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NEWTON'S SECOND LAW OF MOTION

OBJECT: To measure experimentally the linear acceleration of a system moving under the influence of a constant force; and to study the relationships between force, mass, and acceleration.

METHOD: The moving system consists of a loaded car on a horizontal track, connected with a falling mass by means of a flexible cable passing over a pulley. The accelerating force is the pull of gravity on the suspended mass and the total inertia of the system is composed of the combined mass of the car and the falling body. The acceleration is obtained from an electrical record of the distances traveled in successive equal intervals of time.

THEORY: The basis of our present system of mechanics rests upon three postulates advanced by Sir Isaac New-ton in his *Principia* (1686), and known as Newton's laws of motion. These laws may be stated as follows:

I. By virtue of the inherent and universal property of matter called inertia, every body tends to maintain its existing state of rest or uniform velocity unless acted upon by some external force.

II. The time rate of change of the momentum of any body is directly proportional to the force acting upon it, and takes place along the direction of that force.

III. The force exerted by one body upon another is always accompanied by an equal and opposite force exerted by the second body upon the first one.

The term *momentum* was coined by Newton to signify what he called the "total motion" of a body, which involves both the amount of matter in motion and the velocity. The defining equation for momentum is thus

$$M = mv \tag{1}$$

where M = momentum, m = mass, v = velocity. Writing the initial momentum M_o as mv_o and the momentum M at the end of an interval of time t as mv, Newton's second law may be written

$$\frac{mv - mv_o}{t} \propto F \tag{2}$$

Factoring out the mass (which is a constant for all ordinary types of motion)

$$m\frac{v - v_o}{t} \propto F \tag{3}$$

Now acceleration is the time rate of change of velocity, i.e.,

$$a = \frac{v - v_o}{t} \tag{4}$$

Hence,

$$ma \propto F \tag{5}$$

This proportionality may be written as an equation by introducing a constant. Thus, finally

$$F = kma \tag{6}$$

where the value of k, the constant of proportionality, depends only upon the units used. An "absolute" system of units is one based upon the fundamentally established units of mass, distance, and time, and in which the force unit is so defined as to make k unity. Thus, when absolute units are used Eq. (6) becomes

$$F = ma \tag{7}$$

An absolute unit of force is, therefore, the amount of force required to impart unit acceleration to unit mass. In the c.g.s. absolute system the *dyne* is the force required to give to a mass of 1gm an acceleration of 1cm/sec^2. In the British absolute system the *poundal* is the force required to give to a mass of 1lb an acceleration of 1ft/sec^2.

A gravitational system is one in which the unit of force is defined as the pull of gravity on a unit mass at sea level. The value of k is then determined experimentally from the acceleration experienced by a body moving under the force of gravity, i.e., under its own weight. The acceleration of a freely falling body has been found to have an approximate value at sea level of 980cm/sec^2 or 32.2ft/sec^2. Thus, the value of k is 1/980 when the c.g.s. gravitational unit of force is used, and 1/32.2 when the British gravitational system is used. It is readily seen, therefore, that the c.g.s. gravitational unit of force (gm-wt) is equal to 980 dynes, and the British gravitational unit of force (lb-wt) is equal to 32.2 poundals.

Eq. (7), which is the customary way of writing Newton's second law, states that the acceleration of a body is proportional directly to the force acting upon it and inversely to the mass of the body. These relationships can be verified by graphing two sets of experimental data, viz., acceleration as a function of the applied force, the mass being constant; and acceleration as a function of the mass, force constant.

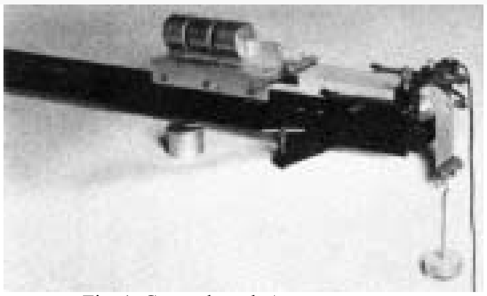

Fig. 1. Car-and-track Apparatus

APPARATUS: The car-and-track apparatus employed in this experiment is illustrated in Fig. 1 and represented diagrammatically in Fig. 2. The track U is a rigid iron casting with machined rails upon which the car travels. Leveling screws LL' provide for a limited adjustment of the inclination. The car A and the falling mass B are connected by a strip of paraffin coated paper tape T upon which the record of the motion is electrically registered. A and B, therefore, constitute a system in which the total mass m is equal to $m_1 + m_2$ and the accelerating force is numerically equal to m_2 in gravitational units or m_2g in absolute units, m_1 and m_2 being the masses of A and B, respectively. Eq. (7) may then be written

$$m_2 g = (m_1 + m_2)a \tag{8}$$

from which,

$$a = \frac{m_2}{m_1 + m_2} g \tag{9}$$

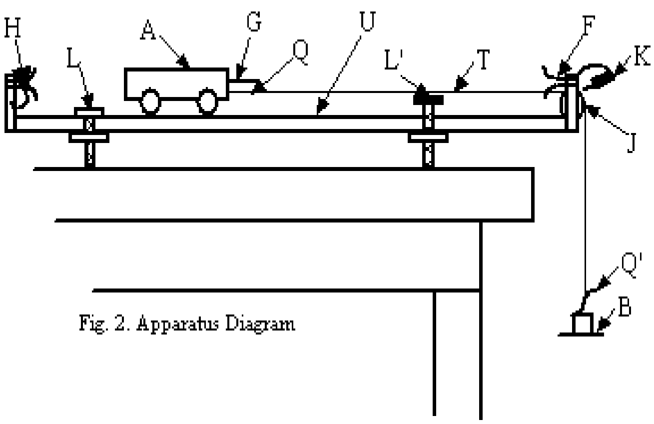

Fig. 2. Apparatus Diagram

The tape passes over a light pulley J and is attached to A and B by means of clips Q and Q'. The shock of impact at the foot of the incline is partially absorbed by a buffer F which coordinates with a bumper G on the car. An insulated spark terminal K may be clamped at various positions along the pulley.

As the system moves under the influence of the pull of gravity on the mass B, the tape passes over the pulley and is punctured periodically by a series of sparks that are caused to pass at regular intervals between J and K. The sparks melt the paraffin coating leaving a sequence of dots to mark the passage of the system. The distance between successive dots, therefore, indicates the distance through which the system moves in the corresponding interval of time. The sparks are generated by an induction coil, one of the secondary terminals being connected to the spark terminal K, the other being grounded.

The spark control system, represented diagrammatically in Fig. 3, contains two specially designed units: the spark timer (Fig. 4) which regulates the spark frequency, and the impulse counter (Fig. 5) which indicates the number of sparks occurring in a given time. The spark timer is an electrically driven vibrating bar V the frequency of which can be varied by altering the amount and position of the loading weight W. Large changes in frequency can be made by interchanging vibrator bars. The vibration is maintained by

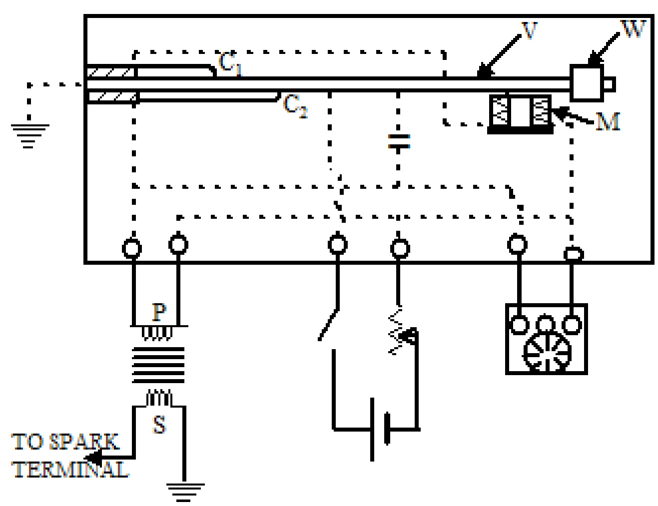

Fig. 3. Spark Control System

an electromagnet M, the current to which is interrupted periodically by means of a pair of contacts C_1. The vibration of the bar operates a second and independent set of contacts C_2 which makes and breaks the current in the primary of the induction coil. The impulse counter is essentially a magnetic escapement by means of which a sweep hand is caused to move over one division of a dial at every magnetic impulse. The mechanism is set in operation by depressing a button which closes the circuit to the counting magnet. The button may be locked down by rotating it slightly. The dial is graduated in 60 divisions and the number of revolutions of the sweep hand is indicated by a small hand moving over a secondary dial. The spark

interval is determined by observing the number of impulses registered in a given time.
Additional apparatus consists of a 6volt battery, an induction coil, a 20ohm rheostat, a switch, a clock, a meter stick, a vernier caliper, a platform balance and a set of weights.

PROCEDURE:

Experimental: Place the track with its lower end projecting well over the end of the table, and with the leveling screws LL' adjust the inclination until the car moves without acceleration when given a slight downward velocity. This adjustment is a partial compensation for friction. With the car resting against the buffer at the foot of the incline, attach a strip of tape and suspend a weight hanger from the other end of it. It is well to double the paper at the ends where the clips are attached. The length of tape should be such that the weight hanger strikes the floor just before the car reaches the foot of the incline. Place the car in the starting position at the top of the incline and anchor it with the hook H.

Fig. 4. Spark Timer

Connect the electric circuit as shown in Fig. 3 grounding the track, the spark timer, and one of the secondary terminals of the induction coil. *Have the circuit inspected by the instructor before closing the switch*.

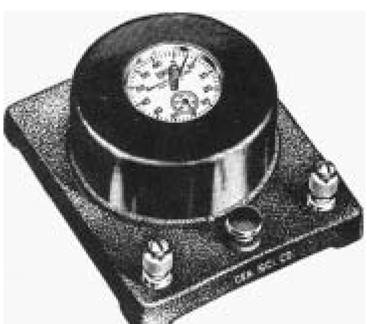

Fig. 5. Impulse Counter

Adjust the frequency of the timer so as to give approximately 10 sparks per second, and determine the exact value of the time interval by observing the number of impulses registered in 1 minute. Make three observations and take the average value. Place the spark terminal K near the side of the track so that several records may be taken on one tape.

A. Variation of a with F, m constant. In this part of the experiment the total mass $m = m_1 + m_2$ is to be kept constant, the accelerating force being changed by shifting mass from A to B. Determine the masses of the car and the weight hanger, including the clips Q_1 and Q_2. Add weights until m_1 = 3900gm and m_2 = 100gm. After making one or two trial runs, close the switch and note whether or not the spark is occurring regularly. When everything is working properly, release the car and make a run. As soon as the run is completed, open the switch and examine the record. Replace the car in its starting position and shift the spark terminal K about a centimeter or so. Transfer 100gm from A to 9 and make another run. Continue in this way, each time transferring 100gm from A to B until a total of 4 records have been obtained. Tabulate the data as shown in Table I. Remove the tape, replace it with anew one, and prepare for the second part of the experiment.

B. Variation of *a* with *m*, *F* constant. In this part of the experiment m_2 is to be kept constant at 200gm. Remove all the weights from the car and take a run with the empty car. Add a 500gm mass and repeat. Continue increasing mL in 500gm steps until a total of 4 observations have been made. Tabulate the data.

Analysis of Data:

Required Analyses: The distance between any two successive dots divided by the time interval between sparks gives the average velocity of the system for the particular interval concerned. If the spark interval is taken as an arbitrary unit of time, the average velocity in centimeters per interval is numerically equal to the distance in centimeters between successive dots. The acceleration is to be determined from two such measurements of velocity, one (v_1) near the beginning of the record, and another (v_2) near the end. Using a vernier caliper, determine the extreme values of v_1 and v_2 for all the records. Enter these values in columns 3 and 4 of Table I. In each case count the number of time intervals during which the velocity increased from v_1 to v_2, remembering that the average velocity is the actual velocity at the *middle of the time interval*. Enter these values of the time in column 5. Dividing the difference between corresponding terms of columns 3 and 4 by the number of time intervals from column 5 gives the acceleration in *centimeters per interval per interval*. Enter these experimentally determined values of acceleration in column 6. Express the experimentally determined values of the acceleration in cm/sec^2 and record in column 7. Compute the theoretical acceleration by Eq. (9), and enter in column 8.
The data can best be interpreted by plotting two curves. From the first half of the data plot ***a*** as the ordinate and m_2 as the abscissa. From the last half plot ***a*** as the ordinate and 1/*m* as the abscissa, remembering that *m* represents the total mass of the system.

Optional Analyses: 1. On one of the best records choose one of the first dots as an initial point and measure the distances, s_1, s_2, s_3, etc., traveled in 1, 2, 3, etc., units of time. Plot the data on log-log graph paper, or on Cartesian paper plot the logarithm of distance against the corresponding logarithm of time. From the slope of the curve determine the relationship between distance and time. Also find the value of the acceleration from the curve. In making this analysis the student will find it profitable to read the instruction sheet on graphical representation of data.

2. On Cartesian coordinate paper plot a curve of s versus t_2. Determine a from the slope of this curve.

3. Compute the tension in the tape for the case in which the acceleration was a maximum.

QUESTIONS: 1. Are the curves consistent with Eq. (6)? Explain.

2. Explain the discrepancy between observed and expected values of a.

3. Is there any advantage in making the pulley J and the car wheels as light as possible? Explain.

4. What happens to the tension in the tape when the car is released? Explain.

5. What is the weight in absolute units of a mass of 5lb?

6. Express the value of g in cm/int^2.

7. As the system travels what effect does the increasing amount of tape below the pulley have upon the acceleration of the system? Is this effect important in this experiment?

TABLE I

1	2	3	4	5	6	7	8
m_1 gm	m_2 gm	v_1 cm/int.	v_2 cm/int.	t intervals	a cm/int.^{2}	a cm/sec.^{2}	a (theoretical) cm/sec.^{2}