Average the scale readings for each load and make a graph to a large scale showing the total shift as a function of the load. Draw the straight line which best represents the plotted points, disregarding if necessary the first one or two points. From the graph compute the shift in centimeters per thousand grams. From Eq. (4) calculate the stretch per thousand grams. Using a sheet of ruled paper, place the points A and B of the optical lever on a line and press all points into the paper. Measure in centimeters the distance d from the line AB to the point C.

Calculate Young's modulus of elasticity using the data obtained and Eq. (1).

QUESTIONS: 1. In case the first one or two of the plotted points did not fit the curve, what explanation could be offered?

2. The length of a wire 200cm long can be measured to an accuracy of ±1mm. If the diameter of the wire is 0.50mm, to what accuracy should the diameter be measured in order to produce the same error in the result?

3. Recalculate the value of the modulus, using a value for the length 1mm greater than the one first used. Make a second calculation increasing the value assigned to the diameter of the wire 0.001cm. Which produces the greater change in the result?

4. How long a piece of wire of the same material and cross section as used in this experiment would it take if the stretch were to be exactly 1.4mm with a load of 4kg?