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LENS STUDY BY RAY TRACING

OBJECT: To study the path of light rays passing through lens sections and producing images.

METHOD: Narrow beams of light from an intense straight-filament incandescent lamp fall on a sheet of paper held on the platen of a ray tracing apparatus. Lens sections placed in the path of these "rays" refract them to form images. The lens equation for the relation among image distance, object distance, and focal length is applied to the data taken.

THEORY: Except for normal incidence, the path of a beam of light is bent when it crosses the interface between materials of different optical densities. Thus a ray of light traveling from

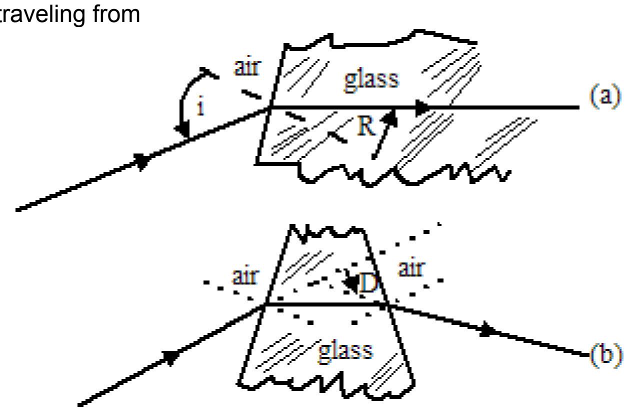

Fig. 1 - Refraction of light

air to glass is refracted as shown in Fig.1 (a). The relationship of the incident and refraction angles, i and R respectively, to the speeds of light in the two media is given by

$$\frac{V_{air}}{V_{glass}} = \frac{\sin i}{\sin R} = n \qquad (1)$$

The constant n, for a given monochromatic ray, is called the *index of refraction*. The value of n for an air-glass interface is approximately 1.5. Should the ray pass through the glass and emerge from the second face, as shown in Fig. 1 (b), the ray suffers another refraction at the second interface. Unless the two faces are parallel, the ray is deviated or bent through an angle D called the *angle of deviation*.

A set of glass "wedge sections" such as S_1 and S_2 of Fig. 2 may be arranged so that light rays from an object source O passing through these wedges will intersect, or focus, at a common point to form an image I of the object. An infinite number of such sections form the characteristic lens section of Fig. 2. The lens shown is called a converging, or a double

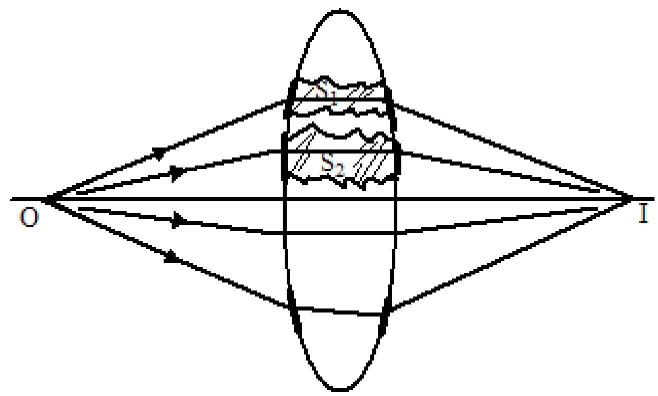

Fig. 2 - Focusing of light by lens section

convex, or a positive (+) lens. When the periphery of the lens is thicker than the center, the lens is a diverging, or a concave, or a negative (-) lens.

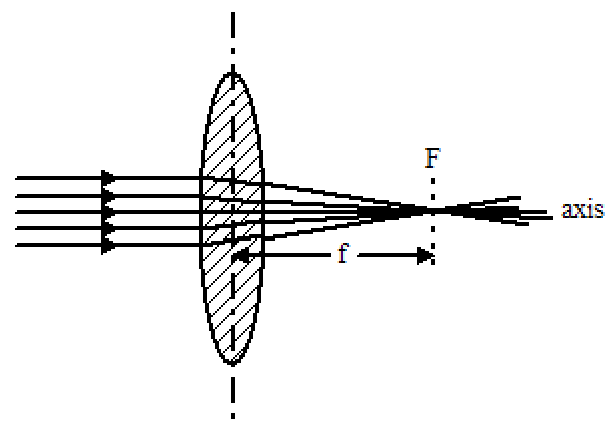

Fig. 3 - Focal length f and principal focus F of lens

Focal Length of a Lens. When the incident rays are parallel to the principal axis, as drawn in Fig. 3, the rays focus at a point F, called the *principal focus*. The distance f from the lens center to the principal focus is the *focal length* of the lens. The value of f depends on the index of refraction n of the glass and on the radii of curvature, r_1 and r_2, of the lens face. This is expressed in the "lens-maker's equation,"

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \qquad (2)$$

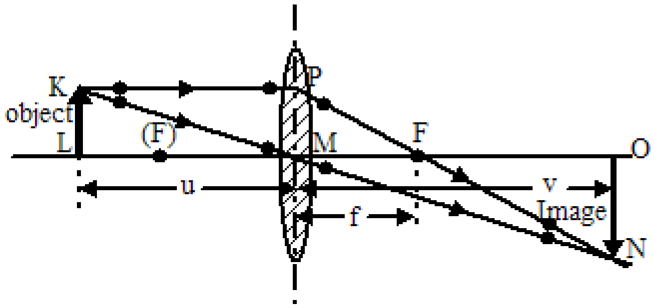

Fig. 4 - Real image formed by a converging lens

Ray Diagram. A ray diagram may be used to locate graphically the position of the image of an object which is produced by a lens. Figure 4 shows the ray construction for a thin converging lens. The drawing follows the convention of locating the entire deviation on the midline. The ray *KP,* parallel to the axis, is refracted to pass through the principal focus *F*. A second ray *KM* is chosen as the one which passes through the lens center. For this ray the refraction angle at the first face of the lens is essentially offset by that at the second face. Thus the ray may be drawn as a straight line. These two rays from the arrowhead intersect at *N* to form its image. Rays from other points on the object *KL* will similarly intersect or focus to form the total image *ON*.

Derivation of the Lens Equation. The triangles *KLM* and *ONM* of Fig. 4 are similar, as are the triangles *PMF* and *ONF*. Equating the corresponding sides of similar triangles, (note that *PM* = *KL*), the following proportions are true:

$$\frac{MO}{ML} = \frac{NO}{KL} \qquad (3)$$

$$\frac{FO}{FM} = \frac{NO}{KL} \qquad (4)$$

Hence,

$$\frac{MO}{ML} = \frac{FO}{FM} \qquad (5)$$

Substituting the notation,

u = object distance = *ML*
v = image distance = *MO*
f = focal length = *MF*

in Eq. (5) gives

$$\frac{v}{u} = \frac{(v-f)}{f} \qquad (6)$$

Clearing fractions and dividing by *u, v*, and *f* gives the lens equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (7)$$

Note that the object and image positions may be interchanged.

Real and Virtual Images. The image shown in Fig. 4 is a real image - the rays of light transmitted by the lens converge to a focus and the image may be projected on a screen. A diverging lens forms a virtual image of a real object, as shown in the ray diagram of Fig. 5. A virtual image cannot be projected on a screen, the light rays do not come to a real focus. It is, however, visible when the emerging diverging rays fall into the eye. The eye receives the diverging rays which appear to come from the virtual image. Note that the real image of Fig. 4 is inverted whereas the virtual image in Fig. 5 is not inverted.

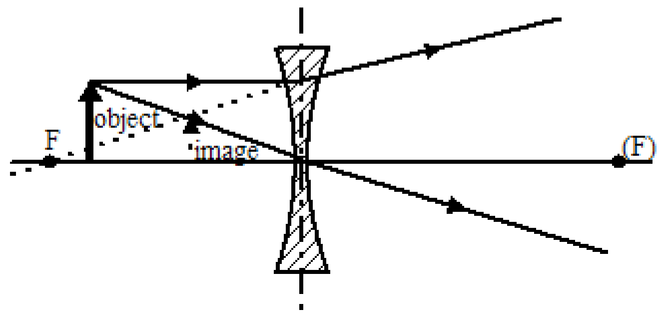

Fig. 5 - Virtual image produced by a diverging lens

Magnification. The ratio of the size of the image *NO* to the size of the object *KL*, Fig. 4, is called the magnification *m*. It follows from Eq. 3 that

$$m = \frac{MO}{ML} = \frac{v}{u} \qquad (8)$$

Thus the magnification is given by the ratio of image distance to the object distance.

Aberration. In the foregoing derivation it was implicitly assumed that all rays from the object point *K* which pass through the lens would strike the point *N*. Unless a relatively small section of a lens with spherical surfaces is used, the image is not sharp, and, therefore, the assumption is proved to be wrong. Peripheral rays focus at a slightly different distance than do a bundle of rays passing through the lens center. This image defect, or aberration, due to the spherical surfaces, is called *spherical aberration*. There are other types of aberration also.

Sign Convention. The equations,

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

and

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

are applicable to all lenses. In any such equation, it is necessary to follow some convention of signs for mathematical clarity. One method introduces (+) and (-) signs in the equations according to the following plan:

u{(+) for real objects (diverging light)
(-) for virtual objects (converging light)

v {(+) for real images
(-) for virtual images

f {(+) for convex or converging lenses
(-) for concave or diverging lenses

r {(+) for convex glass surfaces
(-) for concave glass surfaces

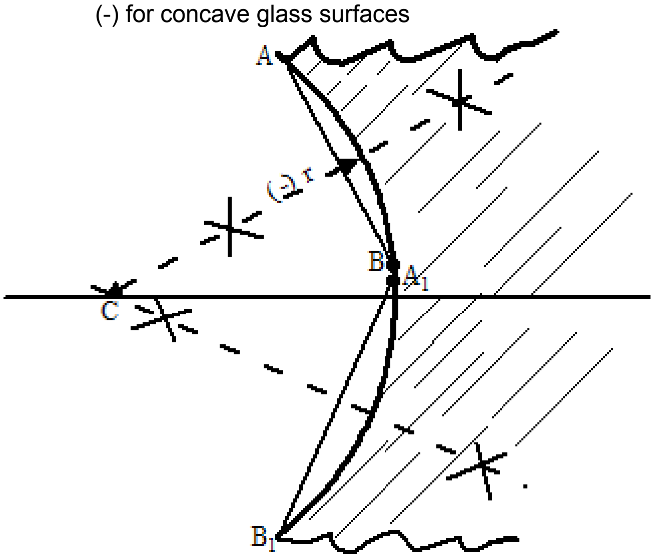

Fig. 6 - Finding the radius of a curvature

The graphical procedure illustrated in Fig. 6 may be used to determine the radii of curvature of the lens surfaces. Draw two chords *AB* and A_1B_1. Bisect each chord. This may be done by using a compass to scribe intersecting arcs of equal radii with the terminal points of the chord as center. The intersection of the two normal bisecting lines locates the center of curvature *C* and hence the radius of curvature *r*. The (-) sign satisfies the convention since this is a concave glass surface.

APPARATUS: Ray Tracing Apparatus with Diverging Ray Attachment, Fig. 7., converging and diverging lens sections, protractor, compass, and ruler.

PROCEDURE: Place a sheet of white paper on the platen of the ray tracing apparatus and clamp it under the edge clips. Set the lens section in the path of the ray, following the instructions given later. Mark the outline of the lens with a sharp pencil. Locate dots in the center of the light rays, incident and emerging. Separate the dots as far as possible. Connect these dots with sharp lines, showing the path of the ray. The work must be carefully done so that accurate data may be obtained. Each of the following exercises is a complete unit.

Part I. Focal length of a lens. Place each lens section so that the ray through the center of the lens is perpendicular to the midline.

(A) Construct ray traces for parallel rays through the center section of the converging lens and determine the focal length. Parallel rays are obtained by shifting the platen.

(B) Repeat I (A) using the diverging lens.

(C) Construct ray traces for peripheral rays through the converging lens. Is the measured focal length of the lens for these rays the same as the value obtained in I (A)?

Part II. The lens equation. Attach the diverging ray shield to produce a set of diverging rays comparable to a real object source. Construct the incident and emerging ray traces using the converging lens. Measure *u* and *v*. Solve for the focal length of the lens and compare with the value obtained in Part I.

Part III. Magnification. Use the light filament as source object. Place it at *L*, see Fig. 4, and locate its image at *O*. Now shift the filament from *L* to *K*. Its image is now at *N*. Test the magnification expression, Eq. (8). Repeat, using a different value for the object distance.

Part IV. Lens-maker's equation. Measure the radius of curvature r_1 and r_2 of each face of the converging lens, following the directions given with Fig. 6. Using the mean

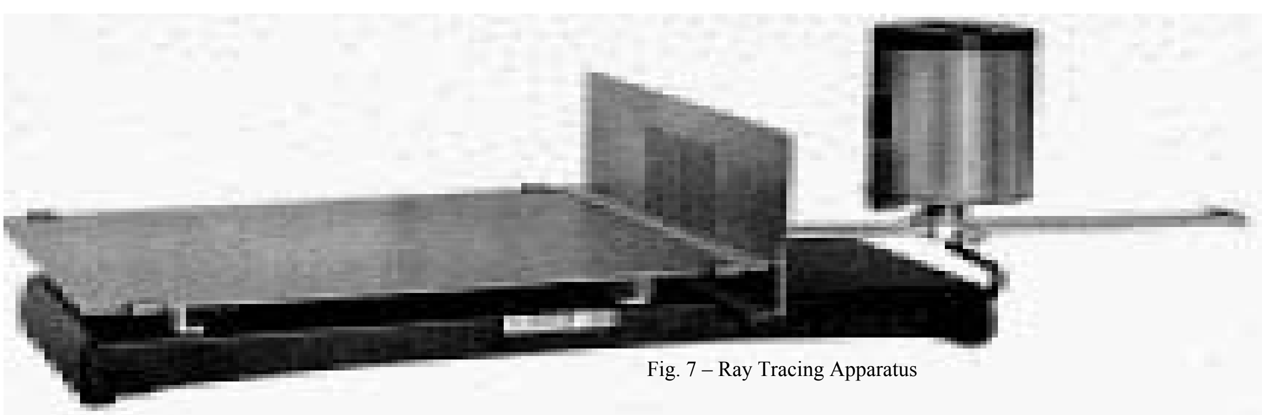

Fig. 7 – Ray Tracing Apparatus

value of the focal length of this lens obtained earlier, compute the index of refraction *n* of the glass.

QUESTIONS: 1. Under what conditions will a converging lens produce a virtual image of a real object? Construct the ray diagram.

2. How must the surface of a spherical lens be modified so that it will not show spherical aberration? Show by an exaggerated drawing of a convex lens section.

3. In Fig. 4 the ray *KMN* is drawn as a straight line. The theory states that this is not quite correct. Show the actual path of this ray for a center section of a *thick* lens.

4. A lens forms an image (picture) of a man. If the lower half of the lens is covered, how will the picture look?

5. Focusing a camera when taking a picture means that the image distance is adjusted to satisfy Eq. (7). Sharp pictures may be taken with a box camera which can not be focused. Explain.

6. Given a block of glass, n = 1.5, with instructions to produce a symmetrical lens of f = +10cm. To what radius of curvature must the two faces be ground and polished?

7. An object is placed 4 inches from a converging lens having a focal length of 10 inches. Locate the image. Is it real or virtual? Is it erected or inverted?