Selective Experiments In Physics

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SIMPLE TELESCOPES – MAGNIFYING POWER

OBJECT: To determine experimentally the magnifying power of an astronomical telescope and of a Galilean telescope and to compare these values with those computed from the focal lengths of the component lenses.

METHOD: The focal lengths of a long-focus and a short-focus lens are determined on an optical bench. These lenses are combined to form an astronomical telescope and the magnification of this instrument is determined experimentally. To determine the magnification the observer looks through the telescope with one eye at a distant scale and directly at the same scale with the other (unaided) eye. Since the images of the scale are seen apparently superimposed the magnification may be determined by direct comparison. This value of the magnification is compared with the value determined from the focal lengths of the component lenses.
The focal length of the diverging lens is determined, this lens is combined with the long-focus converging lens to form a Galilean telescope and the experiment is repeated with this instrument.

THEORY: Whether a body is self-luminous or whether it is made visible by the light diffusely reflected from it, each point

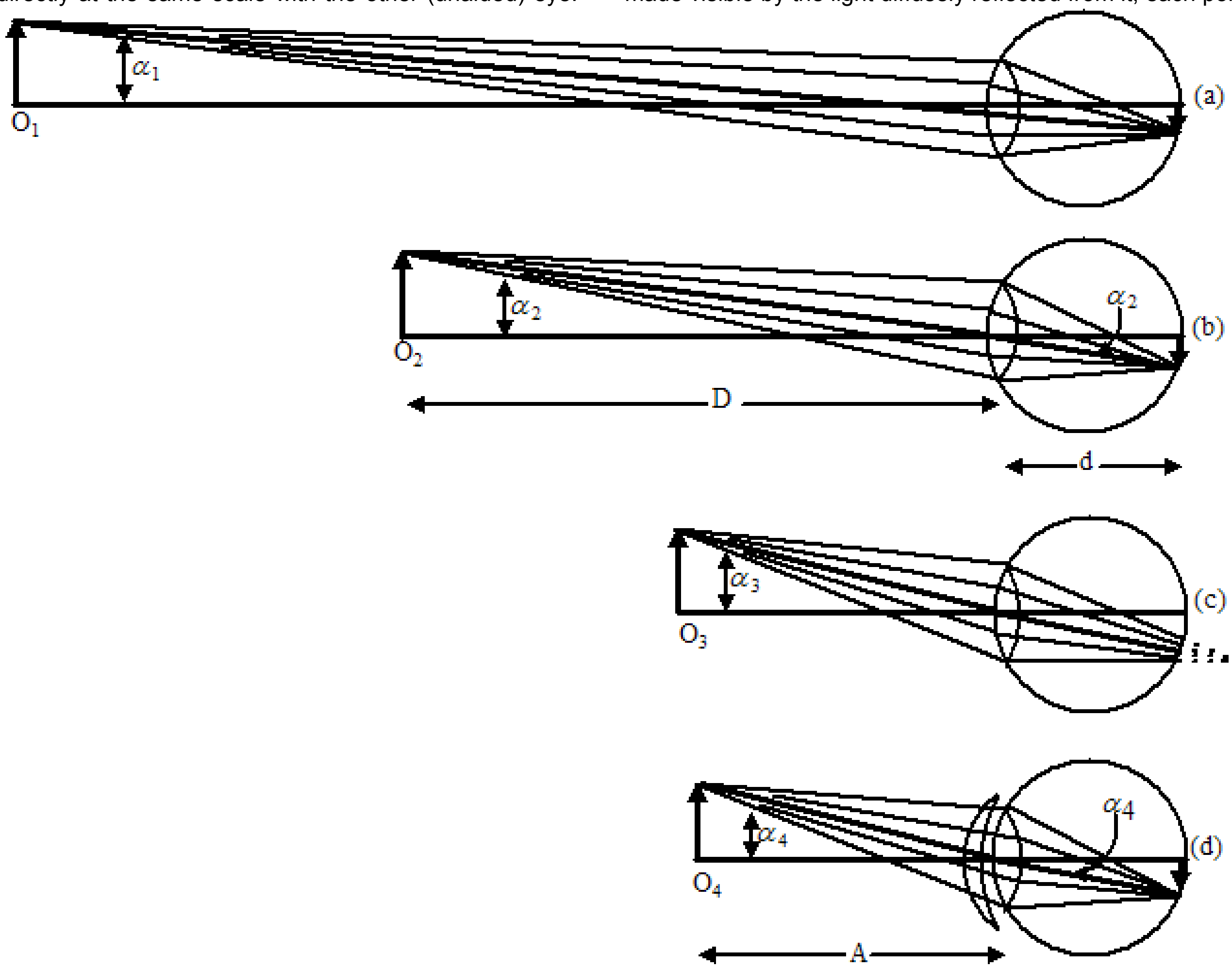

Fig. 1. As an object is brought closer to the eye, the image on the retina increases in size.

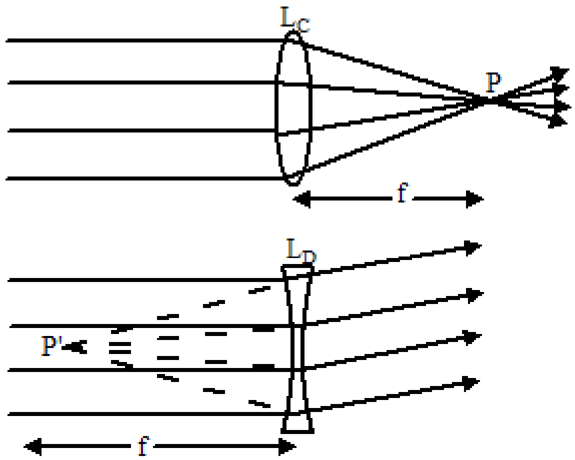

Fig. 2. The focal length of a lens may be either positive or negative.

on the body serves as a point source of light and the light waves are radiated in all directions from it. When the object is viewed, the lens of the eye takes a portion of these light rays and brings them to a focus on the retina. Obviously, all of the rays used by the eye lie within a cone and the size of this cone is determined by the size of the pupil. Figure la shows how the light from one point (the tip of the arrow) on the object 0_1 is brought to a focus on the retina. Of course, the light from each point on the object will be brought to a focus at a different point, bright spots on the object will be represented by bright spots on the retina, and the object will be reproduced point-by-point. In other words, *an image of the object is formed on the retina of the eye*.

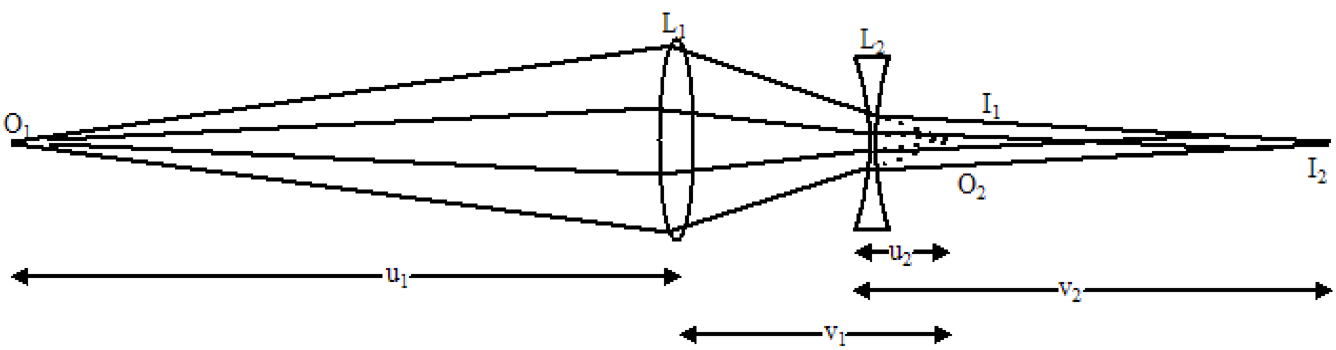

Fig. 3. The real image I_1 of lens L_1 is the virtual object O_2 of lens L_2.

In using any kind of viewing instrument (magnifying glass, telescope or microscope), the eye is an important part of the optical system, and it is important to appreciate the fact that the ability to see detail in a particular object depends upon the size of the image on the retina. It is seen in Fig. 1 that a central ray (the heavy line that goes through the center of the lens) is not deviated and that the size of the image is equal to *d* tan *a*, where *d* is the diameter of the eyeball and a is the angle subtended by either the object or the retinal image. As the object is brought from position 0_1 to position 0_2, the size of the image increases. If the object is brought still closer to the eye, position 0_3, the image is further enlarged, but now it is so close to the observer that the lens is not able to bring the rays of light to a focus on the retina. If 0_2 in Fig. 1 (b) represents the closest object for which the eye is able to form a sharp image on the retina, the distance *D*, from the object to the lens of the eye, is (for obvious reasons) called the *distance of most distinct vision*. For a normal eye *D* is about 25cm or 10in.

Although the *unaided* eye is not able to see distinctly an object at position 0_3, with the help of a converging lens shown in Fig. 1 (d), the object may be brought to a focus. In other words, the purpose of the magnifying glass is to permit the observer to bring the object closer to the eye. When used in this way, the lens is called a "magnifying glass" or a "simple microscope." To simplify the discussion, the lens in Fig. 1 (d) is shown extremely close to the eye but in practice, the lens-although always held close to the eye-is not as close as shown in the illustration. *The magnification M produced by the lens is the ratio of the size of the retinal image with the lens to the size of the largest image possible with the unaided eye.* Obviously

$$M = \frac{\tan a_4}{\tan a_2} = \frac{D}{A} \qquad (1)$$

where *A* is the distance from 0_4 to the eye. *

*Since the angles are small the magnification may be defined either as the ratio of the angles a_4/a_2 or the ratio of their tangents $\tan a_4/\tan a_2$.

What is the best value of *A*? For a particular magnifying glass where should the object be placed? The answer depends upon whether the observer is more interested in the highest possible magnification or in freedom from eyestrain.

For the maximum magnification, the object is so placed that the virtual image formed by the magnifying glass is at a distance *D* (the distance of most distinct vision) from the eye. If, however, the object is placed at the principal focus of the magnifying glass, the image formed by it is an infinite distance away (the light rays reaching the eye are parallel) and it may be seen with the eyes relaxed. The retinal image is not quite as large as in the previous case, but it frequently happens that the observer is willing to make a slight sacrifice in magnification for the sake of comfort. In this case the distance A is equal to the focal length f of the magnifying glass and Eq. (1) becomes

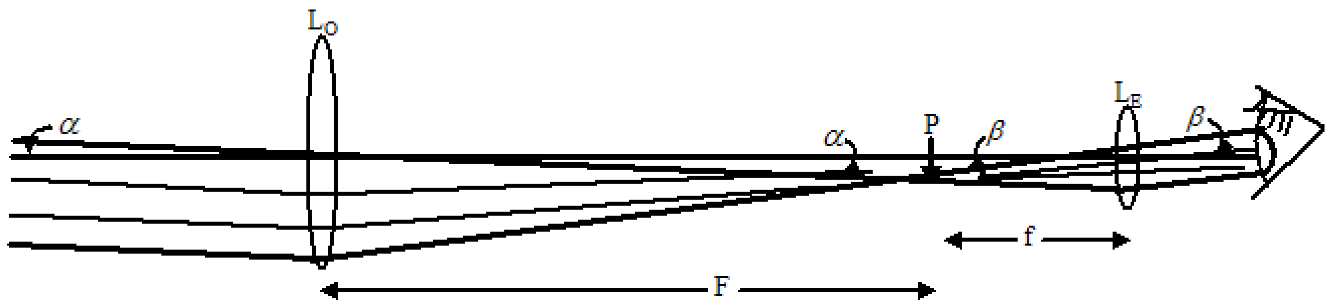

Fig. 4. The astronomical telescope is constructed of a long-focus objective and a short-focus eyepiece.

$$M = \frac{D}{f} \qquad (2)$$

Since a telescope is essentially a system of lenses, it is advisable to start the study of telescopes with a discussion of the lens equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad (3)$$

Although this equation is written with positive signs throughout, it should be remembered that when numbers are substituted for the symbols, these numbers may be -either positive or negative. For example, since parallel rays of light are brought to a real focus (P, Fig. 2) by a converging lens L_C, the lens is said to be positive and the focal length *f* (from *p* to the lens) is a positive number. On the other hand, the diverging (negative) lens L_D causes parallel rays to diverge as if they had originated at some point P' and the focal length (from P' to the lens) is a negative number.

As was pointed out earlier, light radiates in all directions from each point of a material body. If, therefore, this body is used as an object for a lens, the light falling on the lens consists of diverging rays. Consequently, if the incident light is diverging, the object is said to be real and the object distance *u* is positive. Conversely, if the incident light is converging, the object is virtual and *u* is negative. On the other hand, if the light that emerges from the lens is converging, a real image is formed, and the image distance *v* is positive. If the emergent light is diverging, the image is virtual and *v* is negative. For example, suppose the converging lens L_1 forms a real image I_1 of the real object O_1, as shown in Fig. 3. The numerical values of both u_1 and v_1 are positive. If the diverging lens L_2 is interposed, the real image I_1 is a virtual object O_2 for the lens L_2 and the numerical value of u_2 is negative. If the rays of light from L_2 are brought to a real focus at I_2, the image distance v_2 is, of course, positive.

An astronomical telescope is shown schematically in Fig. 4. In its simplest form this telescope consists of a pair of converging lenses, a long-focus objective lens L_o and a short focus lens L_E, called the eyepiece, or ocular. Assume that the object being viewed is far to the left so that the rays coming from it are essentially parallel and that the angle subtended by this object is *a*. Obviously, a real image P of this distant object will be formed a distance *F* to the right of L_o, where *F* is the focal length of the objective lens. The lens L_E serves as a magnifying glass to view P and for a relaxed eye the distance from P to L_E is equal to the focal length *f* of the eyepiece. Without the telescope the size of the retinal image is *d* tan *a*, (where, as before, *d* is the diameter of the eyeball) but when the telescope is used the size is $d \tan \beta$ and the magnification is $\tan \beta / \tan a$. If the length of P is called *s*, it follows that $\tan a = s/F, \tan \beta = s/f$ and

$$M = F/f \qquad (4)$$

From Fig. 4 and from the discussion above it should be clear that, for the astronomical telescope, the following statements are true: For large magnification the focal length of the objective lens should be long and the focal length of the eyepiece should be short. For viewing distant objects the distance between the two lenses should be *F* + *f*. Since the

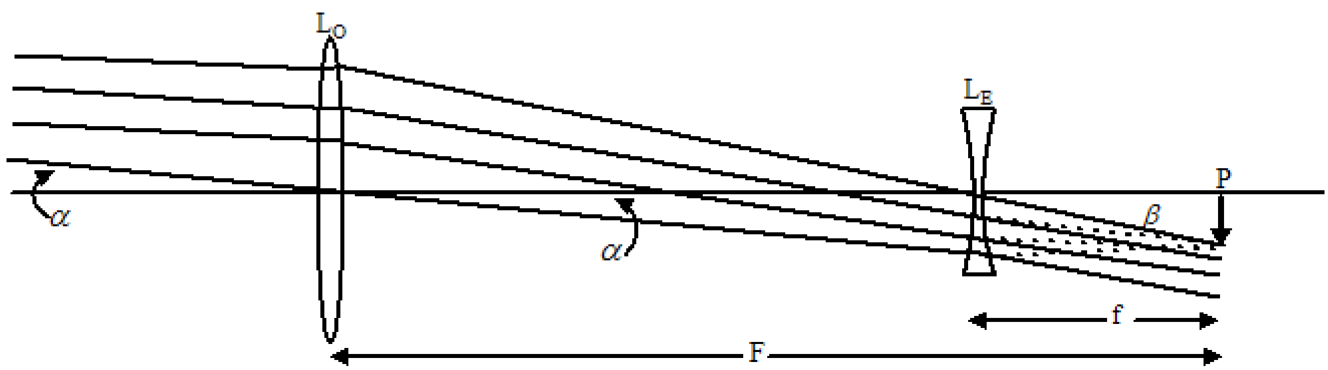

Fig. 5. The eyepiece of the Galilean telescope is a diverging lens.

body is seen inverted this particular telescope should be used for astronomical, rather than for terrestrial, observations.
A telescope may be constructed by combining a converging lens L_o and a diverging lens L_E as shown in Fig. 5. Since this type of telescope was first constructed by Galileo (1609) it is called a Galilean telescope. An opera glass is a Galilean telescope.
If a distant object subtends an angle *a* at the telescope (Fig. 5) the light rays from this object are essentially parallel and, if these rays were not intercepted by L_E, a real image P would be formed at the principal focus of L_o. The distance from L_o to P is equal to the focal length *F* of the objective lens. If the ocular L_E is so placed that its principal focus coincides with P the rays are rendered parallel and the image formed by L_E is an infinite distance away. In other words, the real image P formed by L_o is a virtual object for L_E and, since this virtual object is at the principal focus, the final image is at an infinite distance. It should be clear from Fig. 5 that the lengths of the retinal images, with and without the telescope, are $d\tan\beta$ and $d\tan a$, respectively, and that the magnification *M* produced by the instrument is given by the equation

$$M = -\frac{F}{f} \qquad (5)$$

Since *f* is negative the magnification is, as it must always be, positive.
From Fig. 5 and from the discussion above it should be clear that, for a Galilean telescope, the following statements are true: For large magnification the focal length of the objective lens should be long and the focal length of the eyepiece should be short. For viewing distant objects the distance between the two lenses should be equal to the *algebraic* sum of their focal lengths. Since the image formed by L_E is erect, this telescope may be used for terrestrial observations.

APPARATUS: A long-focus converging lens, a short-focus converging lens, a short-focus diverging lens, a white screen, supports for mounting each of these items on a meter stick, and access to amounted scale are required.
A simple inexpensive optical bench, showing the method of mounting lens and screen, is shown in Fig. 6.

PROCEDURE: Place the long-focus lens and the screen on the optical bench as shown in Fig. 6, point the apparatus toward some distant object (for example, a distant building or electric lamp) and adjust the position of the screen so that a sharp image of the distant object is formed on it. Record the position of both lens and screen and determine the focal length *F* of the lens. In a similar manner determine the focal length *f* of the short focus converging lens. Place the long focus lens L_1 near one end of the optical bench and adjust the position of the screen so that the image I_1 of an electric lamp (a few meters away) is in sharp focus on it. After recording the position of the screen, move it farther (20cm or more) away from L_1 and record its new position. Interpose the diverging lens L_2, as shown in Fig. 3, and adjust its position so that a sharp image is again formed on the screen. Note the position of L_2. From the position of L_2 and

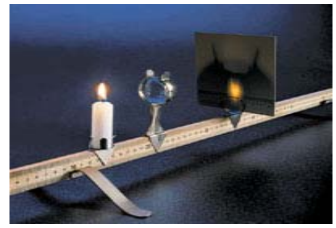

Fig. 6. One form of optical bench.

the two positions of the screen compute, with the aid of Eq. (3), the focal length of the diverging lens.
Arrange the lenses on the meter stick, as shown in Fig. 4, to form an astronomical telescope. Look through this telescope with one eye and along it with the other (unaided) eye at a scale on a distant wall. So adjust the position of the eyepiece that the scale seen through the telescope and the scale seen by the unaided eye appear equally distant. By comparing the relative sizes of the scales as seen by the unaided eye and through the telescope determine the magnification of the telescope.

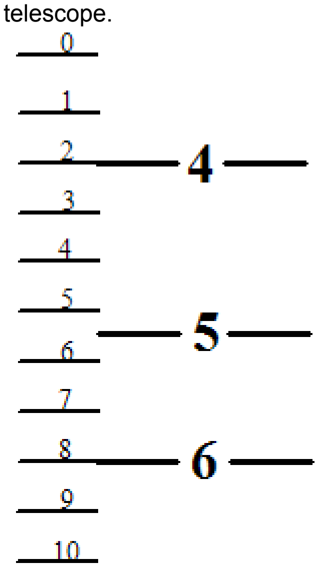

Fig. 7. The magnified scale is compared with the unmagnified scale.

For example, if the two images should appear as shown in Fig. 7, the magnification is 3.5. Compare this magnification with that computed from Eq. (4). Determine the distance apart of the two lenses and compare this distance with the sum of the focal lengths of the lenses.
In a similar manner, determine the magnification of a Galilean telescope and compare this result with that computed from Eq. (5). Compare the distance apart of the lenses with the algebraic sum of their focal lengths.

QUESTIONS: 1. A man sees a 2in x 8in brick, 200 yards away. What is the size of the image on the retina? Although the outside diameter of the human eye is 24mm, the effective diameter (from the optical center of the lens to the retina) is 17mm.
2. Make a diagram to illustrate the case where a virtual image of one lens is a real object of a second lens.

3. Show that the maximum magnification of a magnifying glass is $1 + D/f$.

4. An astronomical telescope and a Galilean telescope have identical objective lenses and each has a magnification of three. Show that the astronomical telescope is twice as long.

5 A small hole punched in a card held close to the eye may be used as a magnifier. Explain.

6. Although prism binoculars contain converging lenses only, objects viewed with them are seen erect. Explain how this is accomplished.

7. A magnifying glass (focal length f) is held a distance x from the eye. Show that the maximum magnification possible with this glass is $1 + (D - x)/f$.