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VISCOSITY OF A LIQUID

OBJECT: To make an experimental determination of the viscosity of a liquid, and to study the variation of viscosity with temperature.

METHOD: The liquid to be studied is contained in the space between two co-axial cylinders, the inner one being so arranged that it can be rotated by the action of a fall-in mass. The angular velocity of the rotary cylinder under the influence of a given torque depends upon the viscosity of the liquid and the geometry of the apparatus. From observations upon the times required for various masses to fall a measured distance, a curve is plotted from the slope of which and the constants of the apparatus the coefficient of viscosity is computed. The method, of course, is applicable only to liquids which adhere to the walls of the cylinders. With some modifications, the co-axial cylinder method can be used to measure the viscosity coefficient of gases.

THEORY: The essential difference between solids and fluids lies in the nature of their response to a shearing stress. Whereas in the former, elastic forces place a limit upon the amount of shear produced by a given shearing stress, in the latter the deformation resulting from a constant shearing stress of any magnitude, however small, increases without limit. In other words, the shear modulus for fluids is zero and they may be said to offer no permanent resistance to shear.
Fluids do, however, differ in their rate of yield under the influence of a shearing stress. Common experience teaches, for example, that some liquids pour more readily than others. The movement of a fluid may be thought of as the slipping of adjacent layers over one another, and the internal friction between contiguous layers is called viscosity. Thus, while a fluid in motion resists a shearing stress with a frictional force which tends to retard the flow, this force disappears when the flow ceases and there exists no elastic force tending to restore the fluid to its original configuration.
In Fig. 1 let the parallelepiped represent a small element of volume in a fluid which is flowing horizontally. The shearing stress on this element of volume is *F/A* where *F* is the horizontal force on the top surface and *A* is the area of the horizontal cross section. For small deformations the shearing strain, or the amount of shear, is equal to the lateral displacement between two surfaces divided by the perpendicular distance between them. Thus, if *v* is the difference in velocity between the upper and lower faces, the amount of shear occurring in unit time is *v/r*. This term is also called the *velocity gradient*, since it is the variation in velocity of parallel layers with distance between them. For streamline motion (no turbulence the ratio between the shearing stress (tangential force per unit area) and the rate of shear (velocity

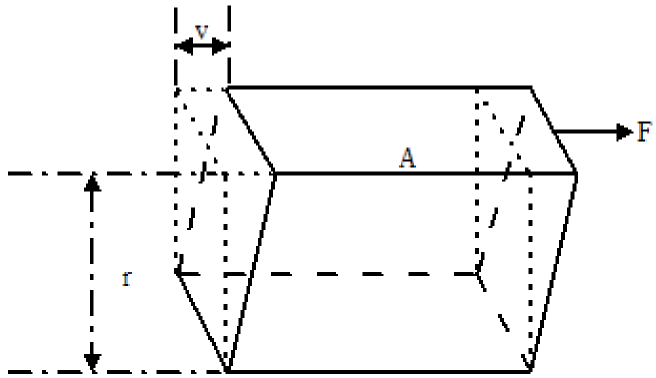

Fig. 1. Small element of volume in a liquid under shearing stress.

gradient) is a constant for a given fluid and is called the *coefficient of viscosity*, or simply the *viscosity*. Designating the coefficient by η,

$$\eta = \frac{\text{shearing stress}}{\text{rate of shear}}$$

$$\eta = \frac{F/A}{v/r} = \frac{F \cdot r}{A \cdot v} \qquad (1)$$

The velocity gradient is expressed more precisely in the calculus notation as the derivative of velocity with respect to distance. Thus

$$\eta = \frac{F/A}{dv/dr} \qquad (1a)$$

Another way of stating the definition is to define the coefficient of viscosity as numerically the tangential force per unit area in a fluid when two parallel surfaces at unit distance apart are slipping over one another with a relative velocity of unity. The c.g.s. unit of viscosity is called the poise; it is the viscosity of a substance that acquires a unit velocity gradient under the influence of a shearing stress of 1 dyne/cm^2.
The foregoing simple treatment is valid only when the volume concerned is of such size that the movement may be considered as taking place in parallel planes. In the co-axial cylinder method of determining the coefficient of viscosity, it is convenient to take as an element of volume a cylindrical section instead of a parallelepiped. The movement then consists of the rotation of concentric cylindrical layers about one another. In Fig. 2 let the dotted line SS' represent an imaginary cylindrical boundary lying within the liquid

enclosed between the two cylinders A and B. For simplicity, consider the inner cylinder to be stationary and the outer one

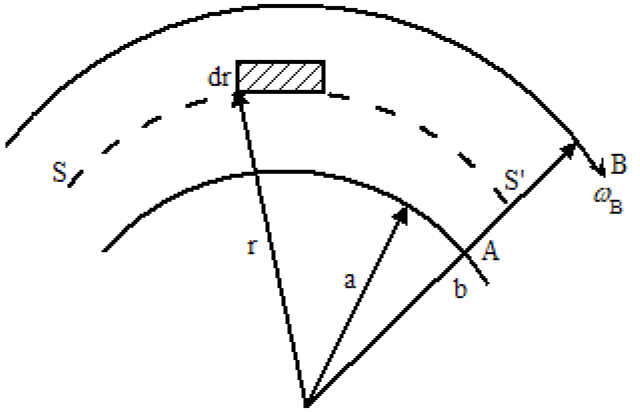

Fig. 2. Shearing stress in a liquid confined between concentric cylinders. (Velocity at A is zero.)

to rotate with an angular velocity ω_B. If the liquid adheres to the walls of the cylinders, a shearing takes place in which concentric cylindrical layers of the liquid slip over each other, the angular velocity increasing progressively from zero at the stationary cylinder to ω_B at the rotating one. The linear velocity of the intermediate surface SS' is $v = \omega r$ where $0 \langle \omega \langle \omega_{\mathbf{B}}$. The velocity gradient at SS' is then

$$\frac{d}{dr}(\omega r) = \omega + r\frac{d\omega}{dr} \tag{2}$$

The first term on the right of the equality sign represents the rate of increase in *v* with *r* when all portions of the substance are moving with the same angular velocity, i.e., when no shearing takes place. When shearing occurs, however, each cylindrical layer has an angular velocity greater than that of the one just inside it; and the second term on the right represents the variation in *v* due to the variation in ω. In a rigid body this term would be zero; in a fluid it represents the velocity gradient due to relative movement of adjacent layers. Thus, while the complete expression for the velocity gradient of a rotating fluid contains two terms, only the second one is concerned with viscosity since it is the only one that involves relative movement of contiguous parts. Substituting this term in Eq. (1a) and separating the variables

$$\eta = \frac{F/A}{r d\omega/dr} \tag{3}$$

or

$$\eta d\omega = (F/A)(dr/r) \tag{4}$$

If the torque applied to the rotating cylinder is *L*, the tangential force sustained by the layer of liquid in contact with the cylinder is *L/b*, and that at any boundary SS' is *L/r*. Since the area of this cylindrical boundary is $2\pi r l$, the tangential force per unit area is $\dfrac{L}{2\pi r^2 l}$ where *l* is the length of cylinder in contact with the fluid. Substitution of this expression in Eq. (4) yields

$$\eta d\omega = \frac{L}{2\pi l} \cdot \frac{dr}{r^3} \tag{5}$$

Integrating between the limits *r* = *a* and *r* = *b* gives

$$\eta\omega_B = \frac{L}{4\pi l}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tag{6}$$

or

$$\eta = \frac{b^2 - a^2}{4\pi a^2 b^2 l} \cdot \frac{L}{\omega_B} \tag{7}$$

The coefficient η is thus determined from the constants of the apparatus and from the experimentally determined ratio L/ω_B. The above treatment holds equally well regardless of which cylinder is rotated, since it is the relative velocity that is important.

In the foregoing discussion it was assumed that the only viscous resistance involved is that exerted by the liquid between the cylindrical surfaces. However, when corresponding ends of both cylinders are closed there is an additional torque due to the viscous drag between the ends. The magnitude of this effect depends upon the radii of the two cylinders and the distance between their closed ends. While a mathematical expression for this factor in terms of the dimensions of the apparatus could be deduced, its precise formulation is complicated by the irregularity at the edges of the cylinders. The problem is simplified, however, when all dimensions of the apparatus are fixed, for then the end effect is a constant which can be treated as a correction to be made to the length of the cylinder. The *effective length* is then the immersed length *l* plus a factor *e* to be determined experimentally. Eq. (7) then becomes

$$\eta = \frac{(b^2 - a^2)}{4\pi a^2 b^2 (l + e)} \cdot \frac{L}{\omega_B} \tag{8}$$

APPARATUS: The viscosimeter employed in this experiment is illustrated in Fig. 3 and represented diagrammatically in Fig. 4. It consists essentially of two metal cylinders A and B of different radii mounted co-axially, one within the other, upon a rigid base. The inner cylinder A rests in bearings so as to rotate with very little bearing friction inside the stationary cylinder B, the liquid under investigation being contained in the space between the cylinders. Attached to the shaft of A is a drum D around which is wrapped a fine cord that passes over a pulley W and carries a mass *m*. The shearing torque is given by the product of the gravitational force on the mass *m* and the radius *k* of the drum. The resulting velocity is determined directly from the time required for the mass to descend a measured distance.

The removable cover C consists of an aluminum bracket which contains the upper bearing for the rotary cylinder. Two screw clamps N hold the bracket in place, accurately centering the shaft, the lower end of which rests in a cone bearing in the base of the outer cylinder. The cylinder A may be locked in position by means of a key K which enters a hole in the drum. One model of the viscosimeter is equipped with an electrical heating element enclosed in a jacket surrounding the outer cylinder.

Fig. 3. Concentric Cylinder Viscosity Apparatus with auxiliary

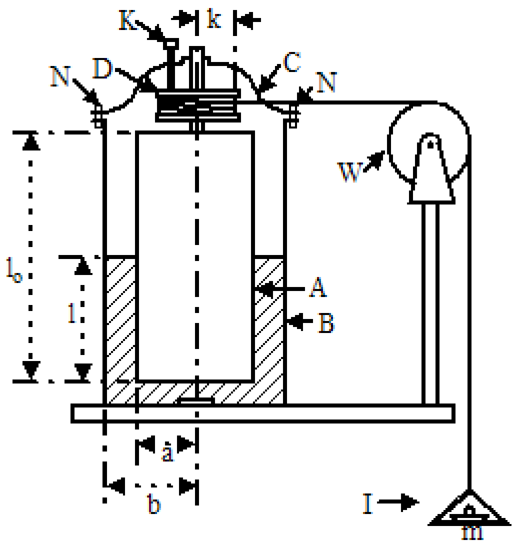

Fig. 4. Vertical section of Concentric Cylinder Viscosity Apparatus.

The relationship expressed by the general equation (8) may be written in terms of the quantities directly measured in this experiment. The shearing torque in absolute units is *L= mgk*, where g is the acceleration due to gravity. The angular velocity is $\omega = \dfrac{s}{kt}$, where *s* is the distance the mass descends in the time *t*. Substituting these relationships in Eq. (8)

$$\eta = \frac{\left(b^2 - a^2\right)k^2 g}{4\pi a^2 b^2 s(l+e)} \cdot mt \tag{9}$$

or

$$\eta = cmt \tag{10}$$

where

$$c = \frac{\left(b^2 - a^2\right)k^2 g}{4\pi a^2 b^2 s(l+e)}$$

is a constant all factors of which can be measured directly with the exception of the end correction *e,* which must be determined from an experimental curve.

In addition to the viscosimeter the following auxiliary equipment will be needed: Set of assorted weights of the smaller denominations (5, 10, and 20gm), small weight holder, vernier caliper, thermometer, small funnel, stop watch, length of thread, and specimen of liquid to be tested. Castor oil and the ordinary motor oils make satisfactory specimens. Glycerin is unsatisfactory because it does not uniformly wet the cylinders.

PROCEDURE:

Experimental:

A. Absolute Value of η***:*** Place the apparatus on the edge of a table- or preferably, on a shelf some distance above the floor- so that the suspended mass has an uninterrupted fall of several feet. Instead of placing the apparatus on a high shelf, the distance of fall may be increased by means of the double-pulley arrangement shown in Fig. 5. Release the upper bearing bracket by loosening the screw clamps N and remove the inner cylinder. With the vernier caliper carefully measure and record the dimensions *a, b, k* and l_o (Fig. 4). In handling the apparatus take care not to injure the cone bearing upon which the cylinder A rests.

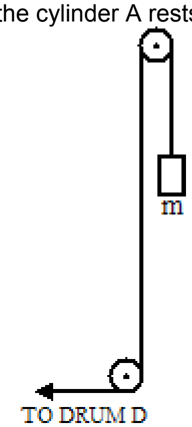

Fig. 5. A method of applying torque to drum of Viscosity Apparatus.

Reassemble the apparatus making sure that the lower end of the spindle is properly seated in its bearing. With a small funnel add liquid until the level is about 1.5cm above the lower end of the cylinder and measure the immersed length *l*. This is a difficult measurement to make and must be done carefully. One way is to make use of the depth gauge on a vernier caliper. The following is a simple and direct procedure: Withdraw the inner cylinder and, holding it so as to drain into the outer cylinder, make a pencil mark indicating the level of the liquid. Repeat this observation several times taking care to keep the cylinder A approximately centered when it is lowered into the liquid. Determine the average value of *l*.

Replace the cover and lock it in place with the screws N. Take a length of thread slightly greater than necessary to reach the floor and loop one end of it over the small pin on the circumference of the drum D. Pass the thread over the pulley W and attach alight (5gm) weight holder to the end. Wind up the drum until the mass is as far from the floor as the arrangement will permit and lock the cylinder in place by means of the key K.

Make the total suspended mass 20gm including the mass of the weight holder. Release the cylinder and take a trial fall, observing closely the motion of the descending mass. It will be noted the velocity increases quickly to a maximum value which is constant during the remainder of the fall. The frictional torque due to the viscosity of the liquid is then equal to the shearing torque produced by the gravitational force on the descending mass. Set up an index I of some sort at a sufficient distance below the initial position of the mass to allow for the region of non-uniform velocity. Measure and record the distance *s* from the index to the floor. Take the temperature of the liquid and repeat the fall, observing the time required for the mass to traverse the measured distance.

Make a series of five determinations, keeping the mass constant and increasing the effective length of the cylinder by adding liquid. The last observation should be made with the apparatus filled just to the top of the inner cylinder. (A thin film of liquid on the top of the inner cylinder will not seriously affect the experiment.) Take the temperature again at the end of the run. Tabulate the data.

With the level of the liquid at the top of the inner cylinder, take a series of six observations varying the mass from 10 to 60gm. Take the temperature at the beginning and at the end of the run. Tabulate the data as before.

B. Variation of η with Temperature: In making this study, the model of viscosimeter which is equipped with an electrical heating unit is most convenient, although not absolutely necessary. With care the heating can be done by means of a flame gently played upon the outer cylinder. *Caution: This product should not be employed when and inflammable liquid is being tested.*

Keeping the mass constant at 15 or 20gm, make a series of observations at increasing temperature up to the limit prescribed by the instructor. Take the temperature immediately before and after each fall and record the average value. The thermometer must, of course, be removed during the fall. If the liquid expands considerably in the temperature range covered, it may be necessary to remove a little with a pipette from time to time in order to keep the level constant. Continue the observations as the liquid cools down until a total of 10 or 12 points have been determined.

Analysis of Data: Before η can be calculated from Eq. (10), the end correction e which enters the constant c must be determined. This is obtained from the first set of data in part A. Eq. (9) shows that the time of descent for a constant mass is directly proportional to the effective length of the cylinder. Plot a curve (hereafter referred to as curve 1) with time of descent as the ordinate and immersed length of cylinder as the abscissa. Clearly, if the end effect were zero length should mean no frictional torque. The fact that t is not zero when *l* is zero indicates that the end of the cylinder is equivalent to a certain additional length. Extrapolate the curve until it meets the *l*-axis. The *l*-intercept then gives the value of the end correction e, which is a positive term to be added to the immersed length *l* to give the effective length (Fig. 6).

From the second set of data in part A plot a curve (hereafter referred to as curve 2) with time of descent as the ordinate and reciprocal of mass as the abscissa. As Eq. (10) shows, this will yield a linear graph passing through the origin (Fig. 7). The slope of this curve gives the average value of the product *mt*.

Having determined the quantity *mt* from the slope of curve 2 and the end correction *e* from the *l*-intercept of curve 1, substitute these values in Eq. (10) and compute the value of η.

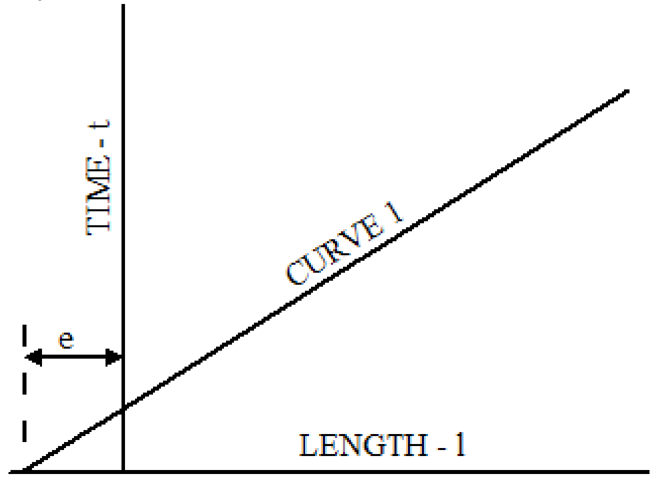

Fig. 6. End correction plot.

Compare the value obtained with that given in an approved handbook and compute the percentage of difference.

From the data of part B and the above determined value of c compute a series of values of η and plot as a function of the temperature *T*.

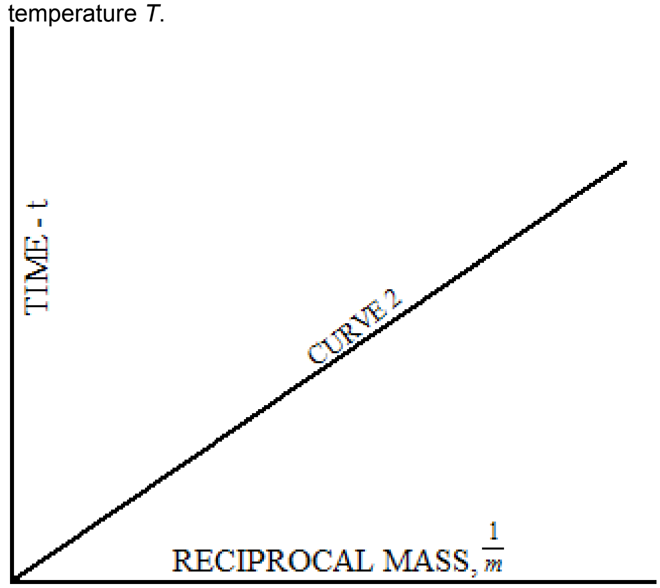

Fig. 7. Curve showing the relationship between the accelerating mass and the time for the mass to traverse a fixed distance.

QUESTIONS: 1. Why are the values of η obtained in part A more reliable than the values used in plotting the curve of part B?

2. What does curve 2 show about the effect of bearing friction? Explain.

3. Does buoyancy affect this experiment in any way? Explain.

4. What percentage error would be introduced into the results by neglecting the end effect?

5. Show that when *a* and *b* are very large in comparison with their difference, Eq. (7) reduces to Eq. (1).