• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

DENSITIES OF SOLIDS AND LIQUIDS

OBJECT:

Part I: To determine the density of two solids, one of which is heavier and the other lighter than an equivalent volume of water.

Part II: To measure accurately the density of various liquids by means of the Westphal balance.

Part III: To measure the specific gravity of these liquids by means of a hydrometer.

METHOD: A body is weighed in air and then immersed in a liquid. The apparent loss in weight of the body when immersed in the liquid is, by Archimedes' principle, equal to the weight of liquid displaced by the body. From these measurements the density and specific gravity of either the solid body or the liquid may be determined.

THEORY: The density of a substance is defined as its mass per unit volume and in the metric system of units is measured in grams per cubic centimeter. The specific gravity of a substance is defined as the ratio of the mass or weight of the substance to the mass or weight of an equal volume of water and is a pure number having zero dimensions.

A convenient method for determining densities or specific gravities is one which uses the principle of Archimedes, namely, that when a body is immersed in a fluid there is exerted on the body a vertical upward force equal to the weight of fluid displaced.

Proof of Archimedes' Principle: Suppose aright circular cylinder of height *h* is immersed in a liquid of density *d* gm per cm^3. The bottom of the cylinder is at a depth of h_2 cm and the top at a depth h_l cm below the surface of the liquid (Fig. 1) where $h = (h_2 - h_1)$.

The pressure on the liquid at depth h_1 is $p_1 = B + h_1gd$ dynes per cm^2, where *B* is atmospheric pressure. The total downward force on the upper surface of the cylinder is $F_1 = p_1A$ dynes where *A* is the area of cross section of the cylinder. Similarly the total upward force on the bottom of the cylinder is $F_2 = p_2A$ dynes.

Since the pressure in a fluid acts at right angles to any surface in contact with it, the pressure on the sides of the cylinder is everywhere horizontal and has no vertical component. Hence the total upward force in dynes on the cylinder is

$$F_2 - F_1 = p_2A = (h_2 - h_1)Agd = hAgd \qquad (1)$$

But *hA* = *V*, the volume of the cylinder, so that

$$F_2 - F_1 = Vgd \qquad (2)$$

or the resultant upward force in dynes is equal to the weight of fluid displaced.

While this proof of Archimedes' principle is for a particularly simple geometrical object, the principle is true for an object of any shape which is immersed in a fluid.

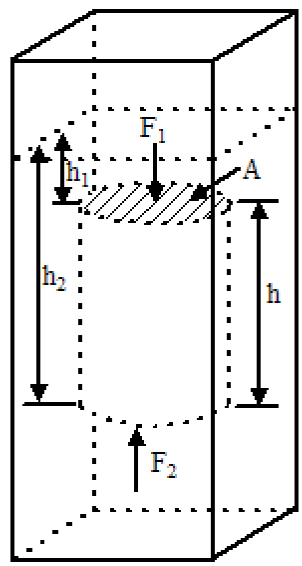

Fig. 1. Vertical forces acting on a right circular cylinder immersed in a liquid of density d gm per cm^3.

Application of Archimedes' Principle: 1. Suppose a body has a density of *D* grams per cm^3 and amass of *M* grams. The volume of the body is $V = M/D$ and its weight in air is *Mg* dynes. The weight (M_Lg) of the body when immersed in a liquid of density *d* is, by Archimedes' principle, $M_Lg = Mg - Vdg$ dynes. Thus $M_L = M - Vd$ or $ML = M - Md/D$.

Since M_L *and M* may be measured by means of a balance, it follows that if the density *d* of the liquid-say water-is known, the density *D* of the body may be calculated from

$$D = \frac{Md}{M - M_L} \qquad (3)$$

2. If the body whose density is to be measured is less dense than the liquid, it is necessary to fasten the body to a sinker so that the two together sink in the liquid. Let M_1g be

the weight in air of the body, say a block of wood, whose density D_1 is less than that of the liquid of density d. Suppose the body of mass M and density D, referred to above, is such that when fastened to the block of wood the two together sink in the liquid. Let the weight of the two together when completely immersed in the liquid be $M_L g$. Then from Fig. 2 it is readily seen that

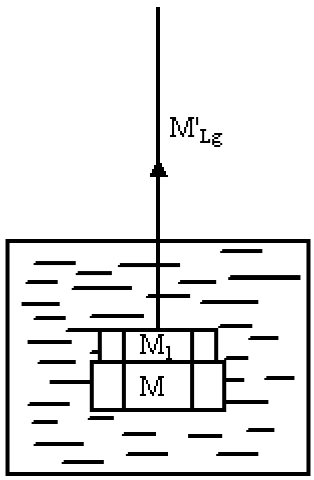

Fig. 2. Block of wood M_1 immersed in liquid, density d, by sinker M.

$$M'_L g = Mg + M_1 g - \frac{Mgd}{D} - \frac{M_1 gd}{D_1}$$

or

$$M'_L = M_L + M_1 - \frac{M_1 d}{D_1} \qquad (4)$$

Hence

$$D_1 = \frac{M_1 d}{M_L + M_1 - M'_L} \qquad (5)$$

3. The relative densities of liquids may be obtained by finding the weight of a sinker in the various liquids. Suppose that when the sinker is immersed in a liquid of density dl its apparent loss in weight is $M_1 g$ and when immersed in liquid of density d_2 the apparent loss in weight is $M_2 g$. By Archimedes' principle $M_1 g = V d_1 g$ and $M_2 g = V d_2 g$ where V is the volume of the sinker. Hence

$$V = M_1 / d_1 = M_2 / d_2$$

or

$$M_1 / M_2 = d_1 / d_2 \qquad (6)$$

M_1 and M_2 may be measured directly by a balance and if one of the liquids is water at a known temperature, its density may be found accurately in tables so that d_2 may be calculated.

PART I

APPARATUS: A balance of the type shown in Fig. 3 is convenient for determining the mass of a body in air or in a liquid. The two bodies whose densities are to be determined are, for example, a block of brass and a block of wood (Fig. 4). A suitable set of weights, a beaker, and a thin piece of cord for tying the two bodies together are necessary.

Fig. 3. Balances which are suitable for density measurements.

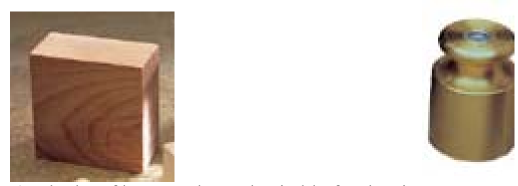

Fig. 4. Blocks of brass and wood suitable for density measurements.

PROCEDURE: By means of the balance determine the mass M of the block of brass in air. Place a beaker containing water on the adjustable platform and suspend the brass block by means of a fine cord or wire from the hook from which the pan is suspended. Completely immerse the brass block in the water. See that no bubbles adhere to the surface of the block. Determine the mass M_L of the brass block in water.

Determine the mass of the block of wood in air. Fasten the wooden and brass blocks together by the cord or fine wire and determine the masses of the two together M'_L when completely immersed in water. The density of water at the particular temperature may be obtained from Table I.

If time permits, determine the density d of alcohol or an oil by weighing the brass block in air and in the liquid. Calculate the density of the brass and wood.

PART II

APPARATUS: A Westphal balance (Fig. 5), which consists of a specially designed balance for measuring the specific gravity of liquids, is needed. A glass plummet, which is

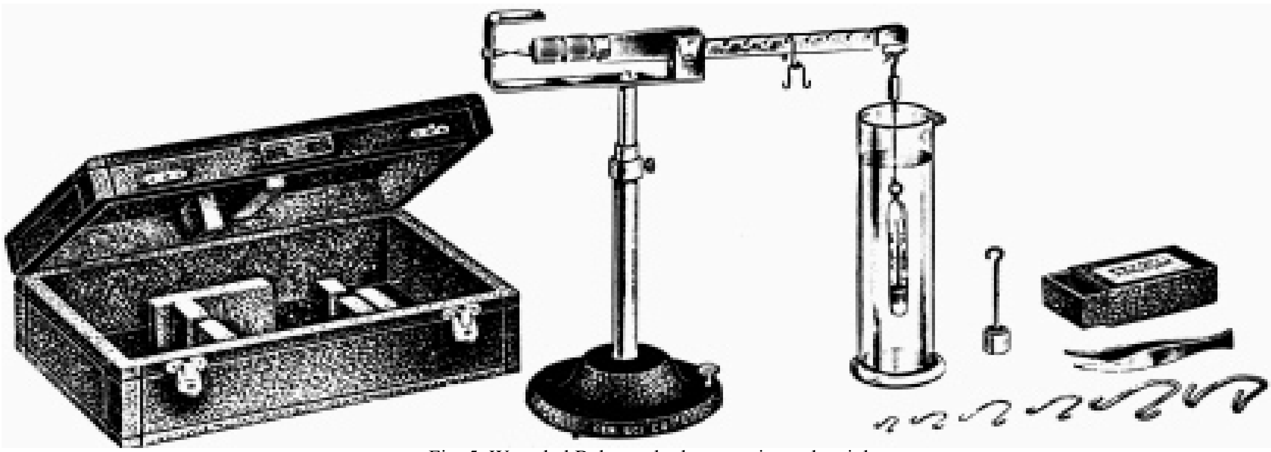

Fig. 5. Westphal Balance, hydrometer jar and weights.

immersed in the liquid whose density or specific gravity is required, is suspended from the end of the calibrated arm of the balance. The apparent loss of weight of the plummet when immersed in the liquid is measured by placing known weights on the arm until a balance is obtained. Four different weights having relative values of 1.0, 0.1, 0.01 and 0.001 are supplied. The glass plummet is constructed so that when immersed in water at 15.5°C the largest or unit weight placed on the hook with the glass plummet produces a balance.

PROCEDURE: Place the leveling screw on the base of the instrument so that it is in the position under the arm of the balance (Fig. 5). Clean and dry the glass plummet and hang it by the fine wire from the hook at the end of the arm of the balance. By means of the leveling screw adjust the arm of the balance so that it is horizontal- that is, so that the two pointers are in line. Clean the glass hydrometer jar and partially fill it with distilled water. Immerse the glass plummet in the water and adjust the height of the instrument so that a single strand of the fine wire cuts the surface of the water. See that no bubbles adhere to the glass plummet.

If the water is at 15.5°C, the unit weight hung at the end of the arm restores the balance. If the glass plummet is placed in other liquids at 15.5°C, the weights required to restore the balance directly measure the specific gravity of the liquids. For high accuracy in measuring the specific gravity or relative densities of liquids it is necessary to have the liquids at the same temperature. This is true not only because the density of liquids changes with temperature but also because the glass plummet changes in volume. If the temperatures are within a few degrees, the error introduced is not large.

Suppose that a balance is obtained when the glass plummet is immersed in water at 20°C with weights of 1.0 and 0.1 at notch 9, the 0.01 weight at notch 8 and the 0.001 weight at notch 5, and when immersed in another liquid at the same temperature a balance is obtained with weight 1.0 at notch 8, weights 0.1 and 0.01 at notch 6, and weight 0.001 at notch 4. The relative densities of the two liquids are, by Eq. (6),

$$\frac{d}{d_\omega} = \frac{0.8664}{0.9985} \qquad (7)$$

The density of pure water at 20°C is given in Table I. Find the density of the various liquids provided, being careful to see that the glass plummet and the hydrometer jar are carefully cleaned each time before anew liquid is used. Read and record the temperature of the thermometer in the glass plummet each time.

PART III

APPARATUS: A hydrometer of the type shown in Fig. 6 and a large hydrometer jar are required.

Fig. 6. Hydrometer for measuring specific gravities of light liquids.

PROCEDURE: Partially fill the hydrometer jar with the liquid whose specific gravity is to be measured and place the hydrometer in it. Clean the hydrometer and jar each time a new liquid is used. In this case the specific gravity of the liquid is read directly from the scale reading on the hydrometer. This method is not capable of giving as high an accuracy as that obtained with the Westphal balance, but the readings may be taken much more rapidly. Determine the specific gravities of the liquids used in the previous experiment.

QUESTIONS: 1. Calculate the density of water in pounds per cubic foot, given 1lb. = 454gm and 1 cubic foot = 28,400cm^3.

2. Given the specific gravity of iron as 7.9, find its density in grams per cubic centimeter and in pounds per cubic toot.

3. Show that density measured in pounds per cubic foot is equal to specific gravity multiplied by 62.4.

4. Explain why it is that when the glass plummet is immersed in a liquid at 15.5°C the weights required for a balance give the specific gravity of the liquid directly.

5. In the construction of a hydrometer is it necessary that the stem have a uniform cross section?

6. Briefly state the theory underlying the use of a hydrometer for measurement of specific gravities.

TABLE I

Density of pure water

10ºC	0.99973 gm per cm^3	18ºC	0.99862 gm per cm^3
12ºC	0.99953	20ºC	0.99823
14ºC	0.99927	22ºC	0.99780
16ºC	0.99897	24ºC	0.99732