CENCO Physics — Selective Experiments In Physics

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DEFLECTION OF A BEAM – YOUNG'S MODULUS

OBJECT: To study the manner in which the deflection of a beam depends upon its length, and to determine Young's modulus by the method of flexure.

METHOD: A uniform rectangular bar supported horizontally on two knife-edges is subjected to a vertical force applied midway between the supports. The deflection of the beam at the midpoint is measured by means of a micrometer screw equipped with an electrical contact. A series of observations is made with a constant load and a varying length of beam (distance between supports). From a logarithmic graph of the deflection versus the length, the mathematical relationship between deflection and length is ascertained. A second series of observations is made in which the load is varied, the length remaining constant. From the slope of a graph of load versus deflection, a value for Young's modulus is obtained.

THEORY: The bending of a beam is an example of the application of *Hooke's law*, which states that when an elastic body is deformed in any way the strain (which is a measure of the deformation) is directly proportional to the stress (which depends upon the deforming force), provided the body is not strained beyond its elastic limit. Special definitions of the terms stress and strain are required for each type of deformation. There are three distinct types of strain: *longitudinal strain*, in which a body is lengthened or shortened along one dimension; *shear*, in which contiguous planes in the body are caused to slip over each other, resulting in a change of shape; and *volumetric strain*, in which the volume is increased or decreased, the shape being unaltered. In each case the numerical value of the ratio between the stress and the strain depends upon a physical property of the material called an *elastic modulus*. For example, in the case of a rod or wire under tension (Fig. 1) Young's modulus is defined as the ratio between the force per unit cross-sectional area (stress) and the fractional change in length (strain). Thus

$$Y = \frac{F/A}{e/l} = \frac{Fl}{Ae} \tag{1}$$

where F is the tensile force, l the length, e the change in length, A the area of cross section and Y Young's modulus. Inasmuch as this experiment is concerned only with Young's modulus, the other two moduli will not be defined here. It should be remarked, however, that only in the case of small deformations can the situation be described in terms of a single type of strain.

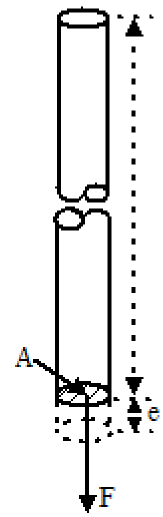

Fig. 1. Elongation of a rod.

Flexure (i.e., the bending of a rod or beam) is a type of deformation that, in general, involves both longitudinal strain and shear. However, in the case of slight bending, the effect of shear is negligible and the problem is essentially one of stretching and compressing. Consider a simple beam of width a (transverse horizontal dimension), height *b* (transverse vertical dimension), and length *l* (distance between supports), supported at each end and loaded at the middle (Fig. 2). When the beam is bent by the application of the load *W*, the layers on the convex (lower) side are lengthened and those on the concave (upper) side are shortened. An intermediate surface, called the *neutral surface,* experiences no change in length; thus, the stress in this portion of the beam is zero. In Fig. 2 the dotted line AB represents the intersection of the neutral surface with the plane of the paper. The longitudinal stress in the successive layers is, therefore, non-uniform, increasing with distance from the neutral surface. Hence, the situation is more complicated than in the case of simple stretching (or compression) represented by Fig.1 where the stress is uniform over the cross section of the body.

The *deflection* of a beam at any point is its vertical displacement from the unstressed position. It can be shown by means of the calculus that the deflection d of the mid-point of a centrally loaded simple beam of uniform rectangular cross section is given by

$$d = \frac{Wl^3}{4ab^3Y} \tag{2}$$

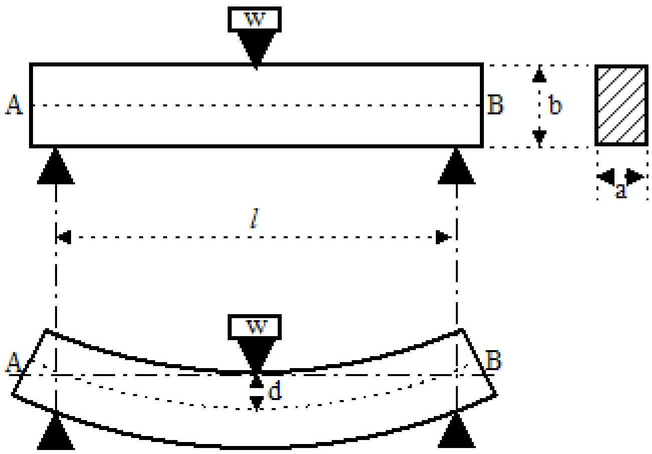

Fig. 2. Deflection of a simple beam.

For a circular beam of radius *r* the expression becomes

$$d = \frac{Wl^3}{12\pi r^4 Y} \tag{3}$$

Eqs. (2) and (3) show that the deflection of abeam loaded as in Fig. 2 is directly proportional to the cube of the length. This relationship has not been derived here mathematically, but it can be deduced from experimental data. Suppose a series of observations of *d* are made for various values of *l*, all other factors remaining constant. Eq. (2) or (3) may then be written

$$d = Cl^n \tag{4}$$

where *C* and *n* are constants. Taking the logarithm of Eq. (4)

$$\log d = \log C + n \log l \tag{5}$$

Thus, if the logarithm of *d* is plotted against the logarithm of *l* on Cartesian paper, or if *d* is plotted against *l* on log log paper, a straight line is obtained the slope of which is the exponent *n*.

Eqs. (2) and (3) also afford a means of determining Young's modulus. Solving for Y

$$Y = K\frac{W}{d} \tag{6}$$

where $K = \frac{l^3}{4ab^3}$ for a rectangle beam

or $K = \frac{l^3}{12\pi r^4}$ for a circular beam.

The slope of a curve of *W* versus d gives the ratio *W/d*.

APPARATUS: The flexure apparatus illustrated in Fig. 3 consists of a cast iron lathe bed with machined ways upon which slide two brackets each carrying a knife-edge. A third bracket located at the middle of the bed supports a micrometer screw. Each bracket is provided with an index which locates the center of the bracket with respect to a centimeter scale attached to the lathe bed. The beam consists of a rectangular metal bar one meter long which rests horizontally upon the two knife edges. A stirrup provided with an inverted knife edge hangs from the mid-point of the beam and carries a weight holder by means of which the load is applied. The tip of the micrometer screw makes contact with the stirrup thereby closing an electrical circuit containing a bell or lamp. A diagram of the arrangement is shown in Fig. 4. Auxiliary apparatus consists of a micrometer caliper, a set of 200gm weights, a dry cell, a switch, and a bell or a 1.5-volt flashlight bulb.

PROCEDURE:

Experimental: Part I. Determination of *n*. Adjust the knife

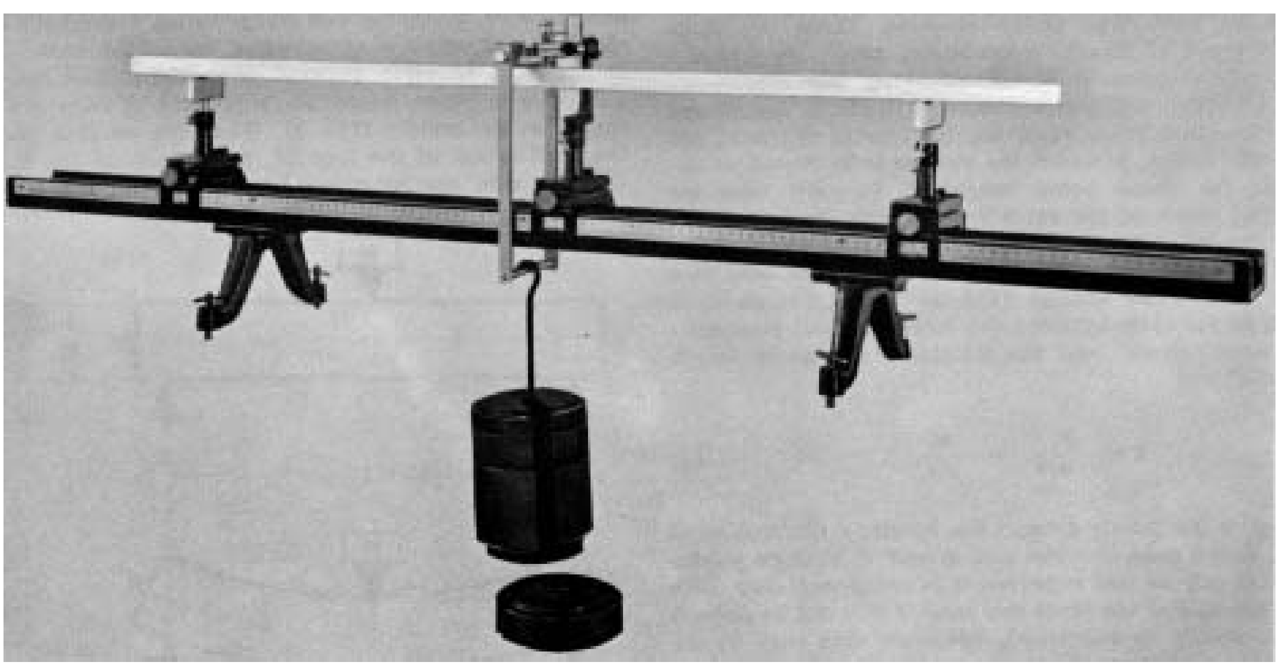

Fig. 3. Flexure Apparatus

edges in their brackets until they are the same height and at right angles to the lathe bed. Locate the brackets so that the knife-edges are exactly 40cm apart and equidistant from the center of the bed. Slip the stirrup on the rectangular bar and place the latter with its broad side resting on the knife-edges and with its center midway between them. Locate the center knife edge C (Fig. 4) exactly half way between the knife edge supports A and Band adjust it so that it is at right angles to the beam. Adjust the center bracket and the micrometer screw M until the latter is directly above the stirrup. Clamp the bracket and micrometer screw securely in place.

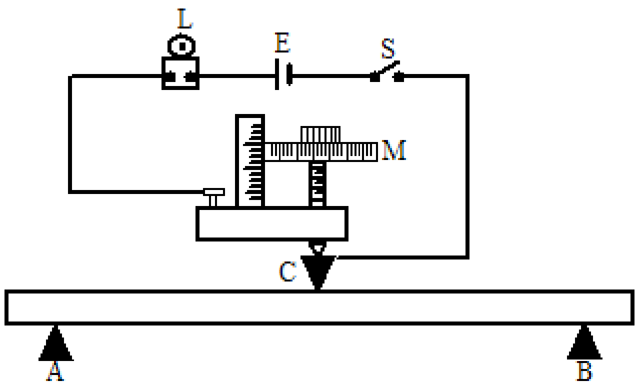

Fig. 4. Diagram of apparatus showing electrical connections.

Connect the battery E, lamp L and switch S in series with the micrometer screw and stirrup as shown in Fig. 4. Owing to a slight oxidation of the contacts it may be necessary to polish them lightly with fine sand paper or emery cloth to insure good electrical contact. All micrometer settings should be made with the screw *advancing*.

Attach the weight holder to the stirrup and adjust the micrometer screw until it barely makes contact with the stirrup as indicated by a signal from the lamp or bell. Read the micrometer and record this reading as the initial (zero load) setting. Place a load of 1000gm on the holder and adjust the micrometer screw for a second reading. Screw the micrometer back, remove the weight and repeat the zero reading, taking the average of the initial and return readings as the no-load setting. From the difference between the loaded and unloaded settings determine the deflection of the beam. Record all the data as indicated in Table I.

Move each knife-edge support 5cm farther out, making the length l 50cm, and repeat the observations using the same load. Continue in this manner, increasing l 10cm at a time until a total of six observations has been made.

Part II. Determination of Y. Keeping the length of the beam constant at 80 or 90cm, take a series of readings of the micrometer as the load is increased in steps of 200gm up to a total of 1200gm. Repeat the readings as the load is reduced in the same steps *being sure to screw the micrometer back before each weight is removed*. Record the data as shown in Table II. With the micrometer caliper measure and record the dimensions *a* and *b* in several places; compute the average values.

Interpretation of Data: From the data of Part I plot on log log paper the deflection in centimeters as ordinate versus the length in centimeters as abscissa. Compare the numerical value of the slope of the curve with the exponent of l in Eq. (2).

TABLE I

Length cm	Micrometer Reading			Deflection cm
	initial	loaded	return	

From the data of Part II plot on Cartesian paper the load in grams as ordinate versus the deflection in centimeters as abscissa. From the slope of the curve determine the ratio *W/d*. Express this ratio in absolute units and compute the value of Young's modulus by Eq. (6). Compare with the handbook value and record the percentage difference.

Optional Procedure and Analysis: 1. Repeat the experimental determinations of Parts I and II with the rectangular bar resting on its narrow side.

2. Repeat the experimental determinations of Parts I and II with the circular bar, see Eq. (2). *Caution: In using the round bar it is important that it should not rotate between observations, because if the bar is not perfectly straight rotation will yield unreliable values of the deflection*. A pencil mark along the diameter at one end will serve as an indicator of the position of the bar.

QUESTIONS: 1. In the determination of *Y*, which experimental measurements must be made with greatest accuracy? Explain.

2. In Part I would there be any advantage in using beams of different lengths instead of one beam? Explain.

3. Would the substitution of a beam of stiffer material have any effect upon the slopes of the graphs of Part I and Part II? Explain.

4. Compare numerically the deflections of a uniform beam having a cross section of 1cm by 2cm when laid on its side and on its edge, the load remaining constant.

5. Describe an experimental procedure for determining the exponent of bin Eq. (2). In what way would the slope of the corresponding curve differ from that of the curved plotted in Part I of this experiment?

TABLE II

Load gm	Micrometer Reading			Deflection cm
	increasing	decreasing	average	