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REFLECTION OF LIGHT – RAY TRACING

OBJECT: To study, by means of ray tracing, the fundamental laws relating to the reflection of light.

METHOD: A narrow beam of light from an intense straight-filament incandescent lamp falls on a sheet of paper placed on the platen of a ray tracing apparatus. The beam is reflected from a plane mirror surface. The resulting light path is studied to determine the law of reflection.
The principal focus of a curved mirror is graphically located by shifting the platen to obtain a set of parallel rays. A pattern of diverging rays, using the Diverging Ray Attachment, gives traces to study image formation for a nearby real object source. The mirror equation for the relation between image distance, object distance, and focal length is applied to the data obtained.

THEORY: When a beam of light in air strikes the surface of a material of different optical density, some of the light is usually reflected at the interface. The fraction of the light reflected depends on the optical conditions of the surface. The paper on which this experiment is printed reflects about 65 percent of the incident light. Polished silvered mirrors reflect over 90 percent of the light which falls upon them. If the surface is relatively rough compared to the wavelength of light, paper for example, the reflected rays diffuse or mix and, thus, no image of the light source is produced. A mirror surface is very smooth and gives specular reflections, that is, the rays are not diffused. Thus a mirror may focus the light from an object to form its image. Regardless of the surface, each ray is reflected in accordance with a simple law of reflection which has been known for many centuries. When alight ray, incident upon a surface, is reflected, the angle of incidence *i* and the angle of reflection *r* are equal, and they lie in the same plane; that is,

$$<i=<r \tag{1}$$

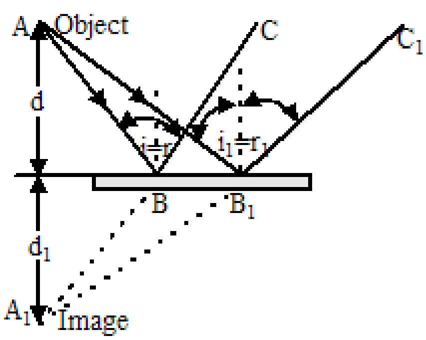

Fig. 1-Reflection of light by a plane mirror.

By convention, the angles are measured from a line perpendicular, or normal, to the surface at the point of reflection. Thus the ray *AB* in Fig. 1 from the source *A* will reflect from the mirror along the line *BC* with <*i* = <*r*. A group of such rays form the reflected beam within the rays *BC* and B_1C_1. When this reflected beam is viewed by the eye, the image A_1 appears to be the source. This image is called a virtual image since the rays do not originate or converge where the image appears to be. The virtual image is beyond (behind) the plane mirror in a position such that the line AA_1, from object to image, is normal to the mirror surface. The distance d_1 equals *d*.
The law of reflection may be applied to any reflecting surface. Figure 2 shows a ray construction which locates an

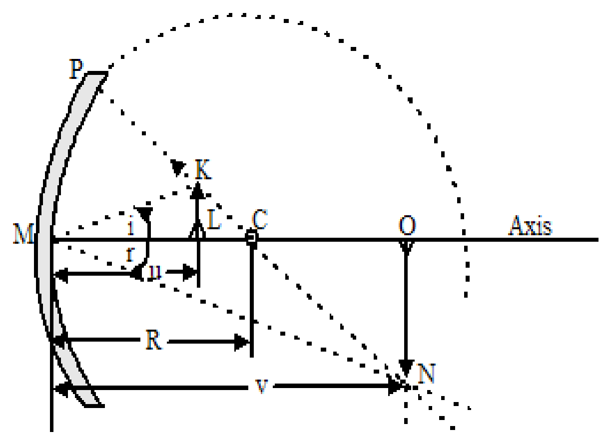

Fig. 2 - Image formed by a curved mirror.

image *ON* of an object *KL* placed in front of a concave mirror surface. The *center of curvature* of the mirror is at the point *C*. The ray *KP,* drawn along a radius from the head of the arrow object, strikes the mirror perpendicularly and, hence, reflects back through *C*. A second ray path *KM* is constructed so that once again <*i* = <*r*. These two rays intersect at point *N* to locate the image *ON*. Other rays from *K* would likewise locate the point *N*.

Derivation of Mirror equation. The triangles *KLM* and *ONM* of Fig. 2 are similar, as are the triangles *KLC* and *ONC*. Taking the corresponding sides of the similar triangles,

$$\frac{MO}{ML}=\frac{ON}{KL} \tag{2}$$

and

$$\frac{CO}{CL} = \frac{ON}{KL} \quad (3)$$

Hence

$$\frac{MO}{ML} = \frac{CO}{CL} \quad (4)$$

Substituting the notation,

u = object distance = ML
v = image distance = MO

and

R = radius of curvature = MC

in Eq. (4) gives

$$\frac{v}{u} = \frac{(v-R)}{(R-u)} \quad (5)$$

Clearing fractions and dividing by uvR gives the mirror equation,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} \quad (6)$$

Note that object and image positions may be interchanged.

Magnifications. The ratio of the size of the image *ON* to the size of the object *KL* is called the *magnification m*. It follows from Eq. (2) that

$$m = \frac{MO}{ML} = \frac{v}{u} \quad (7)$$

Thus magnification is the ratio of object distance to image distance from the mirror.

Aberration. In the foregoing derivation it has been implicitly assumed that all the rays from the object point *K* which strike the mirror will pass through the point *N*. Unless a relatively small section of a spherical mirror is used, the image is not sharp, and, therefore, the assumption is proved to be wrong. Peripheral rays focus at a different distance than do rays striking near the mirror center. This image defect, or aberration, due to the spherical surface is called *spherical aberration*.

Focal length of a mirror. If the incident light is parallel, as from a very distant object, $1/u = o$. Under this condition Eq. (2) becomes

$$O + \frac{1}{v} = \frac{2}{R}$$

and

$$v = \frac{R}{2} = f$$

The distance f is called the *focal length* of the mirror and is the image distance for a very distant object. The position of this image is called the *principal focus F* of the mirror.

Real and virtual images. The image shown in Fig. 2 is a *real image*, since the rays of light converge to a real focus, and the image may be projected on a screen. A concave

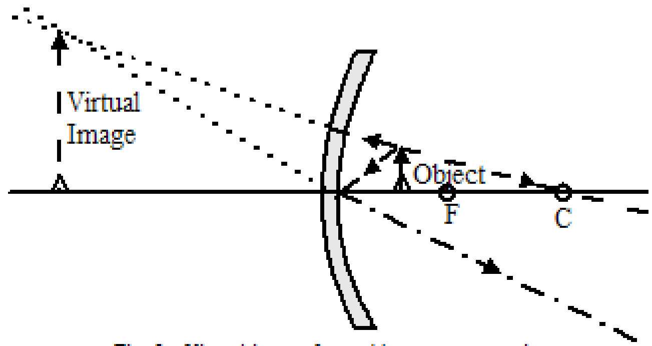

Fig. 3 - Virtual image formed by a concave mirror.

mirror may also give a *virtual image* if the object is closer to the mirror than the focal point, as shown in the ray diagram of Fig. 3. Note that the real image in Fig. 2 is inverted, whereas the virtual image in Fig. 3 is not inverted.

Sign convention. The equation

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} = \frac{1}{f} \quad (8)$$

is applicable to all mirrors. In any such equation, it is necessary to follow some convention of signs for mathematical clarity. One method introduces (+) and (-) signs in the equation according to the following plan:

u {(+) for real objects (diverging light)
(-) for virtual objects (converging light)
v {(+) for real images
(-) for virtual images
R and *f* {(+) for concave mirrors
(-) for convex mirrors.

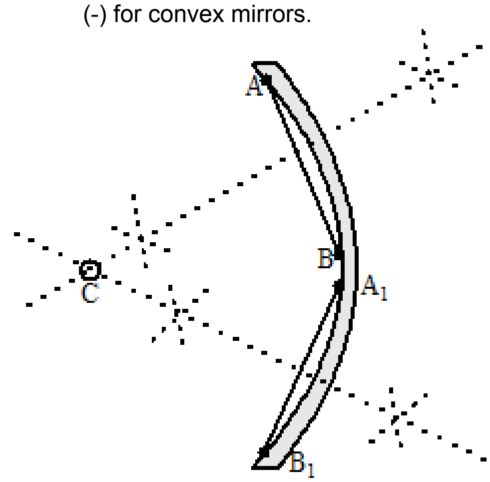

Fig. 4 - Locating the center of curvature.

The graphical procedure illustrated in Fig. 4 may be used to locate the center of curvature of a curved surface. Draw two chords *AB* and A_1B_1 . Bisect each chord. This may be done by using a compass to scribe intersecting arcs of equal radii with the terminal points of the chords as centers. The intersection of the two normal bisecting lines locates the center of curvature *C*.

APPARATUS: Ray Tracing Apparatus with Diverging Ray Attachment (Fig. 5), Optical Disk Accessories (Fig. 6), protractor, compass, and ruler.

PROCEDURE: Place a sheet of white paper on the platen of the ray tracing apparatus and clamp it under the edge clips. Set the mirror in the desired position in the path of the ray, following the appropriate drawing of Fig. 7. In these drawings the outline and position of the mirror is shown relative to the arrow line representing the incoming ray or rays. Mark the outline of the accessory with a sharp pencil. Locate dots in the center of the incident and reflected light rays. Separate the dots as far as possible. Connect these dots with sharp lines, showing the path of the rays, so that accurate data may be obtained from them. The experimental work is directed in the following exercises. Each one forms a complete unit.

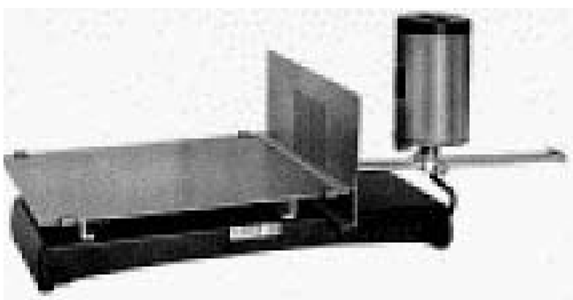

Fig.5 – Ray Tracing Apparatus.

Part I. Reflection by a plane mirror.

(A) Construct the ray traces for the two positions of the plane mirror as shown in Fig. 7 (a). Measure <*i* and <*r* for each trial. Is <*i* equal to <*r*? What is the relation between the angle through which the mirror was turned in the two trials and the resulting angular change in the reflected ray direction? Rotate the mirror to other angles and visually observe the relation between the angle of rotation of the mirror and that of the reflected ray.

(B) Add the diverging ray shield. Allow the divergent rays to fall on the plane mirror as shown in Fig. 7 (b). Construct the ray traces to locate the virtual image. Is the line joining object and image normal to the mirror plane? Are object and image distances equal, that is, does $d = d_1$ (Fig. 1)?

Part II. Reflection by a curved mirror.

(A) Using parallel rays striking near the center of the curved mirror, Figs. 7 (c) and 7 (d), locate the principal focus and measure the focal length of each curved mirror. Shift the platen to obtain these parallel rays.

(B) Measure the radius of curvature *R* of the mirror, following the directions given with Fig. 4. Compare the value obtained to the focal length *f* determined in Part II (A).

(C) Repeat the procedure shown in Fig. 7 (c), using two peripheral rays separated as far as possible. Do these rays focus at the same position as the center rays?

(D) Attach the diverging ray shield. Construct the incident and reflected ray traces, using the concave mirror. Measure *u* and *v*. Solve for the focal length f of the mirror and compare with the value obtained in Part II (A).

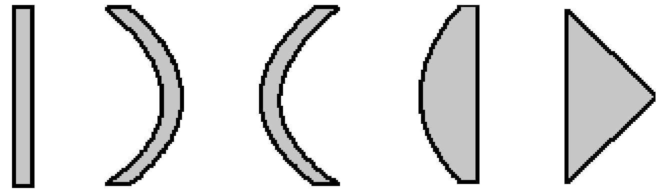

Fig. 6 - Optional Disk Accessories - mirrors and glass sections.

Part III. Internal reflection.

(A) Place the semicircular glass plate in the path of the light ray, as shown in Fig. 7 (e). Slowly rotate the glass plate about *c* as the axis. Record your observations. Does the internally reflected ray obey the law of reflection?

(B) Prism glass sections (see Fig. 7 (f)) are frequently used in optical instruments as internal-reflection surfaces because they are more efficient reflectors than silvered-mirror surfaces. Study and record the internal reflections which may be obtained with a prism.

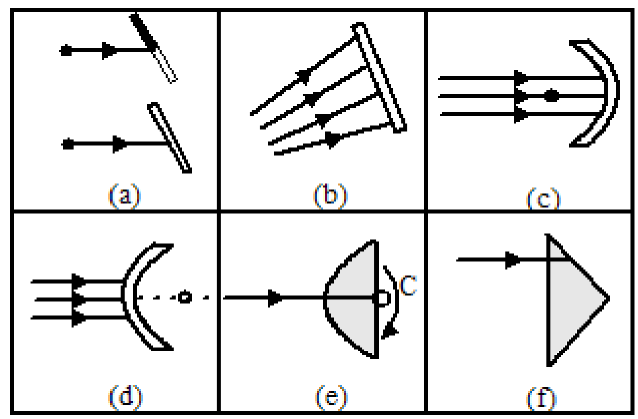

Fig. 7 - Studies of the reflection of light.

QUESTIONS: 1. Reconstruct Fig. 1 substituting a highly magnified rough paper surface for the mirror section shown. Use several incident rays from source A.

2. Prove from the geometry of Fig. 1 that $d = d_1$.

3. Construct graphically a curved mirror surface which does not show spherical aberration.

4. What minimum length plane vertical mirror is required in order that a 6.0-foot man standing in front of the mirror may see his entire body?

5. Does the distance, man to mirror, in problem 4 affect the answer? Prove by a drawing.

6. Using Eq. (8) compute the position and size of the image of a 2.0cm high object placed 12.0cm from a *convex* mirror having a 6.0cm focal length.

7. Construct the scale ray diagram of problem 6. Measure image distance and size. Do the answers agree with the computed values of problem 6?

8. How could a *virtual* object be created in an optical system?