# THIN LENS – GEOMETRICAL OPTICS

**OBJECT**: To study the geometrical optics of thin lenses by the use of a simple optical bench; in particular, to study the formation of images produced by the lenses and to measure the focal lengths of convergent and divergent lenses.

**METHOD**: A thin convex lens (also called a positive, or converging, lens) is mounted on a simple optical bench and used to form an image of a distant object at the principal focus of the lens. The lens then is used to form an image on a screen of a nearby object. The focal length of the lens is calculated from the observed object and image distances, by the use of the thin-lens equation. The magnification produced by the lens is measured from the ratio of the image to object size and compared with the value calculated from the ratio of the image distance to the object distance. The focal length of a concave lens (also called a diverging, or negative, lens) is measured by using the lens in conjunction with a convex lens.

**THEORY**: As shown in Fig 1 the rays of light (solid lines) from a point source diverge radially. The wave fronts are therefore spherical (dashed lines) and are perpendicular to the rays. When the rays pass through a convex (converging, or positive,) lens the change in the speed of the light causes the rays to be bent, or *refracted*. Thus the curvature of the wave front is changed. If this change is sufficiently great the rays will be brought to a focus and a real image will be formed.

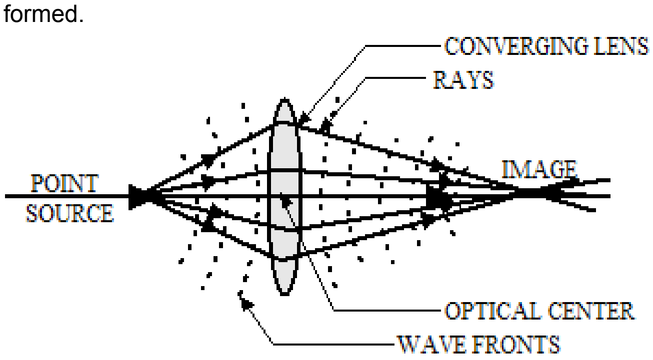



Fig. 1. Refraction of a wave by a lens.

Rays from a very distant source ("infinite" distance) reach a lens as parallel rays. They are brought to a focus, called the *Principal focus F*, Fig 2 (a). The distance from the center of the lens to *F* is called the *focal length f* of the lens. For a diverging (negative) lens, rays entering the lens parallel to the principal axis diverge as they leave the lens in such directions that they appear to come from a point behind the lens. This point is the *virtual principal focus*, Fig 2 (b). For a real source near a lens the rays are divergent as they leave the source.

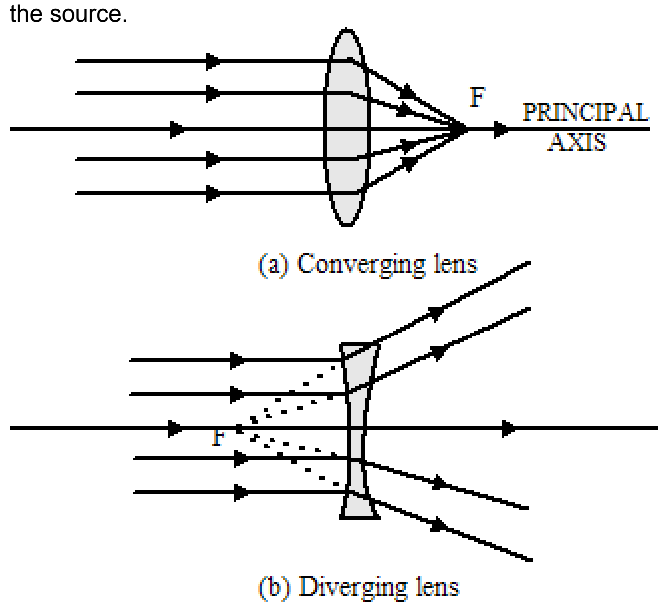


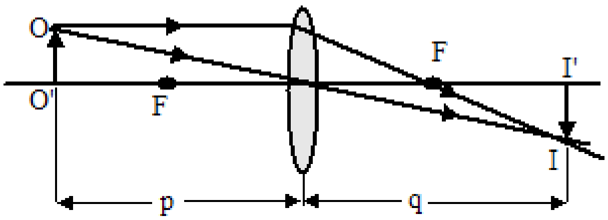

Fig. 2. Principal focus of a lens.

From a simple geometrical construction it is possible to determine the position and size of an image formed by a thin lens. This is done by drawing two rays whose complete paths we know, starting from an object point and focusing at the corresponding image point. In Fig 3 (a) one ray is shown leaving the tip of the arrow object *OO*' and directed parallel to the principal axis. After refraction by the lens this ray passes through F. Another ray also from the tip of the arrow is drawn through the optical center of the lens. For a thin lens this ray is undeviated. The intersection of the two rays at *I* locates the image point which corresponds to the object point *O*. The other image points corresponding to additional object points may be located by similar constructions, thus giving the complete image *II'*.

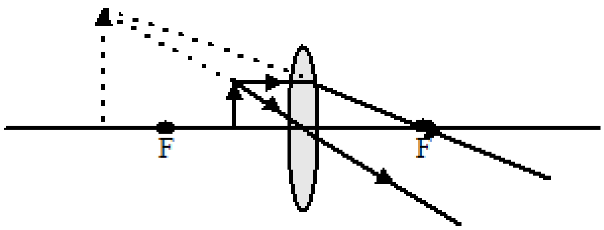
The location of the image for an object placed closer to a converging lens than the principal focus is shown in Fig 3 (b). Under this condition the positive lens cannot sufficiently change the curvature of the wave front to cause convergence that will bring the rays to a real focus. It is seen that the rays from a particular point on the object diverge after passing through the lens. If the refracted rays are traced backward, they intersect at a virtual focus. The entire virtual image is represented conventionally by the dashed arrow. Such a virtual image cannot be formed on a screen,

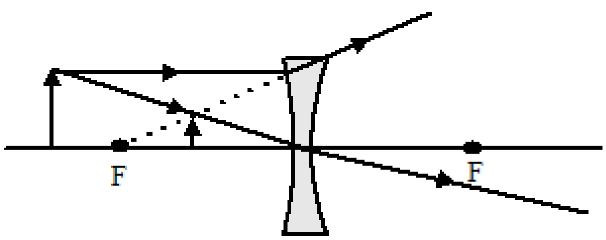
but it may be viewed by looking into the lens, from the right in the figure.
The image of an object formed by a diverging (negative) lens is found by a similar construction, as in Fig 3 (c). Here the ray that is parallel to the principal axis diverges from the virtual focus F after passing through the lens. The image is seen to be virtual in this case.



(a) Converging lens, real image



(b) Converging lens, virtual image



(c) Converging lens, virtual image

Fig. 3. Ray diagram method for location of images formed by lenses.

**The Thin-Lens Equation**. It is possible to find the location and size of an image by algebraic means as well as by the graphical method already outlined. Analysis shows that the focal length *f* of a thin lens, the distance *u* of the object from the lens, and the distance *v* of the image are related by the equation

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{f} \qquad (1)$$

This relation holds for any case of image formation by either a converging or diverging lens, provided that the following conventions are observed:

1. Consider *f* positive for a converging lens and negative for a diverging lens.
2. Object and image distances are taken as positive for real objects and real images, and negative for virtual objects and virtual images. The normal arrangement is taken to be object, lens, and image, going from left to right in the diagram. If *v* is negative, it means that the image is located at the left of the lens, and therefore the image is a virtual image.

**Magnification**. It will be seen from all of the ray diagrams of Fig 3 that the angle subtended at the lens by the image is always equal to the angle subtended by the object. Hence from the graphical construction we can write the following proportion:

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}$$

The first ratio is called the *linear magnification M*, or simply the magnification. Hence in symbols

$$M = \frac{v}{u} \qquad (2)$$

where *u* is the distance of the object from the center of the lens and *v* is the distance of the image from the center of the lens.

**Equivalent Focal Length of Two Thin Lenses in Contact**. This equivalent focal length may be found experimentally by considering the lens combination as a single lens and proceeding as previously described. If the individual focal lengths are known, the equivalent focal length *f* may be calculated from the equation

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (3)$$

where $f_1$ and $f_2$ are the individual focal lengths. Proper regard must be observed for the algebraic signs of these focal lengths.

**Focal Length of a Concave Lens**. Since a divergent lens makes the rays more divergent, it cannot form a real image of a real object, and therefore the focal length of a divergent lens cannot be determined by the methods just described for convex lenses. The focal length of a divergent lens may, however, be obtained by placing it in contact with a convergent lens of shorter and known focal length, measuring the focal length of the combination, and using Eq. (3) for the equivalent focal length of thin lenses in contact.

**APPARATUS**: Simple optical bench and accessories, Fig 4, including: object box with illuminated object, Fig 5; lens holders; image screen; convex lenses, 5cm, 10cm, and 15cm focal lengths; concave lens, 15cm focal length; vernier caliper; source of parallel light rays; ruler; compass. (A more sophisticated form of optical bench is shown in Fig 6.

Fig. 4. Simple optical bench.

Fig. 5. Object box for optical bench.

Instead of the object box illustrated the filament of an unfrosted and shielded lamp bulb can be used as an object.) This experiment should be performed in a room with subdued light.

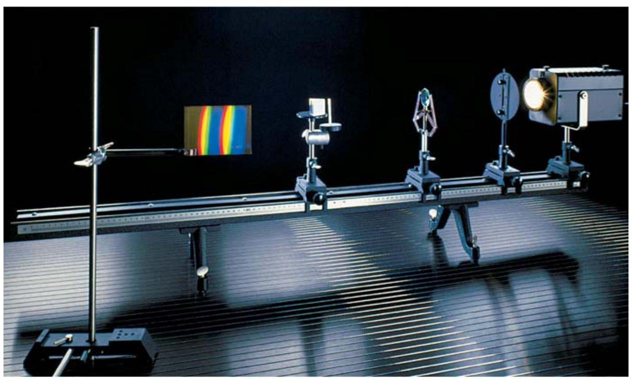


Fig. 6. Lathe bed form of optical bench.

**PROCEDURE**: **I. Focal Length of Converging Lenses**.

1. *Use of distant landscape as a source*. Select the thinnest convex lens available (focal length about 15cm). Mount it in the lens holder on the optical bench and place it near the screen. Point the optical bench through an open window toward a distant building. Place the screen at the zero of the scale on the bench and adjust the position of the lens until the central part of the image of the building is sharply outlined on the screen. Record the distance from the lens to the screen. Show why this distance is the focal length of the lens.

2. (Optional) *Use of parallel-ray illuminator as a source*. Arrange a projector at one end of the laboratory to give approximately parallel rays directed toward the screen of the optical bench. Place the lens on the bench and adjust it to focus the beam of light sharply. Record the distance from the lens to the screen and show why this distance is the focal length of the lens.

3. *Use of thin-lens equation*. Place the illuminated object at the zero position on the optical bench and arrange the screen at a distance of about five times the focal length from the object. Mount the lens near the object and find two positions of the screen for which the image is sharply defined. Choose a position of the lens which will give a sharply outlined image of moderate size (covering about half the screen). In determining the final position of the screen move it from left to right and then from right to left and average the two readings for the final value. Measure $u$ and $v$ and calculate $f$ from Eq. (1). Draw a ray diagram to scale for this case. Draw the diagram to as large a scale as possible.
Repeat the procedure for the measurement of the focal length of one or two other convex lenses.

**II. Magnification**. 4. Using one of the arrangements of Step 3, for which the image is reasonably large, measure with a vernier caliper the sizes of the image and object. Note the percentage difference between this magnification $II'/OO'$ and the value obtained from $v/u$.

**III. Focal Length of Thin Lenses in Contact**. 5. Insert the two thinner convex lenses in a contact in a single-holder and measure their equivalent focal length $f$ as in Step 3. Note the percentage difference between this observed value of $f$ and that obtained from the measured values of the focal lengths $f_1$ and $f_2$ by the use of Eq. (3).

6. Determine the focal length of the concave lens by mounting it in contact with the 5cm convex lens and proceeding as in Step 5.

**QUESTIONS**: 1. Make a rough sketch of the optical features of the human eye and show how this device illustrates many of the principles of geometrical optics studied in this experiment.

2. By the aid of a ray diagram explain the operation of a simple magnifier.

3. Draw a graph to show the variation of $1/u$ with $1/v$ for a converging lens what is the significance of the horizontal and vertical intercepts of this curve?

4. Illustrate by diagrams the effect produced on the curvature of a plane wave by (a) a plane mirror, (b) a convex lens, (c) a concave lens.

5. Explain why the column of mercury in a clinical thermometer looks so much larger than it really is.

6. Under what circumstances does a convex lens form (a) real images, (b) virtual images? Repeat for a concave lens.

7. In this experiment the focal length of a convex lens was obtained by sighting the lens on a distant object and locating the position of the image. If a lens of 16.5cm focal length were used and the "distant" object was 450cm from the lens what percentage error was made by assuming that the object is infinitely distant?

8. Look up in your textbook the equation that gives the focal length of a lens in terms of the index of refraction of the glass and the radii of curvature of the lens surfaces. If the index of refraction of a convex lens is 1.50 and the radii of curvature are equal, how does the focal length compare with the radius of curvature?

9. Under what circumstances are images formed by a convex lens inverted? erect? larger than the object? smaller than the object?

10. Show how Eq. (3) follows from Eq. (1). (Use subscripts 1 and 2 for the first and second lenses in contact.)