

## THE SPHEROMETER

OBJECT: To determine with a spherometer the thickness of a microscope slide, the depth of a depression in this slide, and the radius of curvature of a spherical surface.

METHOD: The thickness of the microscope slide and the depth of the depression are determined directly from spherometer readings. In all these readings the points of the spherometer legs rest on a flat plate. Spherometer readings are taken with the point of the micrometer screw touching the plate, with this point touching the top of the microscope slide, and with the point touching the bottom of the depression.
To determine the radius of curvature of a spherical surface, spherometer readings are taken with the four spherometer points touching a flat surface and with the four points touching the spherical surface. These readings and the distance between the points of the spherometer legs are used to compute the radius curvature of the surface.


Fig. 1. The equilateral triangle formed by the points of the spherometer legs.

THEORY: A spherical surface is a surface all points on which are a fixed distance from a point. This point is called the center of curvature. The distance from this point to the surface is called the radius of curvature R. A spherical surface may be thought of as a section of the surface of a
sphere and the radius of curvature as the radius of this sphere.
Since the points of the spherometer legs are equally spaced at a distance $S$ from each other, an equilateral triangle def may be constructed with these points as vertices, Fig. 1. These three points determine a circle of radius $r$, and when the point of the micrometer screw is in the plane of this figure it is at the center of the circle and a distance $r$ from each leg. It can be shown that

$$
\begin{equation*}
r=\frac{S}{\sqrt{3}} \tag{1}
\end{equation*}
$$

Fig. 2 is a vertical cross section showing the spherometer resting on a spherical surface. The leg $L$ and the micrometer screw $M$ are in the plane of the paper. The other two legs, one behind and one in front of the plane of the paper, are indicated by L. From this diagram it should be evident that the distance $a$ is given by the difference in reading of the spherometer when all four points touch a plane surface and when these points touch the spherical surface. From the geometry of the figure it may be shown that the radius of curvature $R$ of the spherical surface is given by the equation

$$
\begin{equation*}
R=\frac{S^{2}}{6 a}+\frac{a}{2} \tag{2}
\end{equation*}
$$

APPARATUS: Spherometer, plane surface, convex lens, depression microscope slide, and accurate steel rule are required.
The spherometer, Fig. 3, has three arms radiating from the center. The legs, mounted on these arms, and the micrometer screw at the center have hardened and polished points. The micrometer screw is accurately cut and the large graduated head is used to determine fractions of a turn. The vertical scale on the left is used to determine the number of complete turns. It is obvious that the reading of the spherometer, which indicates the vertical position of the point on the micrometer screw, is obtained by combining the readings of the vertical and circular scales.
Assume that the pitch of the screw thread on the spherometer, Fig. 3, is $1 / 2 \mathrm{~mm}$ and that the numbers on the vertical scale represent the number of millimeters (not the number of revolutions). If there are 50 divisions on the circular scale, each one of these divisions represents $1 / 50$ of a revolution or a vertical displacement of the screw amounting to 0.01 mm . Since the circular scale is read to tenths of a division, displacements are measured to thousandths of a millimeter. Since the pitch of the screw is $1 / 2 \mathrm{~mm}$ the vertical scale must be read to half-millimeters,
whether or not the half-millimeter divisions are marked on the scale. For example, if the reading on the vertical scale is between 26.5 mm and 27 mm and the reading on the circular scale is 15.4 divisions, the complete reading is $26.5 \mathrm{~mm}+$ 0.154 mm or 26.654 mm .

Not all spherometers are constructed and calibrated like the one described above. When using a particular spherometer for the first time it is necessary to determine, by careful study, the pitch of the screw and the significance of the numbers marked on each of the two scales.

PROCEDURE: Before proceeding to take observations it is necessary to become thoroughly familiar with the spherometer. Study the spherometer and make sure that the method of combining the vertical and circular scale readings is thoroughly understood.


Fig. 2. Cross section showing the spherometer on a spherical surface.
Place the spherometer on the plane surface and, using the knurled knob, adjust the instrument so that all four points rest equally on the plate. Various methods may be used to indicate when this adjustment has been made properly. The following one is suggested. Holding one of the arms lightly between thumb and finger, push the arm gently back and forth in a direction perpendicular to its length. It is evident from the "feel" of the instrument when it is resting on all four points. If the point of the screw is too low the instrument rotates quite freely around it. Make five observations of the reading on the plane surface. To reduce the chance of error it may be advisable to record the readings of the vertical and circular scales separately, as well as the combined reading. Place the microscope slide on the plane surface and take a set of five readings with the point of the micrometer screw
touching the top of the slide. In a similar manner take a set of readings with the point of the screw touching the bottom of the depression. If the depression is concave, care must be taken that the point is in the middle of the depression.
In a manner similar to the one outlined above take a set of spherometer readings on the spherical lens surface. If both surfaces of the lens are spherical, take measurements on each surface.
Place a piece of smooth paper on the fiat plate and press the points of the spherometer legs against this paper so that these points leave fine punctures in the paper. Use a good steel rule to measure the distance $S$ between each pair of legs. Take three independent sets of readings using anew set of punctures for each.
Compute the thickness of the microscope slide, the depth of the depression in the slide, and the radius of curvature on the lens surface.

Optional: 1. Determine the probable error in the value of $R$ and express the result of this part of the experiment in the form $\mathrm{R}=$ $\qquad$ $\pm$. $\qquad$ .cm. Students who do this part of the experiment should take ten observations each on the plane surface and on the spherical surface.
2. Outline a method for using the spherometer to measure the radius of curvature of a cylindrical surface. Derive the necessary equations.
3. The focal length $f$ of a lens is given by the equation

$$
\begin{equation*}
\frac{1}{f}=(\mu-1)\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) \tag{3}
\end{equation*}
$$

where $\mu$. It is the index of refraction and $R_{1}$ and $R_{2}$ are the radii of curvature of the two surfaces. Determine experimentally the focal length of this lens and compute $\mu$.

QUESTIONS: 1. Derive Eqs. (1) and (2).
2. Classify the following as to whether they introduce systematic or random errors in the determination of the radius of curvature: (a) unevenness of the spherical surface;
(b) a spherometer on which the legs are not evenly spaced;
(c) difficulty in determining when the four legs rest equally on the surface.
3. Show that the first term in the right hand member of Eq.
(2) must be larger than the second term. Hint: first assume that the two terms are equal.
4. Assume that the points of the spherometer legs form an equilateral triangle but that the point of the screw is not quite at the center of this triangle. What effect will this instrumental error have on the computed value of $R$ ?
5. A one per cent error in measuring $S$ is more serious than a one per cent error in measuring a. An error of onetenth millimeter in measuring $S$ is less serious than an error of the same magnitude in $a$. Explain.


Fig. 3. The Spherometer.

