CENCO Physics Selective Experiments In Physics

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REFRACTION THROUGH A SINGLE THIN LENS

OBJECT: To determine the constants (radii of curvature and refractive index) of a single thin lens, and to study the phenomena of chromatic and spherical aberration and astigmatism.

METHOD: A thin lens mounted on an optical bench is used to form an image of an object. From the observed conjugate distances, the focal length of the lens is determined. The index of refraction of the lens material is computed from the focal length and radii of curvature of the lens surfaces. These studies are made for various wavelengths. Chromatic and spherical aberration and the astigmatism of the lens are observed by examining the images formed by the lens through a magnifier.

THEORY:

Definitions and Notation: A spherical lens is a piece of transparent substance having two opposite regular surfaces. These surfaces may both be spherical or one spherical and the other plane.

If the lens is thicker in the center than at its edge, it is called a *convex* lens (the three types are double convex, plano-convex and meniscus convex). If it is thinner at its center than at its edge, it is called a *concave* lens (the three types are double concave, plano-concave and meniscus concave). The line joining the centers of curvature of the two spherical surfaces forming the lens is called the *principal* axis of the lens. In case one surface of the lens is plane, the principal axis is perpendicular to this surface and passes through the center of curvature of the other surface. A *thin lens* is one whose thickness may be neglected in comparison with its focal length. The action of the lens consists in changing the curvature of the waves which pass through it. This is brought about by the fact that the speed VG with which the light travels in the lens is (for glass) less than the speed V_A in air. The speed ratio $V_A/V_G = n$, where n is called the relative refractive index of the lens material with reference to air. The refractive index is a constant for a particular color or wavelength of light. The two other constants which determine the action of a lens are its radii of curvature r_1 and r_2, since these determine the lag (or gain) of the center of the wave front compared with the edge while passing through the lens.

Two points so situated that an object placed at one of them has its image formed at the other are called *conjugate points*, or *conjugate foci*. The distances of these points from the lens are called conjugate distances. The distance from the image to the lens is called the image distance v; the distance from the object to the lens is called the object distance u.

Derivation of the Lens Formula: The formula relating the conjugate distances and the constants of a single thin lens is easily obtained by using the wave fronts. First, however, a geometric relationship should be established between the length of chord, the radius of an associated arc, and maximum radial distances from the chord to the arc. This latter distance is called the sagitta of the arc.

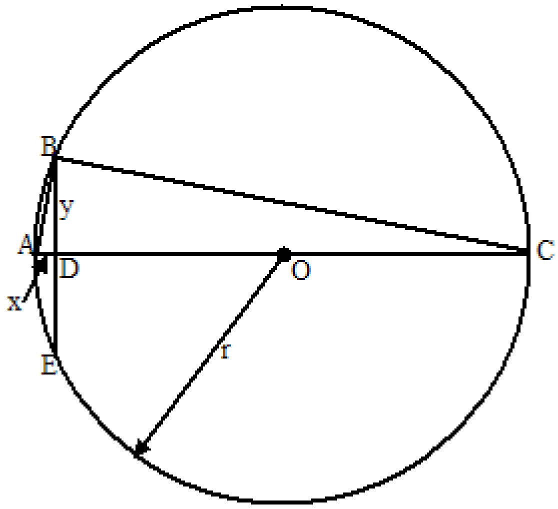

Fig. 1. AD = x is the sagitta of the arc BAE.

In Fig. 1 O is the center of a circle of radius r. A half chord y is dropped to the diameter AC from a point B near A, cutting off a small segment x. Since the triangles ABD and BCD are similar,

$$x/y = y/(2r - x) \tag{1}$$

If now x is small compared with the radius of the curve, x^2 may be neglected in comparison with $2rx$ and

$$x = (y^2/2)(1/r) \tag{2}$$

Hence the curvature l/*r* is proportional to x, the sagitta of the arc BAE.

In order to derive the relation between the conjugate distances of a lens, its refractive index and its radii of curvature, a diverging spherical wave front will be considered to fall upon a thin double convex lens and to be brought to convergence after refraction. By setting up a relation between the sagittas of the incident and emergent

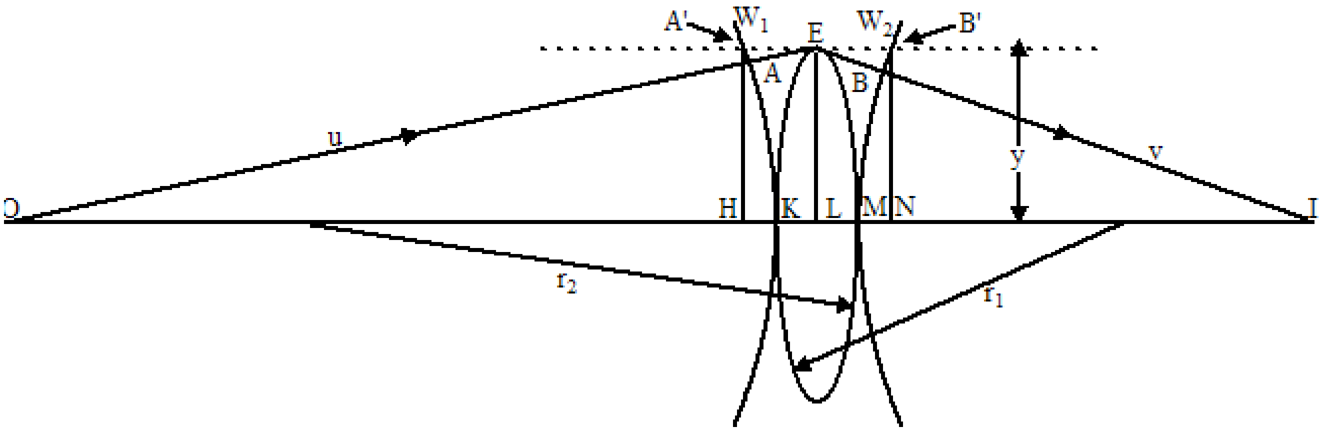

Fig. 2. Illustration of application of the sagittal formula in deriving the lens equation.

wave fronts and of the two surfaces of the lens and applying Eq. (3), the desired expression can be found.

Consider the light to issue from a source at O in Fig. 2 on the principal axis OI of the converging lens, which produces an image of the source at I. The lens has surfaces of radii r_1 and r_2, as shown, and the distance from its axis to its edge E is represented as y. The incident and emergent wave fronts W_1 and W_2 are shown just touching the lens surface at K and M, and extreme rays OE and EI are drawn to the edge of the lens, intersecting these wave fronts at A and B respectively.

The wave advances from all points on the wave front W_1 to the corresponding points on the wave front W_2 in the same time. Consequently, at the edge of the lens the distance AEB (in air) is traversed in the same time t that is required at the center to traverse KM in the lens of refractive index n,

$$t = AEB/V_A = KM/V_G \qquad (4)$$

where V_A and V_G represent the speed of light in air and glass respectively. Since the refractive index

$$n = V_A/V_G \qquad (5)$$

it is seen that

$$AEB = nKM \qquad (6)$$

On the assumption that the edge rays make small angles with the principal axis, the points A' and B' on the wave fronts at a distance y from the axis can be substituted for A and B without appreciable error.

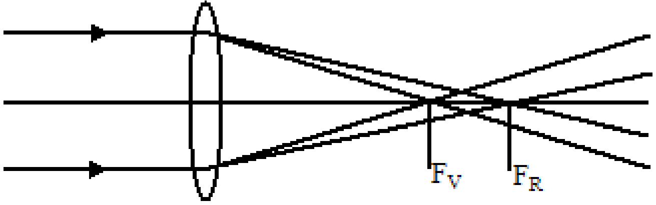

Fig. 3. Chromatic aberration.

The lens surface and the wave fronts can thus be represented by sagittas having the same half-chord y.

Since $A'E + EB' = nKM$, it is found from the corresponding distance along the axis that

$$HK + KL + LM + MN = n(KL + LM) \qquad (7)$$

and from this,

$$HK + MN = (n-1)(KL + LM) \qquad (8)$$

If now each of these sagitta is expressed in terms of half-chord and radius in accordance with Eq. (3),

$$y^2/2u + y^2/2v = (n-1)(y^2/2r_1 + y/2r_2) \qquad (9)$$

Dividing through by the common factor $y^2/2$, the relationship between the conjugate distances and the constants of the lens becomes

$$1/u + 1/v = (n-1)(1/r_1 + 1/r_2) \qquad (10)$$

When the object distance is infinitely great, the image will be at the second principal focus. That is, if $u = \infty$, then $v = f$ and

$$1/f = (n-1)(1/r_1 + 1/r_2) \qquad (11)$$

If, however, the image distance is infinitely great, the object will then be at the first principal focus-that is, $v = \infty$ and $u = f$. This yields the same equation as Eq. (11). This means that for a glass lens in air the two principal foci are equidistant from the center of the lens. Since the thickness of the lens is being neglected, the measurements of u, v and f should be taken to the center of a symmetrical lens. When, however, a plano-convex, plano-concave or a meniscus lens is used, the results are slightly more exact if the measurements are taken to convex or concave surface in these cases.

Chromatic Aberration: The lens formula has been derived on the assumption that the refractive index n is a constant. If, however, the color or wavelength of the light used is varied, the focal length will change with the change in the refractive index. Since the refractive index is greater for the violet end of the spectrum than for the red end, it follows from the lens formula that the focal length of the lens will be less for violet light than for red (Fig. 3)- that is, the focal point F_v is nearer to the lens than F_R. This aspect of dispersion in the lens is called chromatic aberration.

Spherical Aberration: When rays of light parallel to the principal axis of a lens pass through zones near its outer edge, it is found that they are not brought to a focus at exactly the same point as that at which rays meet that have passed through the center of the lens. This defect is called *spherical aberration*. It is not due to any irregularity in the spherical surface of the lens, but occurs whenever the incident or emergent rays-or both-make appreciable angles with the lens faces. The effect of spherical aberration is to produce a blurred image of the source. As the rays come from a point source; a position where the image is blurred the least is called the circle of *least confusion*. The amount of spherical aberration produced by a lens is usually measured by the axial distance F_MF_C between the intersections of the central and marginal rays (Fig. 4).

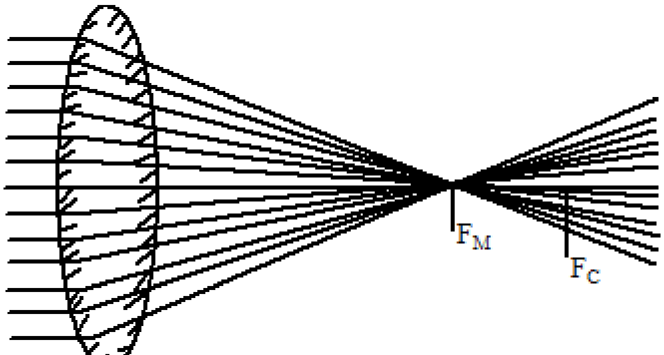

Fig. 4. Spherical aberration.

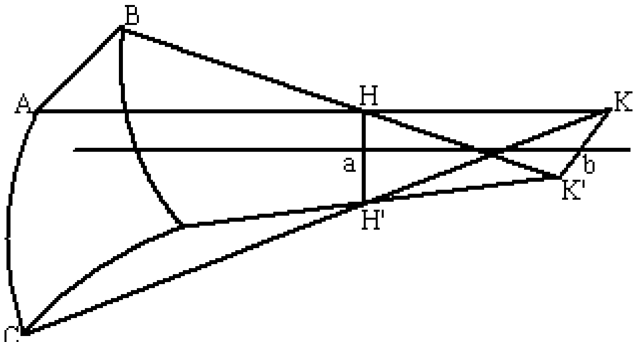

Fig. 5. Focal lines HH' and KK' produced by astigmatism; ab the astigmatic difference.

Astigmatism: While spherical and chromatic aberration are perhaps the most important defects of spherical lenses, there are several other defects which cause loss of definition. One of these is due to the fact that rays of light which pass through the lens obliquely from an object point off the principal axis do not converge upon a common image point. The curvature of the wave emerging from the lens in this case is different in different directions, resulting in the formation of a line image due to the curvature AB and a second line image due to the different curvature AC (Fig. 5). The amount of astigmatism of a lens or mirror for any object point is indicated by the distance between its focal lines as measured along the middle rays from that point.

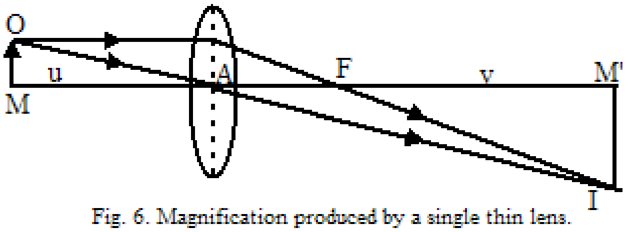

Fig. 6. Magnification produced by a single thin lens.

Magnification: The magnification produced by a lens is the ratio of image size to object size. From Fig. 6 it will be seen that the triangles OAM and M' AI are similar. Hence IM'/OM , the magnification *m,* = *v /u*.

APPARATUS: An optical bench as shown in Fig. 7 together with an extra carriage, a single thin lens, a spherometer, an illuminated object screen and holder (Fig. 8), a hooded screen (Fig. 9), red and blue light filters, image screen with eyepiece, and a circular aperture or iris diaphragm are required.

PROCEDURE:

Experimental: **Constants of the Lens**: Mount the illuminated object, the thin lens and the hooded screen at A, B and C respectively on the optical bench as shown in Fig. 10. Move the hooded screen to the right until the image is sharply defined on the screen. Measure the distances of the object and image from the lens. Next move the hooded screen farther to the right and beyond the position of best definition and approach the image position again, moving the screen toward the lens. Again measure the conjugate distances. The average of the two values obtained for the image distance *v* with the value u constitutes one pair of conjugate distances for the determination of the focal length.

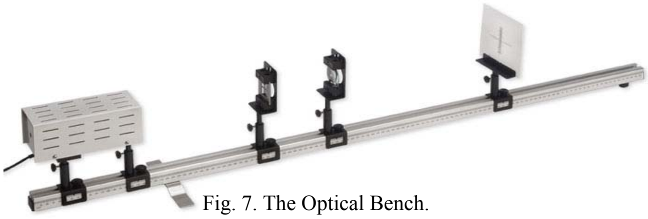

Fig. 7. The Optical Bench.

Determine a second pair of conjugate distances at which *u* and *v* are about equal, and a third pair in which the image distance *v* is larger than the object distance *u*.

Take the necessary measurements with the spherometer for calculation of the radii of curvature of the lens surfaces.

Chromatic Aberration: Remove the hooded screen and replace with the image screen with eyepiece (Fig. 11). Since the focal length of the lens depends on the index of refraction of the glass, which varies with different wavelengths, it is evident that the value determined above will be for the brighter portion of the spectrum of the light used. In order to determine *f* for two fairly different regions of the spectrum, first insert a piece of red glass in the extra slot of the object holder and determine a pair of conjugate foci using the combination image screen and eyepiece. The setting is made by first focusing the eyepiece on the wire screen, then subsequently adjusting the assembly until no

Fig. 8. Object Screen (a) for use in Holder (b).

parallax obtains between the image and image screen as viewed through the eyepiece. In this case the distance *v* should be larger than the distance *u.* Without changing the object distance *u*, replace the red glass with a piece of blue glass and determine a pair of conjugate distances for blue light. As neither of the light filters transmits pure colors, it may not be possible to obtain precise setting for the two image positions.

Fig. 9. Hooded Screen

Spherical Aberration: In order to avoid confusing spherical aberration with chromatic aberration, the observations must be taken with light which is fairly monochromatic. This is best secured by using the red glass, as it transmits a fairly narrow region of the spectrum. The same arrangement of apparatus as used in the previous section may be used with the preferred position of the lens such as to make the image distance considerably larger than the object distance.

Now place a circular aperture or an iris diaphragm (Fig. 12) immediately in front of and to the right of the lens (shown at B in Fig. 10) and adjust the aperture to about 1cm in diameter. The center of the aperture should be on the optic axis of the lens. Using the combined image screen and eyepiece, locate the position of the image and measure the conjugate distances. Raise or lower the aperture until it is near one edge of the lens so that the light which forms the image will pass through the outer position of the lens. As before, locate the position of the image with the combined image screen and eyepiece and record the image distance. The object distance should be the same in both cases.

Astigmatism: To study astigmatism, arrange the apparatus as in the previous section. Filter the light with the aid of the red glass and place the circular aperture or iris diaphragm immediately in front of the lens. The center of the aperture should be on the optic axis of the lens. To facilitate the location of the image points, the aperture should not be larger than 1cm. Locate the position of the image with the combined image screen and eyepiece by testing for parallax between the image and image screen. Now rotate the lens holder in its support approximately 45° about a vertical axis. The wave front emerging from the lens in this position will be astigmatic and the definite focus previously obtained will disappear. Each point of the object will now have two line images but no point image as before. Since the object consists of vertical and horizontal lines, there will be one position of the image screen at which vertical lines are in focus and another at which horizontal lines are in focus. Locate both of these positions for some particular value of the object distance *u*.

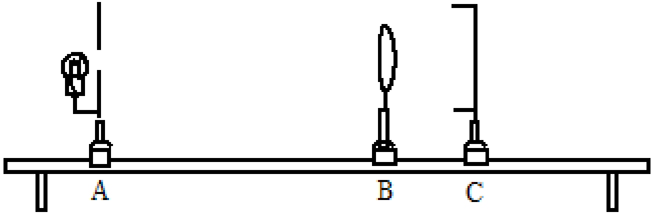

Fig. 10. Schematic diagram of apparatus for measuring the constants of the lens.

Interpretation of Data: From the data calculate the radii of curvature of the lens surfaces, the focal length of the lens and its refractive index for red, blue and white light. Calculate also a measure of the spherical and chromatic aberrations.

Fig. 11. Combination Image Screen and Eyepiece

QUESTIONS: 1. An object is placed so that the image formed by a lens whose focal length is 30cm is real and 120cm from the lens. What is the distance of the object from the lens?

2. An inverted image three times as large as the object is formed by a lens having a focal length of 30cm. How far from the lens is (a) the object? (b) the image?

3. A lens has a convex surface whose radii are 30cm and 60cm. If it is made of glass which has an index of 1.57 for red and 1.61 for blue light, what must be the measure of the chromatic aberration?

4. A converging lens made of quartz has one convex and one concave surface. Compute the focal length of such a lens that has radii of 24 and 40cm. Take the refractive index of quartz as 1.543.

5. A reading glass constructed with convex spherical surfaces each of 26cm radius is found to have a focal length of 25cm. Determine the refractive index of the glass of which it is made.

6. A pair of parallel walls are 8ft apart. Where must a lens having a focal length of 18in. be placed so that objects on one wall are brought to a focus on the opposite wall?

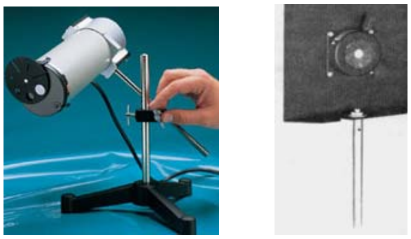

Fig. 12. (a) Circular Aperture: (b) Iris Diaphragm