CENCO Physics Selective Experiments In Physics

• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

INDEX OF REFRACTION OF A LIQUID (LIQUID LENS)

OBJECT: To study the refraction of light at liquid-air surfaces; in particular to measure the index of refraction of several liquids by the use of a liquid lens.

METHOD: A liquid-filled watch glass is used as a lens. By means of this lens a real image of a bright object is formed. Measurements of the object and image distances and the radius of curvature of the watch glass are measured. The measurements provide the required data in the lens maker's equation for the computation of the index of refraction of the liquid.

THEORY: When a ray of light passes from one transparent substance into another in which light has a different speed, the ray is bent if the ray direction is not perpendicular to the interface. This phenomenon is called *refraction*. Refraction is illustrated by the ray *ABC* of Fig. 1 where θ is the incident angle of a ray in air

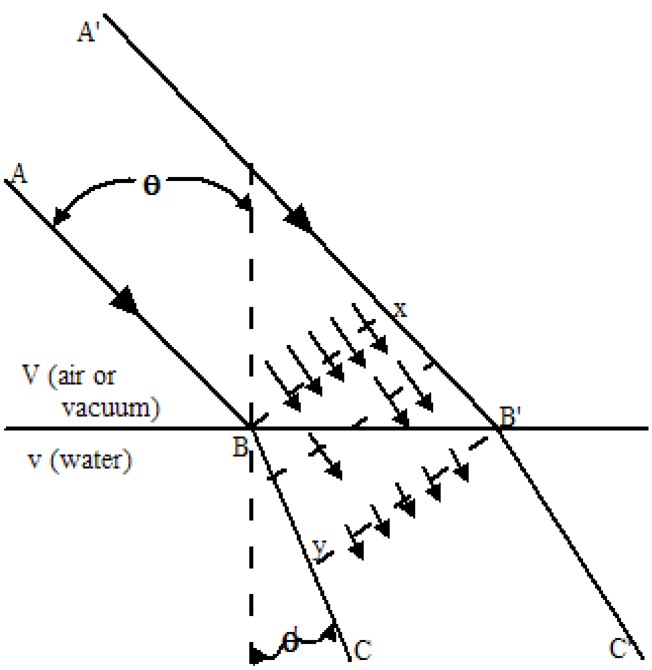

Fig. 1. Refraction of a ray of light.

and θ' is the angle of the ray refracted in water, in which the light speed is lower than in air.

The experimental result that $\sin\theta/\sin\theta'$ is equal to a constant for a given pair of substances under specified conditions is known as the law of refraction. This law, named after Willebrord Snell (1591-1626), is stated algebraically

$$\sin\theta/\sin\theta' = n \qquad (1)$$

where the constant *n* is called the *index of refraction*. Tables of the index of refraction for various substances may be found in handbooks of physical constants. Such tables give the absolute values of the index of refraction; this means that the measurements were made for an interface of vacuum against the stated substance. A few of these constant values of *n* are recorded in Table I.

TABLE I
Index of Refraction

Water	1.3330
Ethyl	1.3605
Glycerine	1.4730
Fused Quartz	1.4585
Crown Glass	1.5172
Diamond	2.4173
Dry Air	1.0003

The measurements were made using yellow sodium light of wavelength 5893-Angstrom units ($1Å=10^{-8}$cm) for substances at 20°C. The preciseness of measurement evidences the importance of this constant in the field of science. The constant value of the index of refraction is one clue to the identity of a substance.

The bending or refraction of a ray of light is due to the fact that a given wavelength of light travels at a unique speed in each substance. One is lead to this conclusion by the following consideration. In Fig. 1 a wavefront of the beam of light bounded by the parallel rays *AB* and *A'B'* is the dash line B_x. The wavefront is the line of disturbance propagating the wave energy and it is perpendicular to the rays or direction of flow of energy.

The wave disturbance along the wavefront travels forward with a speed dependent on the medium. When the beam enters the second medium at *B*, in Fig. 1, that portion of the wave traveling in the second medium moves forward at a slower speed *v*, whereas the portion still in the first medium has a greater speed *V*. Thus, in the time required for it to travel from *x* to *B* the wave moved a shorter distance from B to y at the other side of the beam, producing the new straight line wavefront yB'. Since yB' is not parallel to B_x the direction of the component rays is altered. From the geometry of the figure one may write

$$xB'/BB' = \sin\theta \quad \text{and} \quad By/BB' = \sin\theta'$$

Combining the two equations gives

$$xB'/By = \sin\theta/\sin\theta'$$

Since $xB' = Vt$ and $By = vt$

$$V/v = \sin\theta/\sin\theta' = n$$

where n is now the index of refraction for the combination of these two substances. Thus the index of refraction is also the ratio of the velocities of light in the two media.

The *absolute* value of n given in Table I, where the first medium is empty space or vacuum with light velocity c, relates these velocities as

$$n = c/v \qquad (2)$$

The velocity of light in a vacuum c is accepted as 3 x 10^8m/sec or 186 x 10^3mi/sec. Thus, knowing the absolute index of refraction of a substance permits one to calculate the speed of light in that substance. For example the speed of light in water, v = c/n, v = 186 x 10^3mi/sec/1.3330 = 140 x 10^3mi/sec.

Since the index of refraction of air is 1.0003, it is evident that the velocity of light in air differs from its velocity in vacuum by only 3/100 of one percent. Unless very high accuracy is required the measurements of the index of refraction of a substance may, therefore, be made in air.

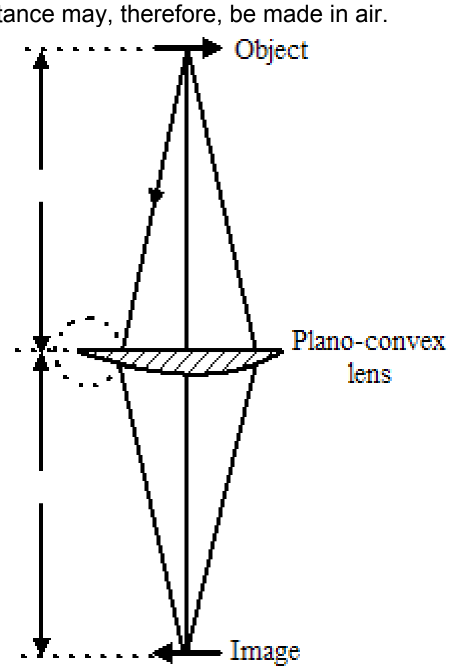

Fig. 2. Formation of a real image by a plano-convex lens.

When the interface through which the light progresses is a curved surface or lens, Eq. 1 still holds for each ray. But a light beam from a source (a beam is a bundle of rays) may have the different rays striking the surface at different angles. Frequently this results in the formation of an image of the source, as shown in Fig. 2 where the lens is a plano-convex lens. The lens equation

$$1/f = 1/u + 1/v \qquad (3)$$

applies where u and v are the object and image distances respectively from the lens and f is the *focal length* of the lens. Note that when the object is at such a great distance from the lens that the incoming rays are essentially parallel lines, $1/u = 1/\infty = 0$. The image distance from the lens for parallel light is thus the focal length of the lens.

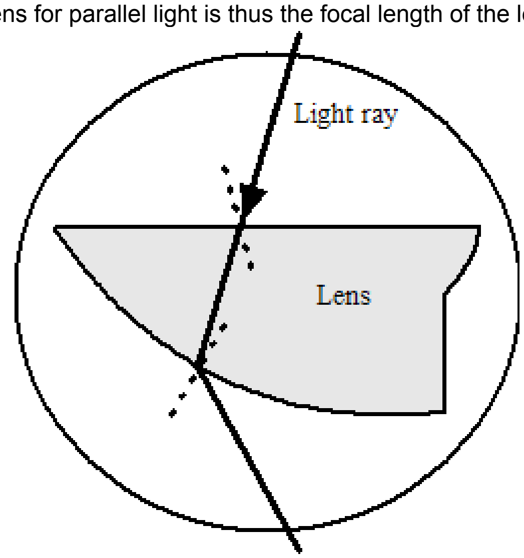

Fig. 3. Exaggerated drawing of the circular area in Fig. 2 to show refraction at each interface.

Figure 3, an exaggerated drawing of the circular area of Fig. 2, shows the application of Snell's law to alight ray at both the entering and emerging surfaces of the lens. It is evident that the total bending or refraction of the ray, which determines the focal length of the lens, must depend on the curvature of the two surfaces and on the index of refraction of the lens material. This is expressed in the *lens maker's equation*

$$1/f = (n-1)(1/R_1 + 1/R_2) \qquad (4)$$

where R_1 and R_2 are the radii of curvature of the lens surfaces. Combining Equations 2 and 3 gives

$$1/u + 1/v = (n-1)(1/R_1 + 1/R_2).$$

Note that the same ray would strike a convex and a concave surface at a different angle θ and thus the algebraic sign of the two R-values may be unlike. The top surface in Fig. 2, being a plane, has a value of R which is infinity; consequently, its reciprocal $1/R$ is equal to zero.

Therefore when applied to a *plano-convex* lens, the above equation reduces to

$$1/u + 1/v = (n-1)/R \qquad (5)$$

where *R* now is the radius of curvature of the convex surface. Thus with *u, v*, and *R* determined experimentally, one may calculate the index of refraction of the lens material. In this experiment a liquid plano-convex lens is made by filling a watch glass with the liquid whose index of refraction is to be found.

The curvature of the lower surface of the liquid is that of the watch glass. Its magnitude may be experimentally measured by several different methods. Three methods are described in the following section.

(A) *Measurement of R with a spherometer.* The spherometer shown in Fig. 4 has three pointed legs spaced so as to form an equilateral triangle. At its center there is an accurately cut micrometer screw leg with a large graduated disc head. This disc, being adjacent to a graduated post, permits reading of full and fractional turns of the screw. The *zero reading* of the spherometer is obtained by placing it on a flat glass plate and adjusting the center leg until all four legs touch the glass. When the center leg exceeds the plane of the outside legs the spherometer will wobble. The screw is then reversed until the wobble disappears.

Fig. 4. The Cenco No. CP73360-00 spherometer.

To measure the thickness of a thin object place the object under the center leg of the spherometer and turn the screw until the three outer legs are in contact again with the flat plate. The thickness is the difference between the new reading of the spherometer scale and its zero reading.

If the spherometer is used to measure the elevation or the depression of the center of a spherical surface, such as a watch glass, the spherometer reading is related to the radius of curvature of that surface.

The diagram in Fig. 5 pictures the four legs of the spherometer in contact with the spherical surface, center at *C*, which has a radius of curvature *R*. In this figure *S* is the micrometer screw leg and *F* is one of the three fixed legs. Leg *F* is in the same vertical plane as *S*. Let the elevation of *AB* = *h*, and the distance from the center leg to one of the fixed legs *AD* = *d.* Let *R* be the radius of curvature of the surface.

From the well-known property of a circle, namely that any angle inscribed in a semicircle is aright angle, the angle *BDE* is a right angle. One may write the following three equations-

$$(AB)^2 + (AD)^2 = (BD)^2$$

$$(AD)^2 + (AE)^2 = (DE)^2$$
$$(BD)^2 + (DE)^2 = (BE)^2$$

Solving these three equations reduces them to

$$(AB)^2 + 2(AD)^2 = (BE)^2 - (AE)^2$$

Substituting h, d, and R in this equation gives

$$R = (d^2 + h^2)/2h \qquad (6)$$

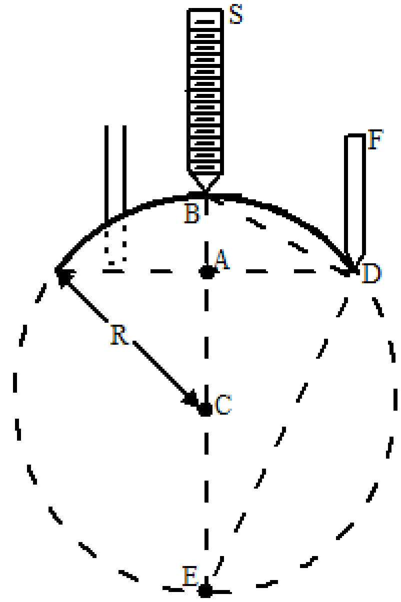

Fig. 5. Spherometer used to measure radius of curvature.

Thus measurements of *d* and *h* provide the needed data to compute *R*, the radius of curvature of the spherical surface. When the surface is concave the procedure is similar except that *h* now represents the distance the micrometer screw must be turned from its zero reading (in the opposite direction) to put the four legs in contact with the concave glass surface.

(B) *Measurement of R with the watch glass as a concave mirror.* The watch glass can be made into a mirror by coating the surface with a reflecting material. A simple coating method is to press a piece of very bright aluminum foil over the convex surface. When this is done carefully, the surface will reflect sufficient light to produce a real image of a bright source. A mirror equation which applies is

$$1/u + 1/v = 2/R \qquad (7)$$

(C) *Radius of curvature by graphical method.* As noted in Fig. 5, *R* is the perpendicular distance from *C*, the center of the "sphere", to any segment. Since all radii lines pass through *C*, this center of curvature may be graphically located by the method shown in Fig. 6. Let *A A' B B'* be the sphere segment curvature.

Draw the chord *AB*. Using a compass with a fixed radius, and with centers at *A* and at *B*, strike the intersecting arcs *D* and *D'*. The line joining *D* and *D'* will be perpendicular to the curve at the point where it crosses the curve. Repeat for the chord *AB.* The intersection of the two bisecting lines locates the center of curvature *C*; consequently *R* may be measured.

APPARATUS: Watch glass, preferably about 9cm in diameter and with a 9cm radius of curvature. Spherometer. Light source as an object, image screen and several transparent liquids. A 150cm rod. Suitable clamps to mount the light source, watch glass, and the image screen. Meter stick, four-inch square of bright aluminum foil, and a five-inch length of 1mm diameter solder wire.

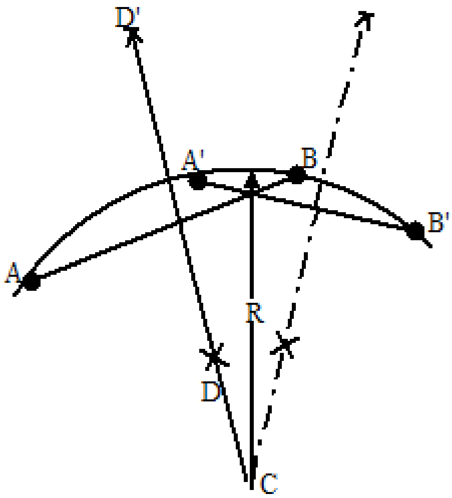

Fig. 6. Graphical method for finding the center of curvature C.

PROCEDURE:
(I) Carefully measure the radius of curvature of the watch glass supplied according to method (A) and either method (B) or (C). See THEORY for additional details.

Method (A). Place the spherometer on a flat glass plate and find the zero reading. Obtain a good average value. Take several measurements of the distance *d* from micrometer screw to the fixed legs to obtain a mean value. When the four legs are in the same plane, place the spherometer on a sheet of paper and press down lightly until a small pinprick is produced by each leg. The measurements of *d* can be made from the pinpricks on the paper. Place the spherometer on the center of the watch glass and adjust the micrometer screw so that all four legs touch the watch glass. Take several measurements for a mean value of *h*. Compute the value of *R* and express it so as to show its degree of accuracy. Find the value of *h* when the spherometer is placed off center on the watch glass. Does it agree with the former value? Draw conclusions.

Method (B). Press the bright side of apiece of aluminum foil in close contact with the convex surface of the watch glass. Using this as a concave mirror, locate the real image produced by a bright light object or preferably the sun. Compute *R* using the mirror equation. Show that when the sun is used as the object, the radius of curvature is equal to twice the image distance. Is it necessary to take the mean of several readings to obtain a respectable value?

Method (C). The curve *A A' B B'*, of Fig. 6, is a cross section of the watch glass through its midpoint. To obtain this curve proceed as follows. Use a solder wire, about one-mm in diameter, whose length is equal to the distance AB'. Place the wire across the midpoint of the watch glass and press the wire to the glass until the curve of the wire conforms accurately to the curve of the glass.
Transfer the wire carefully onto a carbon paper which covers a sheet of white paper. Place a ruler over the wire. Then strike down on the ruler to impress an outline of the wire on the white paper. Proceed, as outlined under THEORY, to find the value of the radius of curvature. Should this value conform closely to the value obtained in (A)? Justify your answer.

(II) The next task is to use the liquid lens to determine experimentally the value of *u* and of *v* for Eq. 4. Set up the apparatus as shown in Fig. 2. Use a vertical rod approximately 150cm long. Place the object, a bright light source, at the top of the rod and the watch glass, with its concave side up, about half way down the rod. Half fill the watch glass with water. Locate the image of the object. Carefully measure *u* and take the mean of several readings of *v* to solve for the value of *n* in Eq. 4. Use the most appropriate measured value of *R*.

(III) Repeat procedure (II), but this time use another liquid supplied by the Laboratory Instructor.

(IV) Optional Experiment. The purpose of this experiment is to determine the effect, if any, on the value of the index of refraction of water and, thus, on the speed of light, of sugar and salt put into solution in the water. Make a saturated sugar solution and a saturated salt solution. Use each solution, in turn, in the watch glass for experimental data. Compute the value of the index of refraction and the speed of light in each solution.

QUESTIONS: 1. Assume that the watch glass has a uniform thickness. Give reasons why (or why not) the glass contributes to the focal length of the liquid lens.

2. Identically shaped pieces of diamond and of glass are viewed when immersed in a transparent liquid having an index of refraction of 1.52. Describe the appearance of each. Assume that all the substances have no color.

3. Compute the speed of light in diamond.

4. Calculate the focal length of a crown glass plano-convex lens having the curvature of the liquid lens used in this experiment.

5. What would be the focal length of a double-convex lens of crown glass when both faces have the curvature of the liquid lens used?

6. A 20cm focal length symmetrical double-convex lens is to be constructed of fused quartz. Calculate the radii of curvature of the surfaces.

7. Would it be possible to construct a 20cm focal length quartz lens if one curvature is specified as *R* = 5cm? If so, calculate, interpret, and draw the lens cross section.

CENCO CATALOG NOS. OF MAJOR APPARATUS
Spherometer, No.73360
Watch Glass, No. 15850-90
Support Ring/Clamp, No. 18005-2
Small Object Box, No. 86027