CENCO Physics Selective Experiments In Physics

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LINEAR EXPANSION OF A SOLID
OPTICAL LEVER METHOD

OBJECT: To measure the coefficient of linear expansion of several metals.

METHOD: The length of a rod is measured at some known low temperature. The change in length for a definite increase in temperature is also measured by means of an optical lever and from these measurements the coefficient of linear expansion is calculated.

THEORY: With few exceptions solids increase in size with an increase in temperature. This increase is not only in length but in width and thickness so that when the entire solid has reached the new temperature its density has decreased.

as the defining equation of *a*. From this it is seen that *a* is the ratio of the fractional change in length to the temperature change and its value is expressed in *reciprocal degrees centigrade* (or Fahrenheit). The coefficient *a* is *numerically* equal to the fractional change in length produced by a one degree change in temperature. Since in most experiments L_o is approximately equal to L_1, the fractional change in length may be expressed in terms of the length at t_1, hence Eq. (2) may be written

$$a = \frac{L_2 - L_1}{L_1(t_2 - t_1)} \tag{3}$$

and

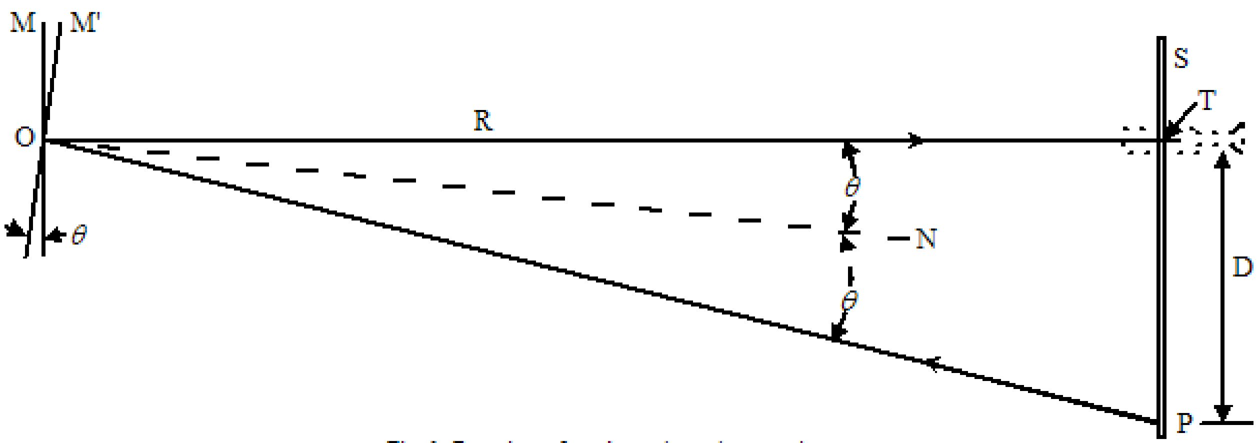

Fig. 1. Rotation of a mirror through an angle θ turns the reflected ray through an angle 2θ.

It is found experimentally that, if the change in temperature is not great, the change in length (L_2 - L_1) is proportional to the length L_o at a temperature of zero degrees and to the change in temperature (t_2 - t_1), or

$$(L_2 - L_1) = aL_o(t_2 - t_1) \tag{1}$$

where the constant of proportionality *a* is called the coefficient of linear expansion. Solving for *a* gives

$$a = \frac{(L_2 - L_1)/L_o}{(t_2 - t_1)} \tag{2}$$

$$L_2 = L_1[1 + a(t_2 - t_1)] = L_1(1 + at) \tag{4}$$

where t is (t_2 - t_1). For a wide range of temperature the coefficient of linear expansion is not constant but varies with the temperature range.

Tremendous forces are involved in the expansion and contraction of solids. If a bar is heated and at the same line prevented from expanding, the force necessary to prevent expansion is the same as that required to compress the bar by an amount equal to the expansion produced by the change in temperature. In the case of rods this force can be calculated from Young's modulus of elasticity.

The Optical Lever: The optical lever furnishes an interesting application of the law of reflection of light. If a beam of light

TO (Fig. 1) originating at a point T on the scale S meets the mirror perpendicularly, it is reflected back on its path; but if the mirror is turned through an angle θ (NOT), an incident beam PO originating at a point P on the scale Swill on reflection take the direction OT where the angle of incidence θ (NOP) is equal to the angle of reflection, ON being the normal to the mirror in the position OM'. By the motion of the mirror through the angle θ, the reflected beam has therefore been turned through an angle 2θ. If a telescope is mounted adjacent to the scale at T and if the mirror is properly adjusted, both reflected beams will enter the telescope and the angle 2θ may easily be determined from the distance D and the scale distance R.

The rods are heat insulated from the end pieces, and they are protected from radiation by heat insulating sleeves which entirely cover the rods. The lower end of the expansion rod rests on a heat-insulating plug in the case, while the upper end actuates the optical lever. The test rod is enclosed in a steam jacket.

As auxiliary apparatus a telescope and scale, a thermometer, a meter stick, a vernier caliper, a barometer, a steam generator with Bunsen burner and tubing, a clamp stand and clamps, rubber tubing, and access to water supply are needed.

PROCEDURE: Measure with the meter stick the length of the rod whose expansion coefficient is to be determined.

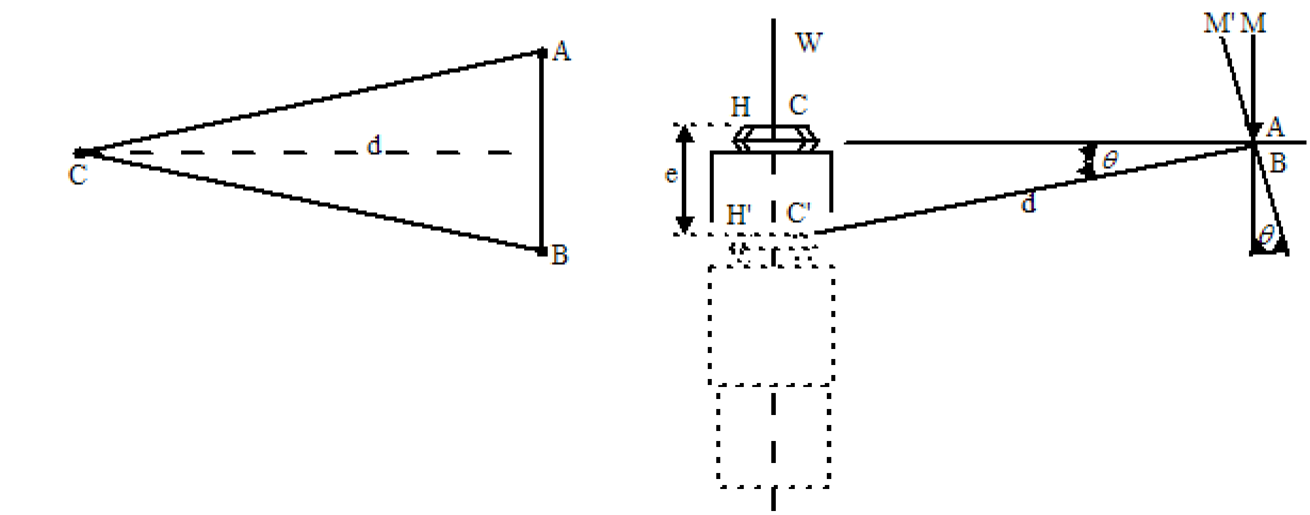

Fig. 2. The Optical Lever as used to determine the expansion of a rod.

In use the optical lever stands upon three hardened steel points, A B C (Fig. 2). Two of these, A and B, resting in a groove, form the fulcrum of the lever, while the third, C, rests on the flat horizontal surface of the rod under test. The lever carries a mirror which is adjustable about a horizontal axis. From the figure it is seen that the elongation *e* is given by the relation

$$e = d\sin\theta \qquad (5)$$

where *d* is the perpendicular distance from the line AB to the point C.

The double angle 2θ may be determined from the distance R from the mirror to the scale and the average distance *D* between readings in the telescope produced by the temperature change ($t_2 - t_1$) by means of the relation

$$\tan 2\theta = D/R \qquad (6)$$

Since the angle θ is small, $\frac{1}{2}\tan 2\theta = \tan\theta = \sin\theta$. In this case

$$e = \frac{dD}{2R} \qquad (7)$$

APPARATUS: A heavy iron base supports two rods, which carry an iron plate for the support of the optical lever (Fig. 3).

Place it in the jacket and install it in the apparatus. Insert the thermometer in the center of the jacket, making sure that the bulb of the thermometer comes fairly close to the rod under test. To the lower opening in the jacket connect a long piece of tubing for conveniently taking care of the waste steam and connecting to the water supply. Attach a piece of tubing about two feet long to the upper opening in the jacket.

Hold it horizontally in a separate clamp stand about six inches away from the jacket So that making connections to the steam generator or sink will not move the jacket. See that the steam generator can be reached by this tubing. Insert a nipple in the free end of this tubing and add on a sufficient length of tubing to reach the sink.

Turn on the water supply and after making sure that the rate of flow is small enough to be cared for by the upper tubing, Connect the tubing which is attached to the *lower* opening in the jacket to the water supply.

Place the optical lever on the apparatus with the mirror vertical. Set the telescope and scale (Fig. 4) at least a meter away from the mirror and at about the same height. Adjust the eyepiece of the telescope so that the cross hairs are clearly seen as soon as the eye looks into the instrument. The main fact to be kept in mind in endeavoring to locate the image of the scale in the telescope is that it cannot be seen in the instrument unless it is possible to see it *with the unaided eye* by sighting along and over the telescope. To do this, get near the mirror and move the head until the image

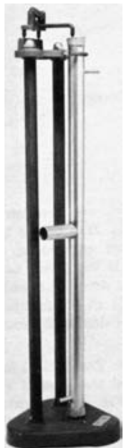

Fig. 3. Apparatus for determining the coefficient of linear expansion using the Optical Lever.

of the eye is seen in the mirror. Keeping the image of the eye in sight, move away from the mirror at least one meter. Still keeping the image of the eye in sight, move the telescope until it is directly under the line of sight. Then move it slightly until the scale can be seen in the mirror while looking over the telescope along the barrel. Next, without touching the eyepiece, pull out the draw tube carrying

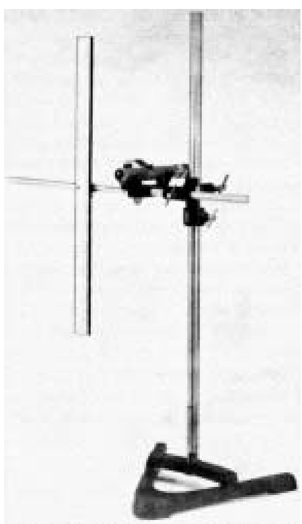

Fig. 4. Telescope and Scale

the eyepiece nearly to its full extent. Place the fingers of the two hands against the sides of the telescope and while looking through it slowly push in the drawtube with the thumbs. The first thing to come in focus should be the mirror; further motion should bring the scale in sight. Center the scale in the field of view. Tilt the mirror until the part of the scale seen is that on the same horizontal line as the center of the telescope. To complete the adjustment, note whether there is any relative motion of the scale image with reference to the cross hairs when the head and eye are moved from side to side. If the image appears to move *with the eye*, and the cross hairs move in the opposite direction, then the image is in front of the plane of the cross hairs and it will be necessary to push the tube containing the cross hairs *in* in order to bring them into a common plane. If, however, the image moves in the opposite direction to the eye and the cross hairs appear to move with it, then the drawtube should be pulled out.

As soon as the temperature of the rod has become constant, the scale reading in the telescope will stop shifting. Then read the temperature of the rod as given by the thermometer and note the position of the cross hair on the scale. If possible read this to 0.01cm.

Disconnect the lower tubing from the water supply. Separate the upper tubing at the nipple and connect the upper inlet of the jacket to the steam generator. If the jacket is held in position by means of a screw, loosen this screw slightly while the heating is taking place, otherwise the expansion of the jacket may tilt the bracket on which the optical lever rests. Shortly after free steam issues from the lower outlet of the jacket, the final position of rest will be reached by the cross hairs as they move along the scale seen in the telescope. Record this final position. Disconnect the steam generator and repeat the initial reading with water in the jacket in place of steam.

Read the barometer and find from tables the temperature of the steam for this pressure. With a meter stick measure the distance from the mirror of the optical lever to the scale. Using apiece of ruled paper, place the points A and B of the optical lever on a line and press all the points into the paper. Using a vernier caliper, measure in centimeters the distance d from the line AB to the point C. Determine the change in length of the rod $(L_2 - L_1) = e$ from Eq. (7), and then calculate the coefficient of linear expansion using Eq. (3).

QUESTIONS: 1. If a rod of the material used in the experiment measured a meter in length at zero degrees centigrade, at what temperature would it measure 100.13cm?

2. A plate of the same material measures 10cm on each edge at zero degrees centigrade and has a hole in the center 5cm in diameter. Find the area of the plate and the diameter of the hole when the temperature is 35°C.

3. The temperature of a steel rod 3 meters long and $1cm^2$ in cross section is lowered 15°C. How much force will be required to stretch it to its original length? (Young's modulus 18 x 1011 $dynes/cm^2$. Coefficient of linear expansion 0.000012 per degree centigrade.)

4. A 30cm rule of steel and one of brass are both correct at 15°C. The coefficients of expansion are 0.0000121 and 0.0000189 per degree centigrade respectively. How much will the rods differ in length at 35°C?

5. A flask made of the material whose coefficient of expansion has just been determined has a volume of 250 cubic centimeters at zero degrees centigrade. Find its volume at 30°C.

6. Why is it permissible to measure the length L_1 with a meter stick although the expansion is measured with an optical apparatus?

7. Show that for an isotropic substance the coefficient of area expansion is approximately equal to double the coefficient of linear expansion and that the volume coefficient of expansion is approximately three times the coefficient of linear expansion.