CENCO Physics
Selective Experiments In Physics

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REFRACTION OF LIGHT AT A PLANE SURFACE
(Index of Refraction: Plate and Prism)

OBJECT: To determine the index of refraction of glass b observing directly the change in direction of light as it passes obliquely between glass and air, and to determine the index of refraction of a transparent prism by measuring the angle of minimum deviation for light passing through the prism.

METHOD: A rectangular glass plate with polished sides is placed on a sheet of paper tacked on a drawing board. A pin is inserted in the board next to one of the sides for sighting through the glass plate. By placing two pins in the line of sight, it is possible to determine the direction taker by the light that emerges from the plate; when the plate is removed, the complete path of the ray can be drawn. By measuring the angle of incidence and the angle of refraction for this ray and applying Snell's law, we can calculate the index of refraction of the glass. The direction taken by a ray of light that originates at a distance and that passes obliquely through the plate is also ascertained by sighting with pins. Finally, pins are used to determine the path of a ray of light that passes through a prism. Then, the extent that ray is deviated from its original direction is measured. The index of refraction of the glass of which the prism is made is calculated from the observed deviation when the prism is oriented at the angle of minimum deviation.

THEORY: When a stick is thrust into a pond of water, the stick appears to an observer on shore to be bent at the surface of the water. Also, the pond appears to be less deep than it actually is. These phenomena, observed by man for thousands of years, are due to the fact that light changes direction as it goes obliquely from water into air or, for that matter, from air into water. More generally, a change in direction occurs when light travels obliquely between any pair of transparent materials in which the speed of light is different.

Consider Fig. 1 in which a ray of light is shown to change direction as it goes from one transparent medium into another. Angles θ_1 and θ_2 are the angles the ray makes with the normal to the interface in the respective media; these are called the angle of incidence and the angle of refraction, respectively. It should be noted that a ray traveling in the opposite direction follows the same path in reverse.

For a given wavelength of light and for a given pair of transparent materials it was empirically discovered that the ratio of sin θ_1 to sin θ_2 is a constant. That is, the ratio has the same value for a given pair of materials regardless of the angle of incidence. This ratio is called the *relative index of refraction*. It is conventionally assigned the symbol n_{21}

$$n_{21} = \frac{\sin\theta_1}{\sin\theta_2} \qquad (1)$$

This relationship is known as Snell's law in honor of Willebrord Snell (1591-1626).

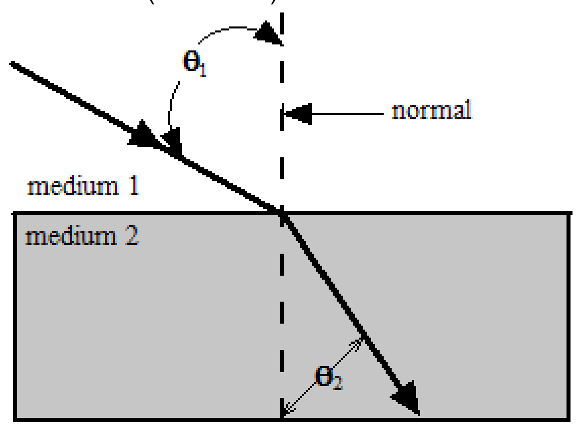

Fig. 1. A ray of light changes direction (is refracted) when it passes obliquely from one transparent medium into another.

The value of the ratio sin θ_1/sin θ_2 when the first medium is a vacuum is called the *absolute index of refraction* of the material of the second medium- or, simply, its index of refraction, and it is assigned the symbol *n*, either with or without an identifying subscript. For practical purposes, the first medium can be air instead of a vacuum, for the difference in the resulting ratio is only about 3 parts in 10,000.

The fact that the ratio sin θ_1/sin θ_2 is a constant for a given pair of transparent materials follows from the wave theory of light. Consider Fig. 2 in which a broad parallel beam of light falls on the plane surface between two transparent materials which differ in the sense that light travels slower in the second material than in the first. Line *OP* is drawn perpendicular to the direction of travel of the beam in the first medium and, thus, represents the intersection of a wave front with the plane of the paper. (By definition, a *wave front* is a surface all points of which are in the same phase of vibration.) According to Huygens' assumption, each point on a wave front is the source of new waves, and the envelope of these from all points gives a new wave front an instant later. Since light travels slower in the second

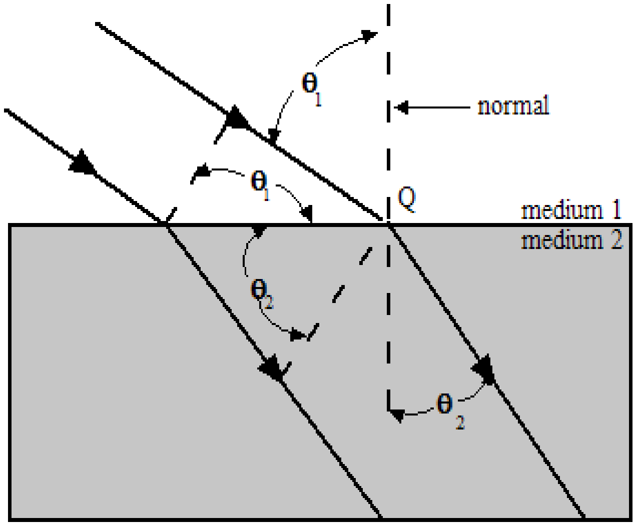

Fig. 2. A broad parallel beam of light passes from one transparent medium into another in which the speed of light is lower than in the first medium. Analysis of the figure enables one to derive Snell's law by applying the wave theory of light.

medium than in the first, by the time the disturbance from *P* reaches the surface at point *Q* the disturbance from *O* will have traveled the lesser distance *OR* into the second medium. The new wave front at that instant will then be represented by the line *RQ* and, since the wave front and its direction of travel are mutually perpendicular, the beam will have changed direction. It will have been refracted toward the normal: angle θ_2 is less than angle θ_1. Note that triangles *OQP* and *OQR* are right angle triangles, that they have a common hypotenuse and that the acute angles *POQ* and *RQO* are equal to θ_1 and θ_2 respectively. Since sin θ_1 = *PQ/OQ* and sin θ_2 = *RO/OQ*, the ratio of these two equations is given by

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{PQ}{RO}$$

Now, since *PQ* and *RO* are distances traveled in the same time, their ratio is equal to the ratio of the speeds of light in the two media. Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \qquad (2)$$

where v_1 is the speed of light in the first medium and v_2 is the speed of light in the second medium. Since v_1 and v_2 are constants their ratio is a constant, in agreement with Snell's law.

If the first medium in Fig. 2 is a vacuum, the ratio is, by definition, the index of refraction of the second medium *n*. The index of refraction of any substance is, therefore, also given by the equation

$$n = c/v \qquad (3)$$

where *v* is the speed of light in the material and *c* is the speed of light in vacuum. (c = 3.00 x 10^8m/sec, to three significant figures.)

It is now apparent that the ratio of the individual indices of refraction 0f two transparent media is equal to the in. verse ratio of the speeds of light in the respective media. For, by Eq. (3), the individual indices have the values $n_1 = c/v_1$ and $n_2 = c/v_2$. Their ratio is thus given by the equation

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} \qquad (4)$$

Furthermore by Eq. (2), when a ray of light passes between these two media

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \qquad (5)$$

This equation can alternatively be written

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad (6)$$

When abeam of light passes obliquely through a plate of glass or any other transparent substance that has parallel sides, the beam that emerges will be parallel to the incident beam, as shown in Fig. 3. This is to be demonstrated in the present experiment. The analytical proof is left for a problem at the end of the experiment.

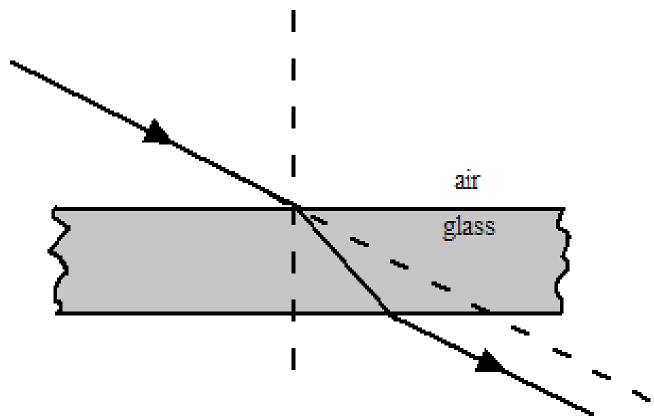

Fig. 3. A ray of light passing through a plate of glass which has parallel sides emerges parallel to its initial direction.

If the two sides of the glass are not parallel, as in the case of a lens or a prism, the beam that emerges is not parallel to the incident beam. The beam is deviated by an amount that depends on the angle between the two sides and upon the index of refraction of the material. Consider Fig. 4 where a beam passes *symmetrically* through a prism; that is, the beam that emerges makes an angle with the second surface that is the same as the angle between the incident beam and the normal to the first surface.

This angle is labeled θ_1 in the figure. The extent that the beam is deviated from its initial direction is indicated by angle *D* in the figure. *D* is the angle between the two dashed lines that depict the initial and the final directions of the beam. It can be shown that the deviation is least when the light passes through the prism symmetrically. Under these conditions, angle *D* is, therefore, called the angle of minimum deviation.

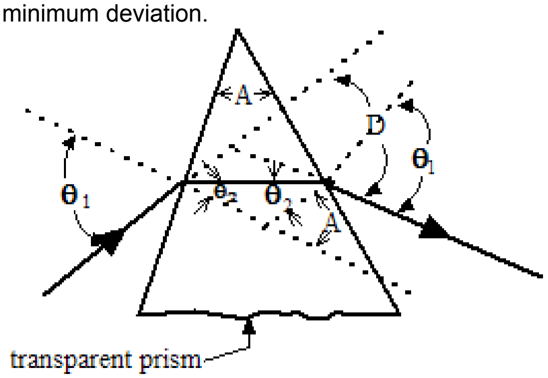

Fig. 4. When a ray of light passes through a prism, it is deviated from its initial direction. The angle of deviation is minimum when the ray passes through the prism symmetrically.

In deriving an expression for the index of refraction in terms of angle *A* of the prism and the angle *D* of minimum deviation, note that the angle between the normals to the two surfaces in Fig. 4 is equal to *A*. By geometry, angle *A* is, therefore, equal to 2 θ_2, where θ_2 is the angle made by the beam inside the prism with each of the normals. Thus, $\theta_2 = \frac{1}{2}A$. By a similar geometrical argument, angle *D* is equal to $2(\theta_1 - \theta_2)$. Thus

$$\theta_1 = \frac{1}{2}D + \theta_2 = \frac{1}{2}D + \frac{1}{2}A$$

By substituting the expressions for θ_1 and θ_2 in Snell's law,

$$n = \frac{\sin\frac{1}{2}(A+D)}{\sin\frac{1}{2}A} \qquad (7)$$

APPARATUS: Rectangular plate of glass that has two plane polished surfaces on opposite sides; glass prism with two plane-polished surfaces; pins; protractor and rule; drawing board.

PROCEDURE: *I. Index of Refraction of a Glass Plate.*
Attach a sheet of paper to a drawing board with thumbtacks. Draw a straight line across the center of the sheet of paper and place the glass plate upon the board with one of its polished sides along this line. Insert a pin vertically at each of three positions, such as at *A, B*, and *C* in Fig. 5.
These pins serve to maintain the orientation of the glass plate and enable one to restore the plate to its initial position in case the plate is moved. If pin A is located near one corner of the plate (as shown), it can be used for determining the index of refraction of the glass according to the method outlined below.

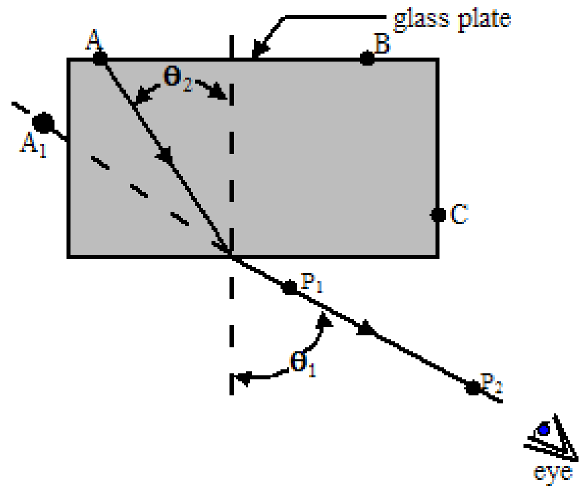

Fig. 5. Illustrating the experimental method of sighting by the use of pins to determine the index of refraction of a glass plate.

The bottom of pin *A* can be seen through the glass plate by sighting along a line that is near the surface of the paper. When the line of sight is oblique to the front surface of the glass, the bottom of the pin appears shifted- as at *A′* in Fig. 5- somewhat closer to and in a different direction than the pin itself. The path that the light follows in reaching the eye can be obtained by inserting vertically additional pins P_1 and P_2 so they are in line with image *A′*. After the plate has been removed from the board, the complete path of the ray can be drawn and a normal to the baseline can be erected where the ray crosses the front surface of the glass. The angles of incidence and refraction can be measured and used in Snell's law to attain the index of refraction of the glass.
Use this method to obtain the path of at least eight rays having various angles of incidence, ranging from a small angle up to the largest angle that will allow pin A to be seen through the glass. Be sure that pins A, B, and C remain securely in position so that the front edge of the glass remains accurately along the baseline. To speed up the process, make the eight sightings one after the other, without removing the plate from the board. Then, after the plate is removed, draw the paths of the several rays and their respective normals. The paths of the rays of light should be drawn as solid lines; normals should be dashed lines to distinguish them from rays. (A well- sharpened pencil should be used throughout.) Identify pinholes by small circles. Use a protractor to measure the several pairs of angles θ_1 and θ_2. Finally, prepare a table in which you enter in successive columns θ_1, θ_2, sin θ_1, sin θ_2 and the calculated ratio of sin θ_1 to sin θ_2. Does this ratio vary appreciably with the angle of incidence? Discuss reasons for observed variation. Obtain an average of the ratio in the last column and take this as the index of refraction of the glass. (If the calculated ratio at small angles differs appreciably from the rest of the entries, it should be discarded in obtaining this average. Why is it reasonable to do this?)

II. *The Path of a Beam of Light through a Glass Plate.* It is instructive to observe what happens to abeam of light as it passes obliquely through a plate of glass that has parallel

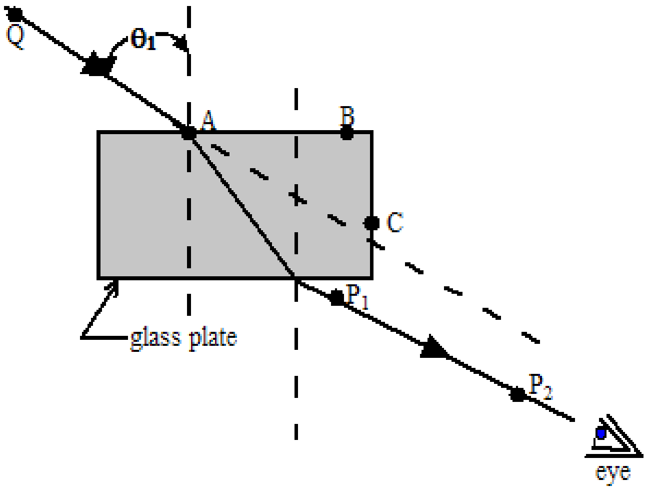

Fig. 6. Experimental method of demonstration that a ray of light incident at an oblique angle on a glass plate with parallel sides emerges in a direction parallel to the incident ray.

polished sides. Again, place the rectangular glass plate on a baseline drawn on a new sheet of paper and insert three pins, *A, B*, and *C*, as before. Insert a fourth pin *Q* at least 5 cm away from pin *A* and at such a point that a line joining *Q* with *A* makes an angle of at least 45° with the normal at the back surface, as in Fig. 6. This line is to be taken as the path of a ray of light that is incident at angle θ_1 at the back surface. The direction that the ray takes as it emerges is then determined by sighting with two additional pins P_1 and P_2 above. Note, however, that it is now necessary to line up these two pins with the image of *Q* as well as with the image of *A*- all four must be in the same line. When this has been done, remove the plate from the board and draw the complete path of the ray. Erect a normal where the ray crosses each surface and measure the angles of incidence and refraction at each surface. Use each pair of angles to calculate the index of refraction of the glass. Compare with results obtained in Part I.

Extend the line *QA* by means of a dashed line into the space on the other side of the plate. Is the emerging ray parallel to this line? When may the two lines not be parallel?

III. *The Path of Light through a Prism; Minimum Deviation*

The path of a ray of light through a prism can also be obtained by the use of pins, as shown in Fig. 7. Place a glass prism that has two polished sides near the center of a sheet of paper on a drawing board and insert two pins, Q_1 and Q_2 about 5cm apart to denote the path of the incident ray of light. The pins should be in such a position that line joining them would fall on one of the polished surfaces of the prism. Then, when looking through the prism along a line that is close to the sheet of paper, some points such as Q_1’ and Q_2’, respectively. Additional pins can then be inserted to obtain the direction of the light that emerges from the prism. However, before doing so, orient the prism for minimum deviation so that the experimental conditions will yield valid data for use in Eq. (7) can be used to calculate the index of refraction of the prism. The following procedure is recommended.

The apex of the prism is held in position by the fingers of one hand while the prism is slowly rotated about a vertical axis

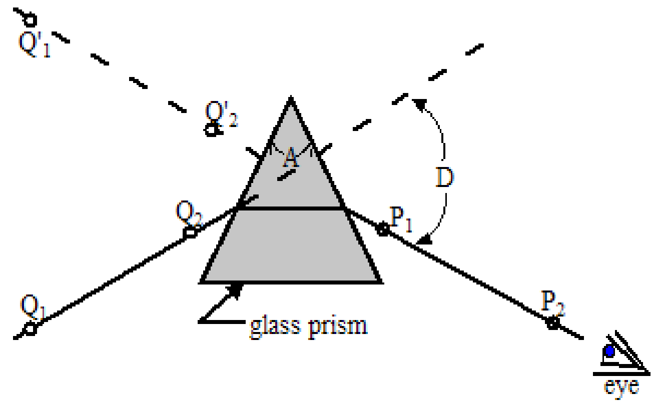

Fig. 7. Experimental method of sighting by the use of pins to determine the angle of minimum deviation for a ray of light that passes through a prism, for the purpose of calculating the index of refraction of the prism.

through the apex. It will be noticed, in general, that it will be necessary to move one’s head either to the right or to the left in order to keep the imagesQ_1’ and Q_2’ in line. The furthest excursion toward the right (for Fig. 7) gives the proper orientation for minimum deviation. When this condition has been obtained, pins P_1 and P_2 are inserted in line with images Q_1' and Q_2'. Before removing the prism, draw a line around it with a well-sharpened pencil for the purpose of measuring angle *A*. Repeat at least twice, with the prism and pins Q_1 and Q_2 on the same sheet of paper but at a different location. For each trial, draw a solid line to show the complete path of the beam of light through the prism. Draw a dashed line to show the initial direction of the beam and extend the emerging beam back to cross this line. Use a protractor to measure the deviation *D* for each trial; measure also angle *A* of the prism. Did the beam pass through the prism symmetrically in each case? If it did, average the three observed values of *D* to give the minimum angle of deviation for this prism. If it did not, use the measured value of D for the most symmetrical case as the minimum angle of deviation. Finally, calculate the index of refraction of the prism by using Eq. (7).

It is instructive to observe that the angle of deviation is significantly greater than the possible minimum when a ray of light passes through a prism in a distinctly non-symmetrical fashion. At one extreme, pins Q_1 and Q_2 can be inserted in such a position that a line joining them is nearly parallel, but not quite parallel, to the first surface of the prism. Try this and, by the use of additional pins, determine the deviation of a beam initially along line Q_1Q_2. Compare this deviation with that obtained for the angle of minimum deviation. Finally, erect a normal to each of the surfaces where the ray crosses that surface. Measure each pair of angles of incidence and refraction and use Snell's law to calculate the index of refraction of the glass from each pair of angles. Compare these values with that calculated from the minimum angle of deviation.

IV. *Index of Refraction of Lucite (Supplementary and Optional.)* If either a lucite block that has polished parallel sides or a lucite prism that has polished sides is available, the index of refraction of lucite can be determined by the methods outlined above.

PROBLEMS: 1. Water has an index of refraction of 1.33. The speed of light in air is 3.00 x 10^8m/sec. Calculate the speed of light in water.

2. A certain prism has an index of refraction of 1.60 and a prism angle of 60°. Calculate the angle of minimum deviation for this prism.

3. Given the prism of Problem 2, let abeam of light fall perpendicularly on the first surface of the prism. Draw a diagram and calculate the angle that the beam makes with the normal to the second surface. Try now to calculate the angle that the emerging beam makes with the normal to the second surface by applying Snell's law. Can the beam get out? (Look up the topic "Total Internal Reflection" in your textbook.)

4. Given a plate of glass of refractive index 1.5, let a ray of light fall on the first surface at an angle of 45°. Calculate the angle that the ray makes with the normal inside the glass.

5. Prove that if a ray of light passes obliquely through a plate of glass that has parallel polished sides, the direction of the emerging beam will be parallel to that of the incident beam.

6. Given a rectangular plate of glass with plane polished sides that are 10cm apart, let a ray of light fall on one of the polished sides at an angle of 45°. By what distance will the incident ray be displaced sideways if the index of refraction of the glass is 1.5?

7. In an early attempt to explain why a ray of light is deviated toward the normal when it enters a pond of water, it was suggested that the particles of light are attracted more by the water than by the air because water is denser than air. Explain what this would predict as to the speed of light in water compared to that in air. How does this agree with observed measurements?

8. A ray of light passes from air into glass. If the angle of incidence is 45°, what is the angle made with the normal in the glass? Use 1.33 and 1.50 as the indices of refraction of water and glass, respectively.

LIST OF CENCO EQUIPMENT

Cenco No.	
85540	Index of Refraction Plate (glass)
85541	Index of Refraction Plate (flint glass), optional
85525	Equilateral Prism (glass)
85336-1	Lucite Prism, optional
85336-2	Lucite Block, optional
72985-2	Protractor