• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

INDEX OF REFRACTION BY APPARENT THICKNESS

OBJECT: To determine the average index of refraction for white light of several specimens of glass and liquid, using the method of apparent thickness.

METHOD: A micrometer comparator is arranged with the axis of the microscope vertical and so adjusted that the movement of the microscope is parallel to its axis. The instrument is focused on a scratch on the top surface of a piece of glass lying in a shallow dish. A specimen of glass to be tested is placed over the scratch and the microscope is raised by means of the carriage screw until the scratch is again in focus. The difference between the two settings of the comparator gives the apparent elevation of the scratch produced by refraction in the glass specimen. The microscope is raised further until the top surface of the specimen is in focus. The difference between the first and last settings gives the thickness of the specimen; and the difference between the second and last settings gives the apparent thickness. From these determinations, the index of refraction is computed. The glass specimen is replaced by a layer of liquid and the experiment repeated.

THEORY: In general, when a beam of light is incident upon a boundary between two media in which the velocity of light has different values, some of the light is reflected back into the first medium and some is transmitted into the second medium. The reflected light obeys the law of reflection, according to which the angle of incidence *i* (Fig. 1) between each incident ray and a normal to the surface is equal to the angle of reflection *i'* between the corresponding reflected ray and the normal; furthermore, the incident ray, the reflected ray, and the normal lie in the same plane. The transmitted light is refracted,

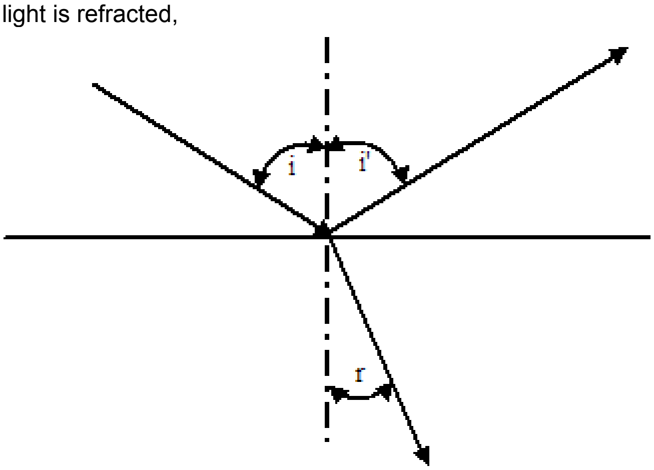

Fig. 1. Reflection and refraction of light.

i.e., it undergoes a change in the direction of propagation, by an amount which depends upon the physical properties of the media on both sides of the boundary. The angle of

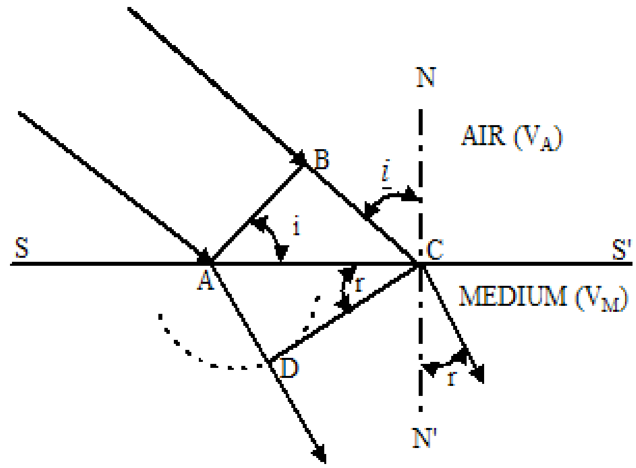

Fig. 2. Refraction at a plane surface.

refraction *r* depends both upon the angle of incidence *i* and upon the relative velocities of light in the two media.
The ratio between the velocities of light in any two media is called *the relative index of refraction* for the two media. In expressing the relative index for light traveling in a given direction, the velocity V_1 in the first medium is divided by the velocity V_2 in the second medium; this ratio is called the index of the second medium relative to the first. Thus

$$n_{2,1} = \frac{V_1}{V_2} \qquad (1)$$

is the expression for the relative index $n_{2,1}$ for a ray passing from medium 1 to medium 2.
The *absolute* index of refraction of a medium is the index of that medium relative to free space, and is defined as the ratio of the velocity of light in free space to the velocity in the medium. It follows, therefore, that the relative index of refraction for any two media is the ratio of their absolute indices. Since the velocity of light in air is very nearly equal to the velocity in free space, and since air is a common medium involved in almost all optical problems, it is customary to define the absolute index n of a medium, practically, as the ratio of the velocity in air to that in the medium. Thus

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$$n = \frac{V_A}{V_M} \quad (2)$$

where V_A is the velocity in air and V_M the velocity in the medium.

From this physical definition of the index of refraction based upon the *velocities* of propagation, a geometrical relationship may be derived between the *directions* of propagation in the two media.

Consider a parallel beam of light incident obliquely upon a plane surface represented by SS' (Fig. 2). The medium above the surface is assumed to be air and that below the surface to have a velocity V_M less than V_A. Let the line AB represent the intersection of the incident wave front with the plane of the paper. The angle of incidence *i* is the angle between the incident wave front and the surface. Since the wave front strikes the refracting surface obliquely, the portion of the wave at A travels a certain distance *AD* in the lower medium in the same time that the portion of the wave at B travels from B to C in air. To determine the direction of the wave front in the second medium, an arc is struck with its center at A and with a radius *AD* such that

$$\frac{BC}{AD} = \frac{V_A}{V_M} \quad (3)$$

The line CD drawn tangent to this arc represents the direction of the wave front in the second medium. The angle *r* between the refracted wave front and the surface is the angle of refraction. In the right triangles ABC and ADC

$$\sin i = \frac{BC}{AC} \quad (4)$$

and

$$\sin r = \frac{AD}{AC} \quad (5)$$

Dividing corresponding sides of Eq. (4) by Eq. (5) and canceling the common term AC,

$$\frac{\sin i}{\sin r} = \frac{BC}{AD} \quad (6)$$

Combination of Eqs. (2), (3) and (6) yields

$$n = \frac{V_A}{V_M} = \frac{\sin i}{\sin r} \quad (7)$$

The relationship expressed by Eq. (7) is known as Snell's law of refraction.

If a normal to the surface, such as NN' (Fig. 2) is drawn at the point of incidence for any ray, the angles which the incident and refracted rays make with the normal are equal, respectively, to the angles which the incident and refracted wave fronts make with the surface SS' (since a ray is normal to the corresponding wave front). Thus, the angles of incidence and refraction may be defined either in terms of the wave fronts and the refracting surface, or in terms of the rays and the normal to the refracting surface. The latter method is the more convenient when the direction of propagation is involved. Equation (7) shows that when a ray of light passes from a medium of relatively high into one of relatively low velocity (e.g., from air to glass) , the ray is bent toward the normal; the converse is true when the direction of the ray is reversed.

In the foregoing discussion it was assumed that the index of refraction is constant for any two media. However, this is strictly true only for monochromatic light.

Although the velocity of all wavelengths (colors) of light is the same in free space (and, practically, in air), in material substances, such as glass, water, etc., the velocity increases with wavelength. Consequently, the index of refraction decreases with increasing wavelength, as may be seen from a consideration of Eq. (2). The variation of index of refraction with wavelength is called dispersion. Owing to dispersion, when a ray of white light is incident' obliquely upon the surface of a transparent medium, the transmitted light is no longer white, for various wave lengths are refracted differently. Dispersion is an important consideration in the design of refracting instruments. For ordinary optical problems, the index of refraction is usually specified for sodium light (i.e., light corresponding to the yellow D lines in the sodium spectrum). The wavelength of sodium light lies in the region of maximum sensitivity of the human eye and is approximately the average wavelength for visible light.

As a consequence, of refraction, an object below the surface of a transparent substance, such as glass or water , appears to an observer above the surface to be nearer to the surface than it actually is. This is a familiar phenomenon to anyone who has ever attempted to estimate the depth of a clear pool of water. The amount of apparent elevation depends upon the depth of the object below the surface, the angle at which the object is viewed, and the index of refraction of the medium. The relationships may be seen from a consideration of Fig. 3. A ray of light OA from a point O,

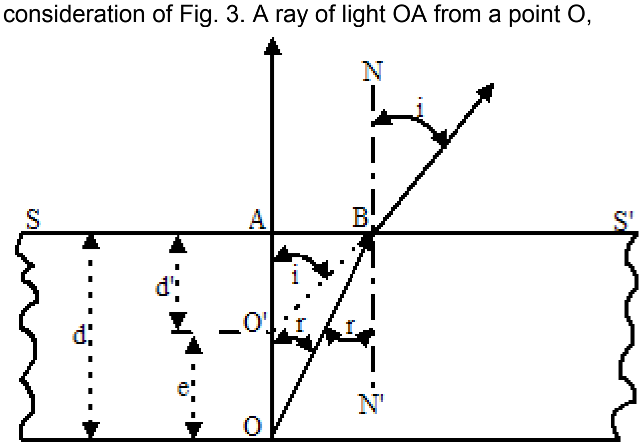

Fig. 3. Apparent thickness

striking the surface SS' normally at A, is undeviated since its angle of incidence is zero. Another ray OB, striking the surface obliquely at B, is refracted away from the normal and appears to an observer above the surface to have the direction O'B inside the medium. If the surface is viewed normally from above, and if the cone of light received by the observer is limited to small angles, all the refracted rays originating at O appear to have their origins at O'. The point O' is then the image of the point O. The distance OO' is the apparent elevation e, and the distance *O'A* is the apparent thickness *d'*. If the angles *i* and *r* are small, the distances *O'B* and *OB* are approximately equal to *O'A* and *OA*, respectively. Then in the triangles AO'B and AOB where the

$$\sin i = \frac{AB}{O'B} = \frac{AB}{O'A} \tag{8}$$

and

$$\sin r = \frac{AB}{OB} = \frac{AB}{OA} \tag{9}$$

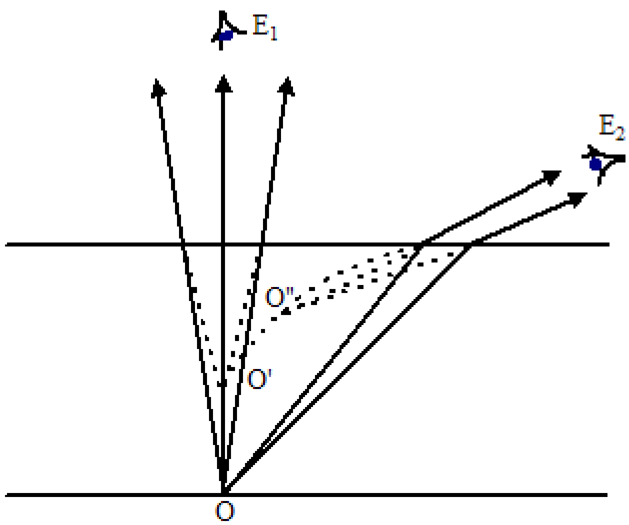

Fig. 4. Effect of viewing angle upon apparent displacement of image.

symbol = means "approximately equal." Dividing the corresponding sides of Eq. (8) by Eq. (9),

$$\frac{\sin i}{\sin r} = \frac{OA}{O'A} \tag{10}$$

Hence,

$$n = \frac{d}{d'} \tag{11}$$

where the depth *d* = *OA* and the apparent depth *d'* = *O'A.* Two essential conditions in the foregoing discussion are that the surface is viewed normally and that the cone of light received by the eye is small. If either (or both) of these conditions is not satisfied, the position and character of the image are affected. As represented in Fig. 4, the images O', O", etc., of the point O lie on a cuspidal curve represented by the heavy dotted line. Thus, when the eye at E_1 receives a small cone of light, the image is at O'; when the eye is at E_2, the image O" exhibits apparent lateral displacement as well as apparent elevation. If a cone of light of wide angle is received, the image is not sharp, since it consists of a series of images differently located.

APPARATUS: The principal piece of apparatus used in this experiment is the micrometer comparator illustrated in Figs. 5 and 6.


Fig. 5. Micrometer Comparator

It consists of a microscope M attached to a metal carriage block which slides in machined ways in a bed B consisting of a heavy iron casting. A vernier V on the sliding carriage coordinates with a 150mm silvered scale S attached to the bed. The vernier can be read to 0.01mm by means of an adjustable magnifier E. The microscope carriage is moved along the ways by means of a carriage screw operated by the knurled head K, and can be locked in position by the lock screw L. The microscope may be arranged with its axis in either of two mutually perpendicular directions in a plane at right angles with the ways, or with its axis parallel to the ways as shown in Fig. 6. A rack and pinion focusing arrangement is operated by the focusing knob F. The bed is machined on two surfaces so that it may rest either on one side or on one end.

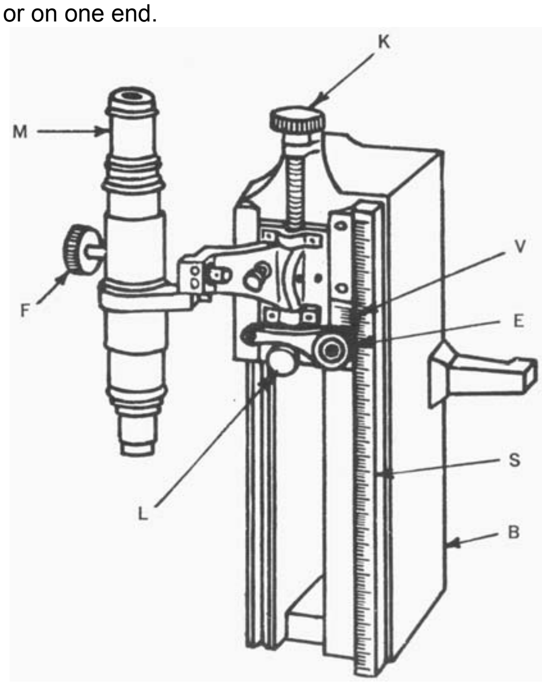

Fig. 6. Vertical arrangement of Comparator.

In addition to the comparator, the only accessory equipment consists of a shallow glass dish, a piece of window glass, lycopodium powder and specimens of glass and liquid to be tested.

PROCEDURE:
Experimental: Place the comparator in the position shown

in Fig. 6 on a table where the light is good. If necessary, a reading lamp provides good additional illumination. Make a fine scratch on apiece of window glass and lay it on the bottom of the dish with the scratch uppermost. Locate the dish so that the scratch is directly below the microscope. Using the rack and pinion, focus the microscope sharply upon the scratch. After this adjustment is made, the rack and pinion is not to be used. Read the vernier and record the reading v_1.

Place a specimen of the glass to be tested over the scratch and slowly elevate the microscope by means of the knurled screw K until the scratch is again in focus. Record the vernier reading v_2. The difference between v_2 and v_1 is the apparent elevation *e* of the scratch. Make a small ink spot on the top surface of the glass specimen. Continue to elevate the microscope with the screw K until the ink spot is in sharp focus. Record the vernier reading v_3. The difference $v_3 - v_1$ is the actual thickness *d* of the specimen, and the difference $v_3 - v_2$ is the apparent thickness *d'*. Several specimens are to be tested, two determinations being made for each specimen.

In making the determination for a liquid, the instrument is first focused on the scratch; three or four centimeters of liquid are then poured into the dish and the same measurements made as before. In order to locate the surface of the liquid, a small amount of lycopodium powder is sprinkled on the surface and the comparator is focused on the floating particles.

Analysis of Data: Compute the index of refraction of each specimen, using Eq. (11). Compare the experimental values with those given in a standard handbook and record the percentage differences.

QUESTIONS: 1. Explain why the method employed in this experiment requires that the cone of light intercepted by the telescope must be limited to a small angle about the normal.

2. Does a pool of water appear deeper when viewed normally, or obliquely? Explain.

3. How deep does a swimming pool 8ft deep appear to be when viewed normally?

4. If the absolute index of refraction of fused quartz is 1.460 and the relative index for light passing from quartz to Canada balsam is 1.048, what is the absolute index of Canada balsam?

5. A plate of crown glass 2cm thick is viewed normally. Find the minimum and maximum values of the apparent thickness when viewed by light at the extremes of the visible spectrum,

6. A plate of glass is covered with a layer of water. Prove that a ray of light emanating from the bottom of the glass plate and passing through the glass and water into the air above has the same direction in the air that it would have if there were no layer of water.

7. A vessel contains water to a depth of 5cm. Floating on the water is a layer of alcohol 3cm deep. What is the apparent distance of the bottom of the vessel below the surface of the alcohol?

8. A swimmer at the bottom of a swimming pool looks at an electric light above the surface. Draw the cone of light in the air and in the water.