

## REFLECTION AND REFRACTION

OBJECT: To study, by means of ray tracing, the fundamental laws relating to the reflection and refraction of light.

METHOD: A narrow beam of light from an intense straight filament incandescent lamp falls on a sheet of paper placed on the platen of the apparatus. The reflecting or refracting optical unit, mirror, prism, or lens, etc., is placed in the path of this "ray" and the resulting light path studied. Measurements from traces of these rays provide data to determine the laws of reflection and refraction and to compute the index of refraction of the medium.
By shifting the platen, a series of parallel rays are obtained for the graphical location of the principal focus of the mirror and lens sections. The Diverging Ray Attachment, a set of parallel slits, provides a diverging-ray light pattern to study image formation for a nearby real object source. The mirror and the lens expression for the relation between image distance, object distance and focal length are applied to data taken when this attachment is added.
A series of suggested experiments are given. Each section is a complete unit. The following topics are treated in the series:

1. The law of reflection applied to plane and curved surfaces.
2. The formation of real and virtual images by reflection.
3. Spherical aberration.
4. The law of refraction.
5. Lateral shift of ray transmitted through a parallel plate.
6. Minimum angle of deviation with a prism.
7. Velocity of light and dispersion.
8. The critical angle and total internal reflection.
9. Use of internal reflection in optical instruments.
10. Formation of real and virtual images by lenses.

THEORY: Reflection: When a beam of light in air strikes the surface of a material of different optical density, some of the light is usually reflected at the interface. The fraction of the light reflected depends on the optical conditions of the surface. This paper reflects about 65 percent of the incident light. Polished silvered mirrors reflect over 90 percent of the light which falls upon them. If the surface is relatively rough, like the paper, the reflection will diffuse or mix the reflected portion of the incident light, and produce no images of the source. For very smooth or highly polished surfaces, for example a mirror, the rays are not diffused and may be focused to form an image of the source. This is called regular or specular reflection.
Regardless of the surface, each ray is reflected in accordance with a simple law of reflection which has been
known for many centuries. When alight ray, incident upon a surface, is reflected, the angle of incidence $i$ and the angle of reflection $r$ are equal and lie in the same plane; that is,

$$
\begin{equation*}
i=r \tag{1}
\end{equation*}
$$

By convention, the angles are measured from a line perpendicular to the surface at the point of reflection. Thus the ray $A B$ in Fig. 1 from the source $A$ will reflect from the


Fig. 1 - Reflection of light by a plane mirror
mirror along the line BC with $i$ equal to $r$. A group of such rays form the reflected beam $B C-B_{1} C_{1}$. When this reflected beam is viewed by the eye, the image $A_{1}$ appears to be their source. This image is called a virtual image since the rays do not actually originate or converge where the image appears to be. It is beyond the plane mirror in a position such that the line $A A_{1}$, from object to image, is normal to the mirror surface, and the distance $d_{1}$ equals $d$.
The law of reflection, being a general law, may be applied to any reflecting surface. If the reflecting surface is a curved mirror, both real and virtual images are possible. Figure 2 shows the ray construction for the real image I of the object O when the rays are reflected from a concave mirror surface. This surface has its center of curvature at C . The angles $i$ and $r$ are measured from the radii of the mirror since these are normal to the curved surface at the point at which the ray strikes.


Fig. 2 - Reflection of light by a curved mirror
The following relationship holds, when all distances are measured from point A on the mirror,

$$
\frac{1}{u}+\frac{1}{v}=\frac{2}{r}
$$

where $u$ is the object distance OA, $v$ is the image distance IA , and $r$ is the radius of curvature CA at the mirror surface. If the mirror used is a rather large section of a sphere the image is not sharp. The rays striking near the mirror center come to a focus at a slightly different distance from the mirror than do the peripheral rays. This image defect or aberration, due to the spherical surface, is called spherical aberration.
A very distant object, for which the rays are practically parallel when they strike the mirror, will produce an image at a point which is one half the distance of the radius of curvature. This is evident in the mirror equation for when the object distance $u=\infty$,

$$
\frac{1}{\infty}+\frac{1}{v}=\frac{2}{r}
$$

and

$$
v=\frac{r}{2}
$$

The position where rays parallel to the principal axis $A O$ focus is called the principal focus of the mirror. Its distance from the mirror is the focal length $f$ of the mirror. Hence the mirror equation may also be written as

$$
\begin{equation*}
\frac{1}{u}+\frac{1}{v}=\frac{2}{r}=\frac{1}{f} \tag{2}
\end{equation*}
$$

Refraction: The velocity of light in a vacuum is 186,000 miles per second or $3 \times 10^{10} \mathrm{~cm}$ per second. This figure, to the accuracy stated, is also the velocity of light in air. Light advances with a lesser velocity in other material substances. In water, for example, the velocity of light is about 140,000 miles per second. A beam of light will therefore bend when it passes from one medium to another of different velocity unless the direction is normal to the interface of these materials. This bending of the light ray is evident from the following consideration. Suppose a monochromatic light
beam in air (velocity $V$ ) enters glass, in which its velocity is some smaller value $V_{\mathrm{x}}$, at an incident angle $i$, Fig. 3. With the wave front at $\mathrm{BB}_{1}$ the ray AB enters the glass and its velocity of advancement is reduced while the other boundary ray $A_{1} B_{1}$ continues to $C_{1}$ at its higher velocity.


Fig. 3 - Refraction of light
When this ray arrives at $\mathrm{C}_{1}$, the first ray has penetrated the glass a lesser distance to C and the new wave front is $\mathrm{CC}_{1}$. Thus the ray is bent making an angle R , called the angle of refraction, in the second medium. This angle is smaller than the incident angle $i$. The relation of these angles to the velocities may be derived in the following way.
In the two right-angle triangles, with $\mathrm{BC}_{1}$ as the hypotenuse of each,

$$
\sin B_{1} B C_{1}=\sin i=\frac{B_{1} C_{1}}{B C_{1}}
$$

and

$$
\sin B C_{1} C=\sin R=\frac{B C}{B C_{1}}
$$

Dividing the first of these expressions by the second gives

$$
\frac{\sin i}{\sin R}=\frac{B_{1} C_{1}}{B C}
$$

Since $\mathrm{B}_{1} \mathrm{C}_{1}=\mathrm{Vt}$
and $B C=V_{x} t$

$$
\frac{\sin i}{\sin R}=\frac{V}{V_{x}}
$$

The ratio $V / V_{\mathrm{x}}$ for monochromatic (one color) light is a constant for a given transparent medium. This constant is usually designated by $n$ and called the index of refraction. Thus,

$$
\begin{equation*}
\frac{\sin i}{\sin R}=\frac{V}{V_{x}}=n \tag{3}
\end{equation*}
$$

Note that Eq. 3 provides a relationship to compute either the velocity of light in a medium $x$ or its index of refraction $n$
provided $i$ and $R$ are known. These angles may be measured on a ray trace. Regardless of the direction of the light ray, $i$ is accepted as the angle for the air surface and $R$ is the smaller angle in the $x$ medium. Suppose a light ray, traveling from glass to air, strikes the interface at increasingly greater angles $R$. Since $i$ increases more rapidly than $R$ (see Eq. 3) there will be a critical value of $R$ for which $i$ has its maximum value, namely $90^{\circ}$. This ray will then graze the surface of the interface. The rays for still greater angles will not be refracted but will be reflected. The $R$ for which $i=90^{\circ}$ is called the critical angle, and Eq. 3 becomes

$$
\begin{equation*}
\sin i_{c}=\frac{1}{n} \tag{4}
\end{equation*}
$$

When the ray passes entirely through the medium $x$, it will be refracted on entering and again on emerging at the opposite face. If these two faces are parallel, the change in direction at the first face is neutralized at the second face. The ray, therefore, maintains its original direction but is laterally shifted in passing through the parallel plate.


Fig. 4 - Refraction of light by a prism
If the two faces are not parallel but have a refracting angle $A$, see Fig. 4, the bending of the ray at one interface is not neutralized at the second interface. The original direction of the ray will be changed by an angle $D$ called the angle of deviation. The size of this angle depends on the refracting angle $A$ of the prism, the index of refraction of the medium, and the direction of the ray through the prism section. Figure 5 is a graph plotted of the angle of deviation angle vs. incident angles. Note that the angle of deviation has a minimum value $D_{m}$. This minimum angle of deviation is achieved when the transmitted ray forms asymmetrical figure with the interfaces. This leads to the expression

$$
n=\frac{\sin 1 / 2\left(A+D_{m}\right)}{\sin 1 / 2 A}
$$

When two prisms are placed as shown in Fig. 6 the rays $A B$ and $A B_{1}$ from the object $A$ will intersect, or focus, at $C$ after refraction by the two prisms. If the surfaces of the prisms are modified as shown by the dash lines, all rays from A may be brought to a focus at the image $C$. This forms a convex lens section. The material and the shape of the lens determine its ability to converge the rays and hence determine the focal length of the lens. This focal length $f$ is related to the object distance $u$ and image distance $v$, for thin lenses, in the lens


Fig. 5 - Plot of angle of deviation vs. incident angle
equation


Fig. 6 - The lens
For a very distant object $u=\infty$ and therefore $v=f$. Thus the focal length of a lens is the distance from the lens to an image formed by a very distant object or by rays parallel to the principal axis. Note the similarity of this discussion to the one given for the mirror.
The value of $f$, depending on the surface curvatures and on the index of refraction of the lens material, is expressed by

$$
\begin{equation*}
\frac{1}{f}=(n-1)\left(\frac{1}{r_{1}}+\frac{1}{r_{2}}\right) \tag{7}
\end{equation*}
$$

where $r_{1}$ and $r_{2}$ are the radii of curvature of the two surfaces of the lens. Thus the required curvatures to produce a given focal length lens from a transparent substance of index $n$ may be computed.
The graphical procedure illustrated in Fig. 7 may be used to locate the center of curvature of a curved surface from which the radius of curvature may be measured.
Draw two chords as shown in Fig. 7. Bisect each chord with a line perpendicular to the chord. The intersection of these two normal lines is the center of curvature C of the arc.


Fig. 7 - Locating the center of curvature
The distance from C to the arc is the radius of curvature $r$. If the modified "prisms" of Fig. 6 were placed with their refracting angles touching to form a concave lens section, the emerging rays from $A$ are diverging instead of converging rays. All the equations are still applicable, but it is necessary to follow some convention of signs for a mathematical treatment to distinguish between these variations. One method introduces (+) and (-) signs in the equations according to the following plan.

```
p {+ for real object
    - for virtual object
q {+ for real image
    - for virtual image
f {+ for convex lens and concave mirror
    - for concave lens and convex mirror
```

APPARATUS: The apparatus required is a Ray Tracing Apparatus, Fig. 8, Diverging Ray Attachment, Optical Accessories, Fig. 9, protractor, and scale.


Fig. 8 - Ray Tracing Apparatus, Cenco No. CP85329-00


Fig. 9 - Optical accessories for ray tracing, Cenco No. CP85252-00

PROCEDURE: Place a sheet of white paper on the platen of the ray tracing apparatus and clamp it under the edge clips. Set the optical accessory in the desired position in the path of the ray following the appropriate small drawing $a, b, c, d$, etc. of Fig. 10. In these drawings the outline and position of the optical unit is shown relative to the line representing the incoming ray. Mark the outline of the accessory with a sharp pencil. Locate dots in the center of the light rays. Separate the dots as far as possible. Connect these dots with sharp lines showing the path of the rays so that accurate data may be measured from the ray traces.
The experimental work is directed in the following parts of seven exercises. Each one forms a complete unit.


Fig. 10 - Reflection and refraction of light
Part I. Reflection by a Plane Mirror: (A) Construct the ray traces for the two positions of the mirror as shown in Fig. 10a. Determine whether the angle of incidence equals the angle of reflection. What is the relation between the change in the angle of the mirror and the change in the reflected ray direction for the two traces? Rotate the mirror to other angles and visually observe the relation between the angle of rotation of the mirror and that of the reflected light ray.
(B) Add the diverging ray shield and allow the divergent rays to fall on the plane mirror as in Fig. 10b. Construct the ray traces to locate the virtual image. Determine whether the line
joining the object and image is normal to the mirror plane and bisected by it.

Part II. Reflection by a Curved Mirror: (A) Using parallel rays striking near the center of the curved mirror, Figs. 10c and d, locate the principal focus and measure the focal length of each curved mirror. Shift the platen to obtain these parallel rays.
(B) Measure the radius of curvature of the mirror, following the directions given with Fig. 7, and compare its value to the focal length determined in Part II (A).
(C) Repeat the procedure shown in Fig. 10c using two peripheral rays separated as much as possible. Do these rays focus at the same position as the center rays?
(D) Attach the diverging ray shield. Construct the incident and reflected ray traces using the concave lens. Measure $p$ and $q$. Solve for the focal length of the mirror and compare with the value obtained in Part II (A) with parallel rays.

Part III. Refraction of Light: Place the semicircular glass plate in the path of the light ray so that the ray strikes the center of the flat face, Fig. 10e. The ray transmitted by the glass will then travel along a radius and thus suffer no refraction on emerging from the curved surface.
(A) Slowly rotate the glass section $90^{\circ}$ about $c$ as the axis and record the observations.
(B) Draw a ray trace for an incident angle of about $30^{\circ}$ and compute the index of refraction of the glass. Repeat for an angle of incidence of about $70^{\circ}$.
(C) Relocate the glass section on either the $30^{\circ}$ or $70^{\circ}$ trace. Now rotate the glass exactly $180^{\circ}$ about $c$ as the axis. Is the ray trace reproduced? Does the direction of a ray of light affect its path?

Part IV. Internal Reflection: Place the semicircular glass plate in the path of the light ray so that the ray enters the curved face and travels along a radius to the center point c , Fig. $10 f$.
(A) Slowly rotate the glass section $90^{\circ}$ about $c$ as an axis and record the observations. Why are the rays not transmitted beyond a definite angle?
(B) Draw the ray trace for the critical angle condition. Measure the critical angle and compute the index of refraction of the glass using Eq. 4. Does the internally reflected ray obey the law of reflection?
(C) Note the color effect in the refracted ray which is especially noticeable near the critical angle condition. This phenomenon is called dispersion.
(D) Internal reflection from an internal glass surface is frequently used in optical instruments since this is a more efficient reflector than a silvered mirror surface. These glass sections usually have the form of a prism. Draw the complete ray traces for the two rays indicated in Figs. 10g, h, and $i$. In each figure one ray is drawn as a dash line to differentiate between the two rays. Continue each in this way.

Part V. Refraction through a Parallel Plate: Place the parallel side glass plate in the path of the ray as shown in Fig. 10k. Is the ray laterally displaced but not deviated? How does the displacement depend on the incident angle?

Part VI. Refraction through a Prism: (A) Place the prism in the path of the light ray as shown in Fig. 10k. Move the platen across the light slit. Is the deviation angle the same for all parallel rays for the refracting angle A? Check with a different incident angle.
(B) Return the prism to the position shown in Fig. 10k with the ray near the apex. Now rotate the prism about the apex point A to increasingly larger incident angles. What happens to the angle of deviation? In what position of rotation does the prism produce the minimum deviation angle?
(C) To determine more accurately the minimum deviation angle a series of readings will be plotted on a graph. Carefully construct a ray trace for each of the following incident angles, $10^{\circ}, 15^{\circ}, 25^{\circ}$, and $35^{\circ}$. Use the same acute refracting angle $A$ for each. Measure the angles $i_{1}, i_{2}$, and $D$, Fig. 101, on all four traces. Plot a graph with angles $i$ as abscissas against $D$ as ordinates. Read the minimum deviation angle $D_{\mathrm{m}}$ of the prism from this graph. Compare angles $i_{1}$ and $i_{2}$ for the minimum condition. Compare the values of $n$ obtained by Eqs. 3 and 5 .

Part VII. Refraction through Lenses: Place the lens section so that a ray through the center of the lens, dash line, is perpendicular to a mid-line drawn in the lens.
(A) Construct ray traces for parallel rays through the center section of the diverging or concave lens, Fig. 10m. Measure the focal length of this lens.
(B) Draw the ray traces indicated in Fig. 10n for the convex lens. Do the center rays and the peripheral rays focus at the same point? Explain. What is the focal length of the lens as measured on the center rays?
(C) Measure the radius of curvature $r_{1}$ and $r_{2}$ of each face of the convex lens, following the directions given with Fig. 7. This glass has an index of refraction of approximately 1.52. Assuming this value and using the measured values of $r_{1}$ and $r_{2}$, compute the focal length of the lens sections. Compare to the measured value from Part VII (B).
(D) With the diverging ray attachment in the light beam construct the ray pattern, Fig. 100, for the convex lens. Measure $D_{0}$ and $D_{\mathrm{i}}$. (See Fig. 6.) Compute the focal length of the lens and compare with the values previously determined.

QUESTIONS: 1 . What is approximately the velocity of light in the glass sections used in this study?
2. What is the magnitude of the lateral shift of a ray of light when it passes through a plate glass window 1 cm thick at an incident angle of $60^{\circ}$ ?
3. Prove in Fig. 1 that the line $A A_{1}$ is normal to the mirror surface and that the distance $d_{1}$ equals $d$.
4. Construct graphically a mirror surface which does not show spherical aberration.
5. In what optical instruments and for what purpose might the internally reflecting elements of Figs. $10 \mathrm{~g}, \mathrm{~h}$ and i be used?
6. Under what conditions will a converging lens produce a virtual image?
7. What minimum length plane vertical mirror is required in order that a 6 -foot man standing in front of the mirror may see his entire body? Sketch to scale.
8. What does the color effect noticed in Part IV (C) show concerning the velocity of light in glass?

