CENCO Physics Selective Experiments In Physics

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NEWTON'S SECOND LAW OF MOTION: ATWOOD'S MACHINE

OBJECT: To study Newton's second law of motion; in particular to show that accelerating force is proportional to the product of the mass by the acceleration of the body acted upon.

METHOD: A modified Atwood's machine, Fig. 3, consisting a cord passing over a light pulley, makes it possible to measure the acceleration of the known masses and thus to check Newton's second law of motion. The unbalanced weights produce an acceleration which is observed by means of the traces formed by a vibrating tuning fork constituting a part of one of the accelerated bodies. The acceleration is measured by noting the gain in distance which the fork passes over during successive equal time intervals. Different accelerations are produced by changing the weight of the bob which balances the tuning-fork frame. By taking observations of the accelerations produced by two different known forces acting upon known masses, it is possible to verify the working equation set up assuming the validity of Newton's second law of motion, and involving only these measurable factors.

THEORY: Most of mechanics is based upon three great experimental principles, first clearly stated by the great English mathematical physicist, Sir Isaac Newton, and commonly known as "Newton's laws of motion." They may be stated in modern form as follows:

1. *Every body continues in its state of eat or uniform motion in a straight line except in so far as it is compelled by some external force to change that state.*
2. *The force required to accelerate a body varies directly with the mass of the body and with the acceleration produced, and the acceleration takes place in the direction of the accelerating force.*
3. *The acting force upon a body and the resulting reacting force which the body exerts are always equal and oppositely directed.*

Newton's second law of motion may be symbolically stated as follows:

$$f \propto m$$

$$f \propto a$$

or $$f \propto ma \qquad (1)$$

whence $$f = kma \qquad (2)$$

where f is the accelerating force acting on a body of mass *m*, and *a* is the resulting acceleration. The proportionality constant k is inserted to change the proportionality sign to one of equality, in accordance with a well-known principle of algebra.

The numerical value of the dimensionless factor *k* depends only on the units selected for the various quantities. If the absolute c.g.s. units are used, namely unit mass as 1gram and unit acceleration as 1cm/sec^2, then the unit force is that force which will impart to a 1gram mass an acceleration of 1cm/sec^2. This unit of force, which makes k = 1, is called the *dyne*.

For much commercial work there has grown up a somewhat misleading custom of using the term "gram" as a unit of *force* as well as of *mass*, although the two concepts are entirely distinct. In this "gravitational" system the gram of force is defined as the weight of a 1gram mass. If this mass is allowed to fall freely under gravity, it moves with the acceleration *g*, the value of which is approximately 980cm/sec^2. Since from the force equation this force, in dynes, equals the mass, in grams, multiplied by the acceleration, in centimeters per second per second, it follows that the weight *W* of a 1-gram mass (which is being called a force of one "gram") is 1 x 980 = 980dynes. Or in general symbols, since *a* = *g* when *f* = *W, W* = *mg*. The term "gram-weight" is used by some physicists for this "gravitational" unit of force, in an endeavor to avoid confusion with the gram of mass. Since this experiment is concerned merely with the ratios of forces, it is immaterial which system of units is used.

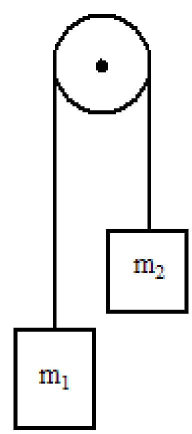

Fig. 1. Idealized diagram of Atwood's Machine

The general outline of the method to be used in showing that $f \propto ma$ may be made clear by an illustration. Consider two equal masses M_1 and M_2 connected by a light flexible cord passing over a practically frictionless light pulley (Fig. 1). Since the masses are equal, the system is in equilibrium and it will remain at rest; or, if given an initial impulse, it will continue to move with uniform speed, i.e., without acceleration. Now let a small object of weight w_1 be taken off M_1. The system is no longer in equilibrium but is acted upon by an accelerating force w_1. The system will, therefore, move with accelerated motion. Let the acceleration be represented by a_l. From the fundamental force equation the following may be written:

Accelerating force = k × (total mass moved) × acceleration

$$w_1 = kMa_1 \tag{3}$$

where M is the total mass accelerated.

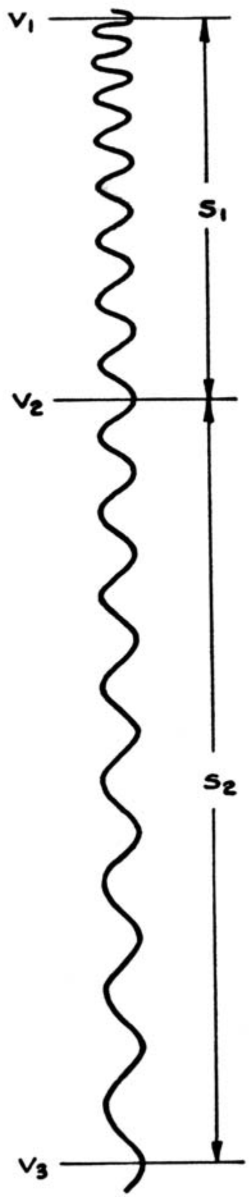

Fig. 2. Symbolic notation used in measuring acceleration.

If a larger object of weight w_2 be taken from M_1, the acceleration will be larger and may be represented by a_2. The corresponding force equation will now be

$$w_2 = kM'a_2 \tag{4}$$

where M' is the total mass moved for this case.

If Newton's second law of motion be true, it should therefore be possible to write from Eqs. (3) and (4) the direct proportion

$$\frac{w_1}{w_2} = \frac{kMa_1}{kM'a_2} \tag{5}$$

This experimental confirmation of the second law will consist in showing the correctness of the proportion given in Eq. (5). To do this it is only necessary to apply two known forces to a balanced system of known masses such as shown in Fig. 1, and to measure the corresponding accelerations thereby produced.

Calculation of the Acceleration: A simple analytical method for determining the value of the acceleration of the tuning fork is as follows: Divide the entire trace made by the fork (Fig. 2) into two equal time intervals. Let v_1 be the velocity at the beginning of the first interval, v_2 the velocity at the end of that interval and also at the beginning of the next, and v_3 the velocity at the end of the second interval. Designate by s_1 the distance fallen time interval t and by s_2 that fallen in the second time interval. A familiar equation for the space traversed in uniformly accelerated motion then lives

$$s_1 = v_1 t + \tfrac{1}{2}at^2 \tag{6}$$

$$s_2 = v_2 t + \tfrac{1}{2}at^2 \tag{7}$$

Subtracting Eq. (6) from Eq. (7) gives

$$s_2 - s_1 = v_2 t - v_1 t = (v_2 - v_1)t \tag{8}$$

Dividing both sides of Eq. (8) by t^2

$$\frac{s_2 - s_1}{t^2} = \frac{v_2 - v_1}{t} \tag{9}$$

But the right-hand member of Eq. (9) 18 the acceleration a, hence

$$a = \frac{s_2 - s_1}{t^2} \tag{10}$$

From this equation the acceleration can be calculated, using the measured values of s_1, s_2, and t. The time interval t is obtained by dividing the chosen number of vibrations of the fork used for the time interval by the frequency of the fork. In Fig. 2 eight vibrations are used. If the frequency were 128vib/sec, t would be 8/128 sec.

APPARATUS: The principal piece of apparatus is a modified form of the device known as Atwood's machine (Fig. 3). It consists of a carriage C containing the time-tracing tuning fork F, with stylus S attached to one prong. The carriage is counterbalanced by the weight of an adjustable bob (not shown in Fig. 3) connected to the fork frame by a strong cord passing over alight and nearly frictionless pulley K. When a part of the mass of the bob is removed and the carriage is released from its uppermost position by withdrawing the catch H, it falls with an accelerated motion between the vertical guides G and G' and is caught in the dashpots D and D'. The friction between the carriage and the guides is small when the apparatus has been properly leveled. The tuning fork is electrically driven so that constant amplitude of vibration is maintained throughout the time of fall. Consequently the waves which traced by the stylus on the coated surface of the plate P are of the same amplitude, which facilitates the location of the crests of the waves and therefore the accurate determination of the distances

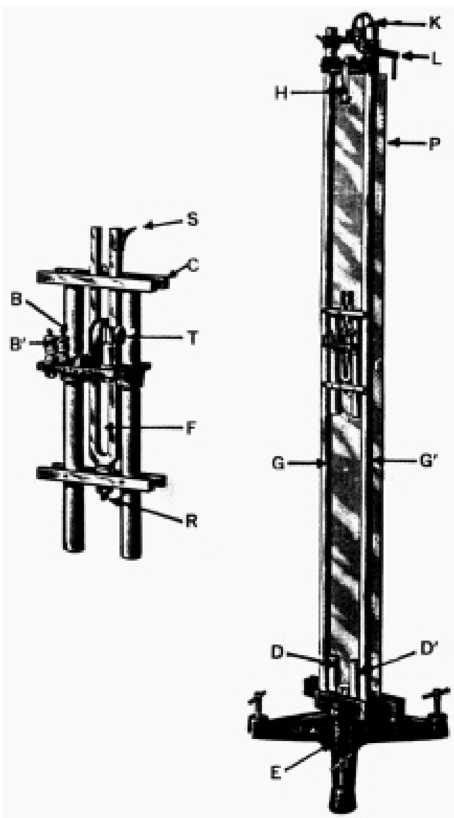

Fig. 3. Modified Atwood's Machine (detail of tuning fork mounting shown at left).

corresponding to the equal time intervals. Typical traces are shown in Fig. 4. The strip of heavy plate glass P is held in grooves at its lower and upper ends. The plate can be shifted horizontally so that a number of tracings can be obtained.

The surface of the plate on which the records are to be traced may be prepared in several ways. It may be coated with carbon from the smoke of a gum camphor flame; or a suspension of whiting or lamp black in alcohol may be applied with a camels' hairbrush. Another form of coating is Bon Ami applied with a moist cloth or sponge.

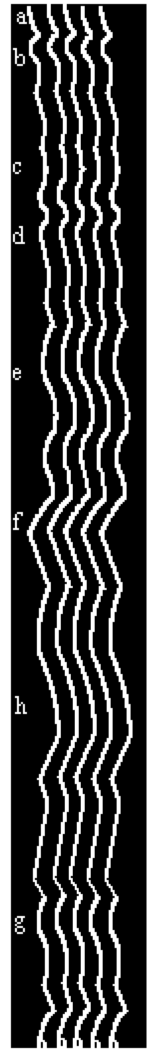

Fig. 4. Typical traces made by a falling fork.

The most satisfactory method of obtaining a permanent record of the traces is the use of two specially-prepared paper strips, held against the glass plate by means of two clamps. These strips are made of a coated paper such that a clear impression is obtained for a permanent record. When the records have been made on a strip it can be torn off at the clamps and removed for the measurements. The additional strip permits a second Student or group of students to use the apparatus without the interruption of having to prepare the surface of the plate. The strips with the tracings constitute permanent records which should be filed with the report of the experiment.

The auxiliary apparatus required includes a four to six volt storage battery, meter stick, double-pole, single-throw switch, plumb bob, small celluloid triangle, about 10 special 100gram slotted weights, a set of slotted iron weights ranging from 5 to 500grams, and trip scales.

PROCEDURE:

Experimental: With the carriage at its uppermost position, attach the plumb bob to R and adjust the leveling screws in the base E until the bob hangs over the center of the bolt head between the dashpots D and D'. While the glass plate is in position and *before* its surface has been prepared, adjust the stylus S so that it presses lightly on plate P. The pressure of the stylus on the plate should be sufficient to produce a good trace; if too much pressure is applied, errors due to friction will be increased. See that the dash pots D and D' are tightened.

Vary the weight of the counterpoise until the system is so adjusted that the fork, after being given a slight downward push, will continue to move *downward* with a *uniform speed*. This means that the weight of the counterpoise is less than that of the carriage by an amount equal to the effects of friction. Note that this adjustment is possible only for a *downward* motion of the carriage; it cannot move equally well up or down, since friction always opposes the motion.

In making the final adjustment of the balancing weight to produce this condition of equilibrium, have the coated plate against the stylus, as its friction is necessarily appreciable.

Arrange the wheel and the stop L, which catches the counterpoise at the end of its upward motion, so that the cord does not rub anywhere. Adjust the length of the cord and the position of the stop until the bob is just caught in the stop as the carriage strikes the dashpots.

To set the tuning fork into vibration, close the switch from the battery to the binding posts B and B'. Give the fork an initial impulse by squeezing the prongs together with the fingers and then allowing them to spring apart. The small screw T, which periodically interrupts the electrical circuit and thus maintains the fork in vibration, should be so adjusted that the sparking at the contact is a minimum. Adjust the voltage of the battery so that an amplitude of vibration of several millimeters is obtained. *Do not keep the switch closed when the fork is not vibrating* as this tends to overheat the coil. As soon as the fork has fallen, open the electrical circuit. Tilt backward before bringing the carriage up again. See that the dashpots D and D' are kept tightened.

Having made the necessary preliminary trial falls with the counterpoise adjusted for equilibrium, i.e., for the carriage to fall at *uniform speed*, remove a suitable accelerating weight from the bob. The weight removed should be approximately half of the total which is removable; 200 or 300grams is suggested. Then release the catch and allow the fork to fall. *Stand by to catch the counterpoise in the hands just at the instant it reaches the top stop*. This will assist in preventing the cord from running off the pulley, thereby allowing the bob to fall with ajar which often results in a broken cord and sometimes the breaking of the expensive glass plate.

Obtain one good trace on each of the two paper strips for the first accelerating force. Then remove a second weight, equal to the first, so that the total accelerating force will be twice as large as in the first case. Obtain one good trace on each of the two strips for the second force.

Record the total mass moved in each case. This will be the sum of the mass of the carriage (stamped on the frame) and that of the counterpoise *after* the accelerating weight has been removed.

At the end of the period leave the glass plate and the balancing bob on the apparatus.

Tabulation and Interpretation of Data: Remove the paper (or the coated plate, if that method is used) and stretch it out, coated side up, on a flat surface in good light. Beginning near the top, pin-mark, as finely and as accurately as possible, the crest of one of the first distinct waves. This can best be done by placing a small transparent right-angled triangle transversely across the crest and scribing with a needle point a fine line bisecting the crest and extending about 1/2cm above it. Count and record the total number of wave crests in the entire trace and divide this number by two, so that two equal time intervals are obtained. Make a line, similar to that on the first crest, at the crest for the end of the first and second time intervals. Place a meter stick edgewise on the trace so that the graduations are directly touching it. Measure carefully the distances s_1 and s_2 between the lines, estimating each to fractions of millimeters. Record these distances for each of the four good traces. Note the frequency of the fork, stamped near its base. From this frequency and the number of waves used in measuring the two distances, compute the time interval t.

Calculate the respective accelerations from Eq. (10). Finally, substitute the appropriate values in the working formula, Eq. (5), and determine the percentage difference between the ratio of the two accelerating forces and the corresponding products of the masses by the accelerations. The working formula may be restated as follows:

$$\frac{w_1}{w_2} = \frac{Ma_1}{M'a_2} = \frac{(M_1 + M_2 - m_1)a_1}{(M_1 + M_2 - m_2)a_2} \qquad (5a)$$

where m_1 is the mass removed from the bob in the first case, m_2 the *total* mass (the sum of m_1 and the second mass) removed for the second case, and the other symbols have the meanings previously given.

QUESTIONS: 1. Is the weight of a body the same thing as its mass? Discuss briefly. Is the weight of a body constant at all places on the earth? Is the mass?

2. In each of the following cases, state how the readings of a spring balance will compare if the balance is attached to the top of an elevator and supports a constant mass. State reason in for each case: The elevator is (a) at rest; (b) moving with constant speed upward; (c) moving with constant speed downward; (d) moving upward with steadily increasing speed; (e) moving downward with steadily increasing speed; (f) moving downward with acceleration g; (g) rising with uniformly diminishing speed.

3. Discuss the relative increase in accuracy in the above experiment by a possible decrease in the error in the measurement of the following: (a) weight of frame and fork; (b) weight of balancing mass; (c) frequency of fork; (d) more accurately calibrated meter stick; (e) effective reduction of friction.

4. Compare the accelerations of two bodies, both starting from rest at the top of a plane inclined at an angle of 30° with the horizontal, if one body falls vertically and the other slides without friction down the plane.

5. Derive an equation which expresses the total downward force on a light pulley over which a pair of masses m_1 and m_2 are suspended by a light cord.