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NODAL SLIDE
A STUDY OF THE PROPERTIES OF THICK LENSES

OBJECT: To determine the principal points, the nodal points and the focal lengths of thick lenses and lens combinations.

METHOD: The characteristic of the nodal points is made the basis of their determination. The thick lens or lens combination is mounted in a nodal slide on an optical bench. The nodal slide provides motion of the lens parallel to the bed of the bench and also rotation about a vertical axis through its point of support. If the lens is placed so as to form a real image on a screen, any rotation of the lens about a vertical axis will, in general, produce lateral motion of the image on the screen. If, however, the rotation of the lens takes place about the second nodal point, there will be no shift of the image, since the ray passing through the first nodal point will emerge from the second nodal point parallel to its original direction. If the incident light is parallel, this will be true regardless of the position of the first nodal point. The distance from the center of rotation to the image screen gives at once the focal length, and the position of the lens or lens system with reference to the center of rotation locates the nodal point with reference to the principal axis of this system. By rotating the nodal slide 180˚, the other nodal point is found in the same way.

THEORY: The conjugate focal relation $1/u + 1/v = 1/f$ in which *u, v* and *f* are measured from the center of the lens holds only so long as the thickness of the lens may be neglected in comparison with the focal length. If the measurements of object and image distances (*u* and *v* respectively) are taken from the vertex of the refracting surface of a lens whose thickness must be taken into consideration, the relation between these distances and the focal length *f* is very cumbersome. It is possible, however, with a thick lens to secure the simplicity of the original conjugate focal relation by making the measurements of the distances *u* and *v* from two selected points on the axis of the lens. These two axial points are called the *principal points* of the thick lens. Planes drawn perpendicular to the axis through the principal points are called the *principal planes* of the lens. These principal planes have been chosen as planes of unit magnification. There are two other points on the axis of a thick lens which possess important properties. A ray of light directed toward the first of these points on the axis will after refraction by the lens proceed from the second point on the axis in a direction parallel to that of the incident ray. These points are termed the first and second *nodal points* respectively. When image and object are situated in the same medium, for instance a glass lens used in air, the principal points and the nodal points coincide.

If the formula for refraction at a single spherical surface is applied successively to two spherical surfaces separated by a distance which is not negligible in comparison with the radii of curvature of these surfaces, a relation may be secured between the object distance measured from the vertex of the first surface and the image distance measured from the vertex of the second surface, in terms of the distance between the vertices of the surfaces and the refractive index. This expression, however, is not convenient to use on account of the number of terms which it contains. If it is asked whether this elaborate expression may be reduced to the simple conjugate focal relation which holds for a single thin lens, the mathematical answer is that this may be accomplished provided the measurements of the object distance u and the image distance *v* are made from *any pair* of conjugate points.

In the Gauss theory of optical imagery the two conjugate points selected were arbitrarily taken as that pair for which the lateral magnification is unity. The *lateral magnification* produced by an optical system is defined 'as the ratio of the size of the image to the size of the object. In Fig. 1 *A'B'/AB=m*, the lateral magnification. In Fig. 2 *K'H'/KH*=1, *K'*

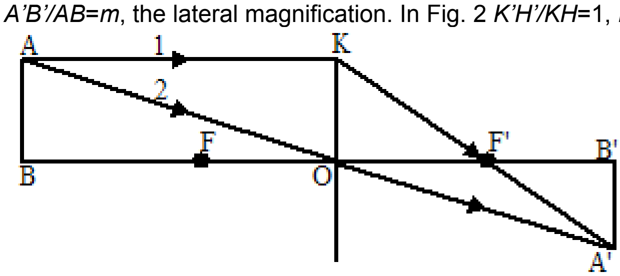

Fig. 1. Image formation in a single thin lens.

being the image of *K*. An object in the first principal plane has an erect virtual image of the same size as the object in the second principal plane. In the graphical construction for finding the image produced by a single thin lens, the image of a point A (Fig. 1) is found by means of two rays. Number 1, parallel to the axis, is refracted so as to pass through the second focal point *F'* when it strikes the plane KO through the optical center of the lens. Ray No. 2 passes through the optical center O without deviation. Their intersection gives the image point *A'*. Imagine the line through the optical center to be cut in two and the parts displaced slightly a distance *HH'* (Fig. 2), the planes through *H* and *H'* being the principal planes of the thick lens. The principal points H and

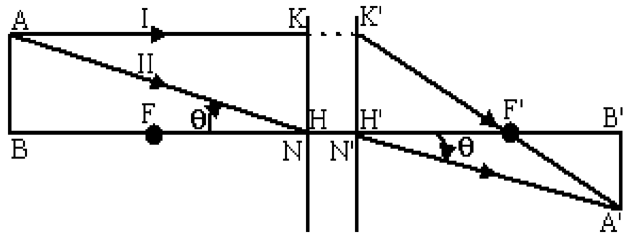

Fig. 2. Image formation with a thick lens.

H' coincide with the nodal points N and N' when the image and object are located in a medium of the same refractive index. Then ray No.1 (Fig. 1) is replaced by ray I (Fig. 2), which passes parallel to the axis from A to K. The point K is imaged in K', ray I passing thence through the second focal point F'. Ray No. 2 is replaced by ray II. It is directed toward N at the angle θ with respect to the axis of the lens and leaves the point N' making the same angle 6 with the axis. The point of intersection A' of rays I and II is the image. In order to secure parallel light, a plane mirror M (Fig. 3) is placed back of the nodal slide S and the slide moved along

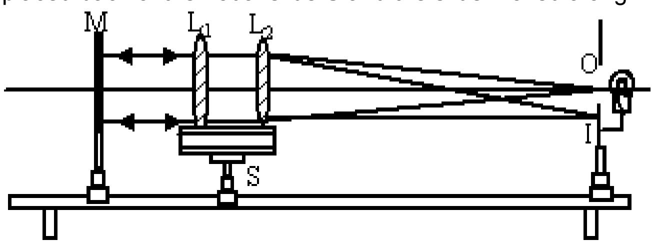

Fig. 3. Arrangement for producing parallel light as used with the nodal slide S and an optical system consisting of two lenses L_1 and L_2.

the bench until an image I of the object O is formed on the white screen in the plane of the object. The source (the object) may then be thought of as being at an infinite distance and the light from M entering the lens system L_1 and L_2 as a parallel beam.

If, in Fig. 4, a ray ac from a distant object is incident through

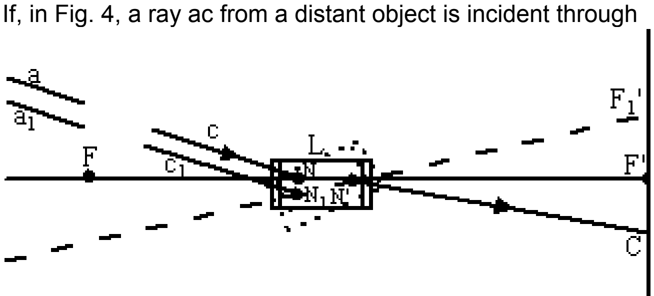

Fig. 4. Rotation of a lens system L about the second nodal point N' produces no shift of the image on the screen at C.

the first nodal point N of a lens system L, it will emerge parallel to its original direction through N' and meet the screen at C. The point C is the image of the object from which ac came. As the object is supposed to be at a great distance, its image will be formed in the focal plane- that is, a plane normal to the axis of the lens system passing through plane normal to the axis of the lens system passing through the second focal point F'.

Now suppose the lens system rotated about the second nodal point N'. As ac is coming from a great distance, the ray a_1c_1 from the same point of the object incident through N_1 will still be practically parallel to ac. Thus the ray that emerges from N' has not moved, and the image at C will remain at rest.

Thus to find the second nodal point N' it is necessary to find the point about which rotation of the lens system produces no movement of the image of a distant object. This is easily accomplished experimentally, because if the second nodal point N' be in front of the axis of rotation O, i.e., too near the screen (Fig. 5a) then a small counterclockwise rotation will

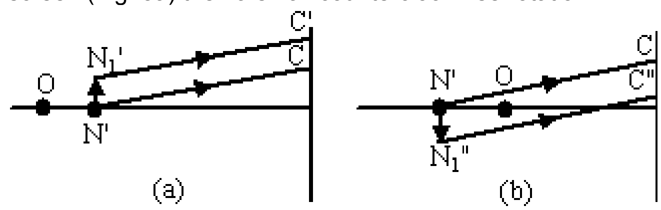

Fig. 5. The reversal of the image motion when the axis of rotation O moves from behind the second nodal point N' (a) to a position in front of it (b).

carry N' to N_1' and therefore C to C'. If, on the contrary, the second nodal point N' is behind 0, i.e., too far from the screen (Fig. 5b), a similar counterclockwise rotation will take it to N_1" and the image C moves to C' '.

The Gauss formulas for the calculation of the focal length and the location of the principal points and focal points of a thick lens. (Fig. 6) are as follows: *

*Southall, *Mirrors, Prisms, Lenses*, Chapter XI.

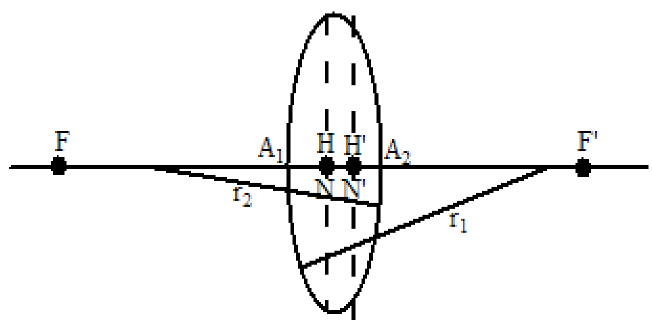

Fig. 6. The principal points H and H', coinciding with the nodal points N and N', and the focal points F and F' of a thick lens.

$$f = (nr_1r_2)/N \quad (1)$$

$$A_1H = -r_1d(n-1)/N \quad (2)$$

$$A_2H' = -r_2d(n-1)/N \quad (3)$$

$$A_1F = -r[nr_2 + (n-1)d]/N \quad (4)$$

$$A_2F' = r_2[nr_1 - (n-1)d]/N \quad (5)$$

where r_1 and r_2 are the radii of curvature of the lens surfaces, *n* the index of refraction of the lens, *d* the distance between the vertices of the lens surfaces A_1A_2 and *N* is a constant defined by the following equation:

$$N = (n-1)[n(r_2 - r_1) + (n-1)d] \quad (6)$$

Fig. 7. The Optical Bench with three carriages for supporting accessories.

In case one surface of the lens is plane, say $r_2 = \infty$, these equations become

$$f = r_1 / (n-1) \quad (7)$$

$$A_1H = 0 \quad (8)$$

$$A_2H' = -d/n \quad (9)$$

$$A_1F = -r_1 / (n-1) \quad (10)$$

$$A_2F' = r_1 / (n-1) - d/n \quad (11)$$

APPARATUS: The apparatus consists of an optical bench (Fig. 7), a combined object and image screen (Fig. 8), nodal

(a) (b)

Fig. 8. Object and Image Screen (a) for use in Holder (b).

slide (Fig. 9), plane mirror mounted on a 10mm rod (Fig. 10), lenses and vernier caliper. The diameter of the lenses should be within the range of 5 to 8cm. The plano-convex lens and the double convex lens should each have a focal length of about 15cm, and the concave lens 30-40cm.

PROCEDURE:

Experimental: Mount a plane mirror M, the nodal slide S, and the object holder with combined object and image screen O and I on the optical bench as illustrated in Fig. 3. Place *one* plano-convex lens in the nodal slide, and adjust the position of the slide until the distance between the lens and object is approximately the focal length of the lens. Orient the mirror until the light from the object O, rendered parallel by the lens, is reflected back normally and forms an image I of the object. Move the nodal slide along the bench until the image I is sharply focused.

By adjustment of the lens in the nodal slide and subsequent refocusing by movement of the nodal slide, locate the second nodal point of the lens using the method outlined in the Theory. Measure the distance from the center of rotation of the nodal slide to the screen. This is the focal length *f*. of the nodal slide to the screen. This is the focal length *f*. Note the position of the lens holder in the nodal slide. Rotate the slide 180° and find the other nodal point, measuring the focal length and noting again the position of the lens holder on the slide.

Measure the thickness of the lens with the calipers.

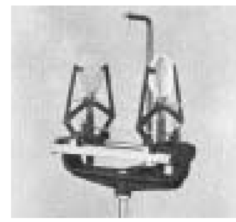

Fig. 9. The Nodal Slide is arranged to hold one or two lenses of which the nodal points may be determined. It consists of a main support bracket with two horizontal rods. The bracket is swiveled on a vertical support rod by means of which the entire slide may be supported on an optical bench clamp. The bracket carries a downwardly projecting inductor, the point of which is in the vertical axis of rotation of the bracket. Either one or two lenses may be supported from lens clamps. The lens clamps may be independently moved laterally, or the combination may be adjusted with reference to the vertical support.

Determine the nodal points and focal length of the combination of a convex and a concave lens separated two or three centimeters. Measure the thickness of each lens used and also the distance apart of the inner surfaces of the two lenses.

Fig. 10. Plane Mirror, mounted.

Interpretation of Data: From the data taken with the single lens, find the distance between the nodal points expressed as a decimal part of the thickness of the lens. Draw an enlarged diagram of the lens, and on the axis of the lens show the position of the lens surfaces and the two nodal points. Show in a diagram drawn to scale the results obtained with the combination of lenses used.

QUESTIONS: 1. Find the focal length and the positions of the focal points and principal points of each of the following glass lenses surrounded by air (N = 1.5). Make an accurate sketch of each lens, marking the position of the points mentioned: (a) A double convex lens of radii 10 and 15cm and of thickness 3cm; (b) A double concave lens whose radii are 10cm and 12cm; (c) A meniscus convex lens for which r_1 = 5cm and r_2 = 10cm, and d is 3cm; (d) A plano-convex lens whose radius of curvature is 10cm, thickness 1.5cm.