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THE DIFFRACTION GRATING

OBJECT: To measure the wavelength of light with a diffraction grating.

METHOD: A slit in an opaque screen, illuminated with a sodium flame or other source of bright line spectrum, is viewed through a diffraction grating held near the eye. With the rulings of the grating parallel to the slit, several orders of spectra are seen on either side of the slit. The various spectral images are located by means of a transverse scale mounted beside the slit. From the known value of the grating space and from the measured distances between the slit and the grating and between the slit and the successive spectra, the wavelength of light is calculated.

THEORY:

The Diffraction Grating: Although structurally one of the simplest of optical instruments, the diffraction grating is the most powerful instrument ever devised for studying spectra. It consists essentially of a grid made by ruling a series of fine lines of uniform width and spacing either on a plate of transparent material or on a polished reflecting surface. Its action is a case of constructive and destructive interference between the light transmitted or reflected by various regions of the grating.

As a preliminary to the study of the diffraction grating, consider the following experiment first performed by the British scientist Thomas Young in 1801. Two narrow slits A and B are cut in an opaque screen and illuminated by a

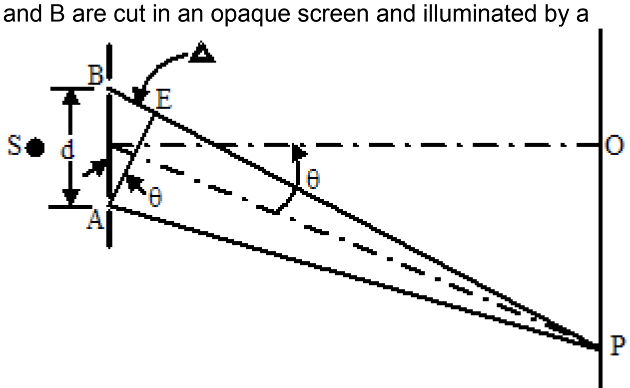

Fig. 1. Interference due to two slits.

centrally located source of light S (Fig. 1). When the light from the two slits is received on a white screen OP the illumination is found to vary in a peculiar manner, consisting of a bright band at O bordered on both sides by alternate dark and bright bands. The bright bands are due to constructive interference of the light from the two slits and the dark bands are due to destructive interference. The central band is bright because it is equidistant from A and B and the light from the two slits is therefore in phase at O. However, at some position P, the distance BP is greater than AP by one half wave length and the light arriving at P from the two slits is opposite in phase, resulting in destructive interference and darkness. As the point P is chosen still farther from O a position is found where BP is one whole wavelength greater than AP and the resulting constructive interference produces a bright band. Extension of this argument shows that when the distance BE = BP - AP is equal to an even number of half wave lengths there is a bright band, and when BE is an odd number of half wave lengths there is a dark band. The distance BE is called the "retardation" and is represented by the symbol Δ. The conditions for constructive and destructive interference can then be written

$$\Delta = n\lambda \text{ (constructive)} \qquad (1)$$

$$\Delta = \left(n + \tfrac{1}{2}\right)\lambda \text{ (destructive)} \qquad (2)$$

where λ is the wavelength of light and *n* is any integer. It is apparent from symmetry that the same situation exists on both sides of O.

A transmission grating consists of a great number of fine slits very close together. The principle may be understood from a study of Fig. 2 in which A, B, C, Dare parallel slits perpendicular to the plane of the paper and separated by a uniform distance d. Each pair of slits acts like the double slit in Young's experiment and, since *d* is a constant, the retardation Δ between adjacent slits is constant. Hence, the effects due to the successive slits are additive and the result is an increase in the amount of light transmitted.

The action of the grating can be described best by making use of a fundamental principle of wave motion advanced by the Dutch scientist Christian Huygens in 1678. According to Huygens' principle, a wave may be regarded as propagating itself by means of an infinite number of secondary wavelets. Every point in the wave front is considered as a source of wavelets and the envelope of all the wavelets at any time constitutes an advancing wave front. Diffraction phenomena are due to interference between wavelets from different portions of the same wave front.

In Fig. 2 let MN represent a plane wave front (surface of constant phase) in a monochromatic beam of light incident normally upon the grating. The interruption of the wave by the grating gives rise to a set of wavelets with their origins at the slits. These secondary wavelets are in phase at their origins. Any envelope of the secondary wavelets constitutes a new wave front indicating the propagation of light along a line normal to the wave front. The first and obvious envelope to consider is the one represented by the line M'N' parallel to

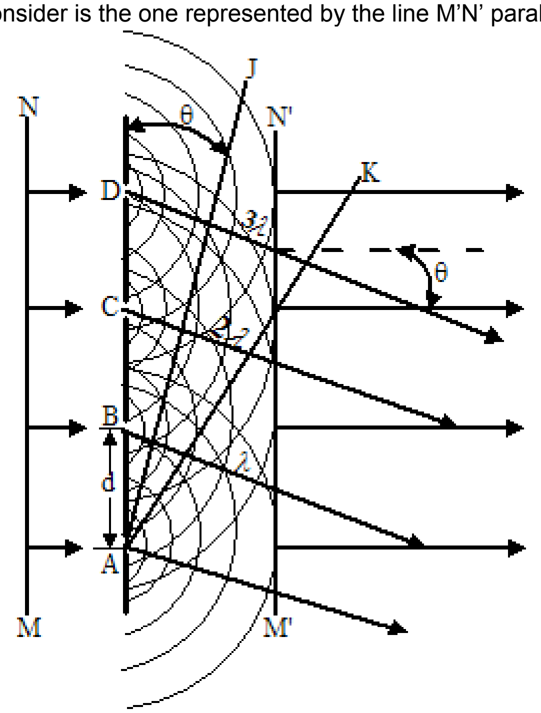

Fig. 2. Theory of the diffraction grating.

the original wave front. The only effect of the grating upon this part of the light is to reduce its intensity by cutting out parts of the beam. Thus, a lens placed after the grating as shown in Fig. 3 would produce a direct image at O precisely (except for the reduction in intensity) as if the rating were not there.

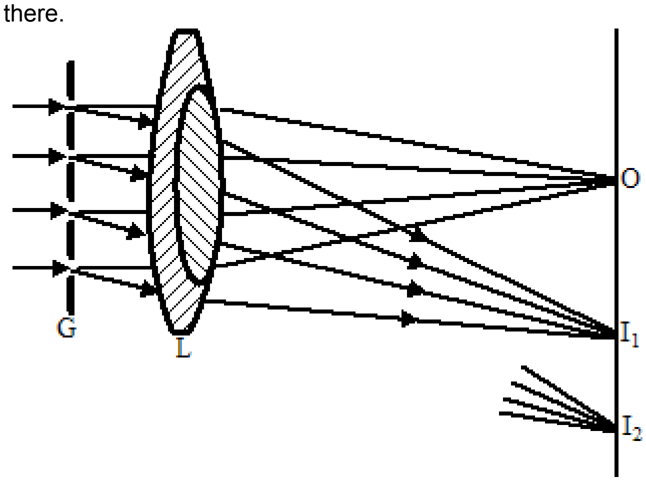

Fig. 3. Formation of first and second order images.

Another envelope, represented by the line AJ (Fig. 2) can be constructed so that it includes a given wave from A, the first preceding wave from B, the second preceding wave from C, and so on. Thus, AJ is also a surface of uniform phase and constitutes a plane wave front traveling in the direction indicated by the normals from the slits to AJ. Therefore, after passing through the grating, light is propagated not only in the original direction but at an angle determined by the condition for constructive interference, Eq. (1). Since $\Delta = d\sin\theta$, where θ is the angle the diffracted wave front makes with the plane of the grating (or the diffracted ray with the original ray), Eq. (1) may be written

$$d\sin = n\lambda \tag{3}$$

If *n* is 1, the wave front AJ gives rise to an image at I_1 (Fig. 3) called a *first order* image. If *n* is 2 the wave front AK is diffracted through a greater angle and a *second order* image is produced at I_2. Thus, immediately on either side of the central (direct) image O are the first order images flanked by the images of successively increasing orders.

If the source is *monochromatic* (one value of λ) the angle θ is definite and monochromatic images of the slit are formed at I_1, I_2, etc. If the source is *heterochromatic* (several values of A) the various wavelengths are diffracted at different angles and each image becomes a *spectrum*, composed of a series of monochromatic images, with the blue end nearest to the central image and the red end farthest away. The central image is not a spectrum but retains the composite color of the source, since at O the retardation is zero for all wavelengths.

Consideration of the grating formula, Eq. (3), shows that the number of orders that can be produced is limited by the grating space *d*, since θ cannot exceed 90 degrees. The highest order is then given by

$$n_{\max} = \frac{d}{\lambda} \tag{4}$$

Thus, a coarse grating (large *d*) gives rise to a large number of orders, while a finely ruled grating may be limited to one or two orders. On the other hand, it can be shown that the spectra produced by a fine grating is spread out more than one formed by a coarse grating. This can be demonstrated readily by looking at a source of light through gratings with different degrees of fineness.

An important property of a diffraction grating is its resolving power, which is its ability to separate two wavelengths closely together. The smaller the wavelength difference that can be resolved, the greater is the resolving power. It can be shown that the resolving power *R* is directly proportional to the total number of lines *N* in the grating and to the order of the spectrum *n*

$$R = Nn \tag{5}$$

Thus, increasing the total number of lines (with a given grating space), has two advantages; it increases the brightness of the spectrum by transmitting more light, and it increases the ability of the grating to analyze the spectrum. Resolving power thus becomes a function of the size (aperture) of the grating. Resolving power of a grating is analogous to the resolving power of a lens in which the aperture determines the brightness of the image and the ability to distinguish detail.

Diffraction gratings are made by means of a very accurate dividing engine which cuts, with a diamond point, very fine lines on a plate of glass or speculum metal. The principle of the reflection grating is the same as that of the transmission grating, the light being reflected from the spaces between the rulings. Often a reflection grating is ruled on a spherical

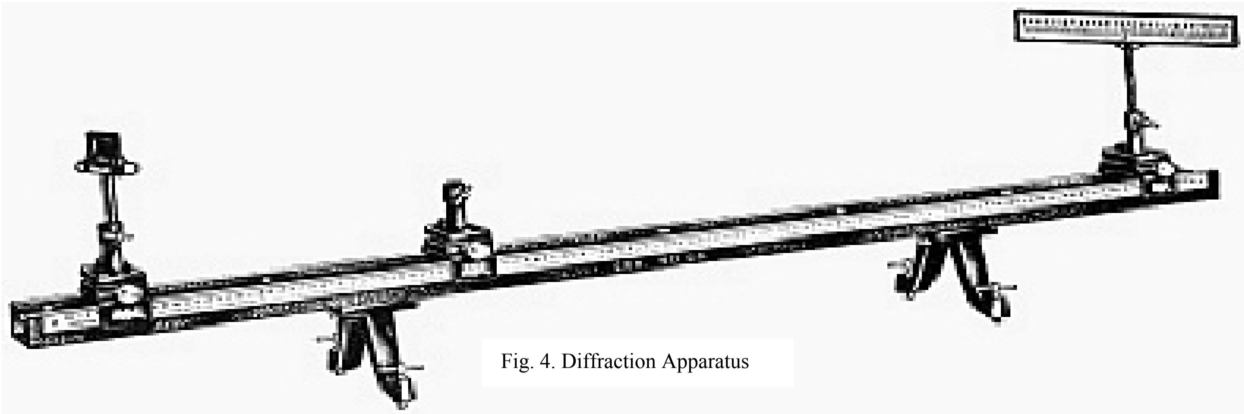

Fig. 4. Diffraction Apparatus

concave surface which acts like a convergent mirror, thereby eliminating the necessity of a lens. Inexpensive replicas of gratings can be made by making impressions in collodion. A specially prepared collodion flowed on the master grating forms a tough film which retains the impressions of the rulings.

After it has dried thoroughly, the film is stripped from the grating and mounted upon glass. Crude transmission gratings can be made by stretching fine wires between the threads of two parallel screws. The first grating ever used was constructed in this manner. The mesh of a wire screen can be used as a simple transmission grating. Apiece of phonograph record can be used as a reflection grating, the grooves acting as the rulings; indeed, surprisingly good measurements can be made in this way.

Spectra: When the spectrum of an incandescent solid such as a tungsten lamp is produced by a diffraction grating or by a prism, it is found to consist of a continuous band of color shading gradually from violet at the short wave length end to red at the long wave length end. Such a spectrum is called a *continuous* spectrum and is emitted by incandescent solids and liquids. "White" light is light containing all visible wavelengths in the relative intensities with which they are present in sunlight. On the other hand, the spectrum of a neon light or a mercury arc is found to consist only of certain colors which appear as bright lines separated by darker regions. Such a spectrum is called a discontinuous, or bright line, spectrum. Bright line spectra are produced by electrically excited gas discharge tubes and by metallic arc and spark discharges. The wavelengths and relative intensities of the lines in a discontinuous spectrum are characteristic of the atoms of the substance emitting the light. For this reason bright line spectra are often called characteristic spectra. The correlation between the characteristic spectra of the elements and their atomic structure is one of the fundamental objectives of the science of *spectroscopy*.

The frequency of vibration *n*, the wavelength λ, and the velocity *V* are related by the fundamental equation of all wave motion

$$V = n\lambda \qquad (6)$$

The wavelength of light may be expressed in centimeters, but it is often convenient to express it in terms of some smaller unit. The *micron* (μ) is defined as one millionth of a meter (10^{-4}cm). The Angstrom unit (Å) is equal to 10^{-8}cm. Thus, the wavelength of a certain yellow light might be given as 0.00006cm, 0.6 μ or 6000Å.

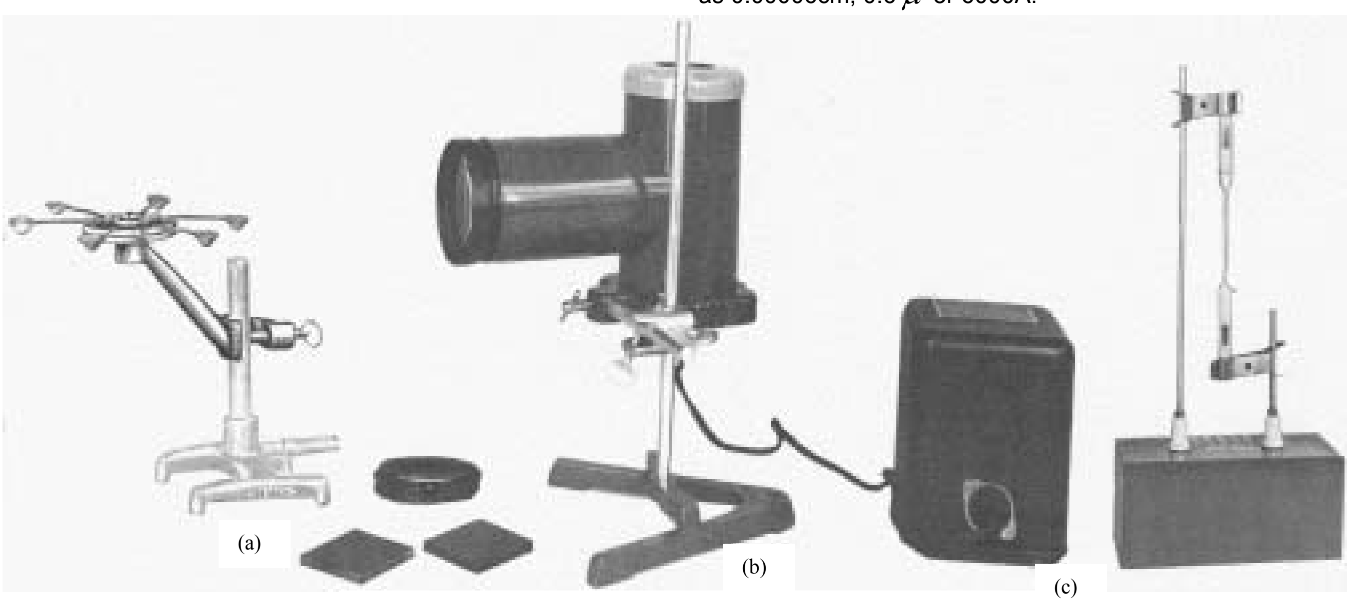

Fig. 5. Light Sources: (a) Flame, (b) Mercury Arc, (c) Gas Discharge Tube

APPARATUS: The apparatus consists of a slit and scale mounted upon one end of an optical bench with a transmission grating upon the other as shown in Fig. 4. Back of the slit is mounted a source of light which may be any of the sources shown in Fig. 5. One of the simplest and most convenient sources is a Bunsen burner provided with a grid upon which may be placed crystals of various salts. The flame spectrum of sodium is ideally suited to the purpose of this experiment since it is monochromatic, sufficiently intense, and easily produced. The mercury arc is an intense and highly satisfactory source, and a variety of characteristic spectra can be obtained readily by means of simple gas discharge tubes excited with a spark coil. The gratings to be used are collodion replicas having 10,000 and 25,000 lines per inch (Fig. 6). One or two coarse gratings are provided for qualitative observation and comparison.

Fig. 6. Diffraction Gratings

PROCEDURE:

Required Experimental: Mount the slit and scale on one end of the optical bench and adjust so that the scale is at right angles to the bench. Mount one of the coarse gratings in the support at the other end. Set up the light source to be used and look toward it through the grating with the eye held close to the grating. On either side of the slit will be seen a series of images of decreasing intensity. In this case the lens and retina of the eye take the place of the lens and screen of Fig. 3. The situation is represented schematically in Fig. 7. Count the number of orders that can be observed.

Replace the coarse grating with the 10,000 line one and adjust it about its vertical axis until the two first order images are equidistant from the slit. With the eye close to the grating observe the positions of all the images on the scale and record their distances s_l, s_2, etc., from the slit. Measure the distance l from the slit to the grating. For any given image the diffraction angle θ is given by the relationship

$$\tan\theta = \frac{s}{l} \quad (7)$$

Making use of a trigonometric table, determine θ_1 and θ_2 from the ratios s_1/l and s_2/l. Substitute in Eq. (3) and obtain two independent determinations of the wavelength.

Repeat the experiment with the finer of the two gratings. Compare the average of these values with the handbook value. Illuminate the slit with white light and determine the approximate limits of the visible spectrum.

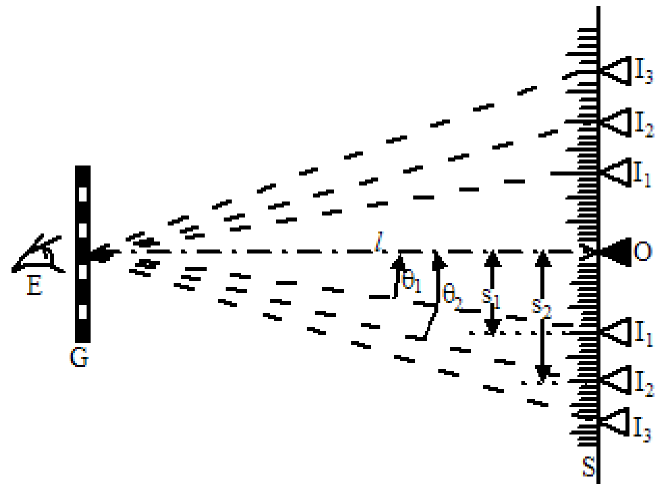

Fig. 7. Direct viewing method of observing diffracted images.

Optional Experimental: Substitute a second source of unknown identity for the first one and, using the fine grating, determine the wavelength of the prominent lines. Attempt to identify the source by comparing the wavelengths of the observed lines with the characteristic spectra of the elements in a handbook.

QUESTIONS: 1. With a grating of 10,000 lines per cm at a distance of 2 meters from a slit illuminated by a mercury arc, what would be the linear separation between the two components of the yellow doublet (5770Å and 5790Å) of mercury?

2. With infrared light of wavelength 8μ how many orders would be produced by a grating having 500 lines per centimeter?

3. What are the maximum and minimum vibration frequencies of visible light?

4. What advantage, if any, would a concave reflection grating have over a plane grating for experiments with ultra violet light?

5. It is customary to speak of lines in a spectrum. Why is each frequency represented by a *line*?