Selective Experiments In Physics

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NEWTON'S SECOND LAW OF MOTION: ATWOOD'S MACHINE

OBJECT: To study the relationship between force and acceleration in a simple dynamical system.

METHOD: An adaptation of Atwood's machine consists of a strip of paper tape passing over an aluminum pulley and carrying known masses at its ends. The acceleration produced when the masses are unequal is measured by means of an electrical recording system. A series of observations is made, the difference between the masses being varied while their sum is kept constant. A curve is plotted showing the relationship between acceleration and accelerating force.

THEORY: Sir Isaac Newton's second law of motion is one of three fundamental postulates advanced by the eminent scientist in his monumental *Principia* (1686). These three postulates form the basis of the present system of mechanics. They may be stated as follows:

I. By virtue of the inherent and universal property of matter called inertia, every body tends to maintain its existing state of rest or uniform velocity unless acted on by some external force.

II. The acceleration experienced by a body is directly proportional to the force acting upon it, inversely proportional to the mass of the body, and takes place in the direction of the applied force.

III. The force (action) exerted by one body upon another is always accompanied by an equal and opposite force (reaction) exerted by the second body upon the first one.

The second of these laws expresses a quantitative relationship which is susceptible of experimental verification. The mathematical expression of the law is

$$a = k\frac{F}{m} \qquad (1)$$

or

$$F = kma \qquad (2)$$

where the numerical value of the proportionality constant k depends only upon the units used. An "absolute" system of units is one based upon the arbitrarily established units of mass, distance and time, and in which the force unit is so defined as to make k unity. Thus, when absolute units are used, Eq. (2) becomes

$$F = ma \qquad (3)$$

An absolute unit of force is, therefore, the amount of force required to impart unit acceleration to unit mass. In the c.g.s. absolute system, a force of one dyne is the force required to give to a mass of 1gm an acceleration of 1*cm/sec*^{2}. In the British absolute system, the poundal is the force required to give amass of 1lb. an acceleration of 1*ft/sec*^{2}.

A gravitational system is one in which the unit of force is defined as the pull of gravity on a unit mass at sea level. The value of k is then determined experimentally from the acceleration experienced by a body moving under the force of gravity, i.e., under its own weight. The acceleration g of a freely falling body has been found to have an approximate value at sea level of 980*cm/sec*^{2} or 32.2*ft/sec*^{2}. Thus, the value of k is 1/980 when the c.g.s. gravitational unit of force is used, and 1/32.2 when the British gravitational system is used. It is readily seen, therefore, that the c.g.s. gravitational unit of force (gm-wt) is equal to 980 dynes, and the British gravitational unit of force (lb.-wt) is equal to 32.2 poundals.

A convenient arrangement for demonstrating experimentally the relationships expressed by Eq. (3) is the Atwood machine illustrated diagrammatically in Fig. 1. Ideally the system consists of two masses m_1 and m_2 attached to the ends of a weightless cord that passes over a weightless and frictionless pulley. Under these idealized conditions the inertia of the system consists of the sum of the two masses $m_1 + m_2$, and the accelerating force, expressed in gravitational units, is numerically equal to the difference of the masses $m_1 - m_2$, or in absolute units $(m_1 - m_2)$ g. By Newton's second law the acceleration is then

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2} \qquad (4)$$

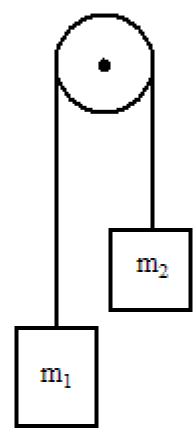

Fig. 1. Idealized diagram of Atwood's Machine

Practically, of course, this ideal situation cannot be realized. It can be approximated roughly by reducing friction to a minimum and by making the masses of the cord and the pulley very small in comparison with the suspended masses. While the mass of the cord may be made almost negligible, the mass of the pulley introduces a complication that must be considered in this experiment. The complication arises from the fact that, whereas Newton's second law as stated above applies to *linear* motion, the motion of the pulley is *angular*. A correction involving the rotational inertia of the pulley must, therefore, be made to Eq. (4). This can be done by considering the energy relationships involved. When the system travels a vertical distance sunder the influence of the accelerating force $(m_1 - m_2)$ g, an amount of work $(m_1 - m_2)$ gs (absolute units) is done on the system. As a consequence, the system, initially at rest, acquires kinetic energy equal to $\frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$ where *v* is the linear velocity of the masses, ω the angular velocity of the pulley, and I the rotational inertia of the pulley. The two terms in this expression for kinetic energy represent, respectively, the translational and the rotational energy. Neglecting friction, the energy equation may then be written

$$(m_1 - m_2)gs = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2 \qquad (5)$$

The relationship between the angular velocity ω of the pulley and the linear velocity of its rim, which is also the linear velocity *v* of the tape and suspended masses, is $\omega = v/r$, where *r* is the radius of the pulley. Substitution of this relationship in Eq. (5) yields

$$(m_1 - m_2)gs = \frac{1}{2}\left(m_1 + m_2 + I/r^2\right)v^2 \qquad (6)$$

from which

$$(m_1 - m_2)g = (m_1 + m_2 + m_o)\frac{v^2}{2s} \qquad (7)$$

where $m_o = I/r^2$ is a correction to the inertia of the system which may be called the "equivalent mass" of the pulley. Substituting $a = v^2/2s$ and solving for *a*,

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + m_o} \qquad (8)$$

Compensation for Friction: Eq. (8) was derived upon the assumption that the system is frictionless. In actual practice, however, it is necessary to take the negative force of friction into account. This may be done by adding to m_1 a compensating mass *x* just sufficient to equalize friction. Since the weight of the mass *x* is used in overcoming friction, x does not enter the expression for the accelerating force, but the inertia of the sys-tem is increased by the amount *x*. The expression for the acceleration thus becomes

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + m_o + x} \qquad (9)$$

Upon casual inspection of Eq. (9) it would appear that the friction correction only affects the total mass of the system, but it must be remembered that this expression does not refer to the same system as does Eq. (8) but to that system with m_1 increased by an amount *x*. If the latter system were frictionless, its acceleration would be

$$a' = \frac{(m_1 - m_2 + x)g}{m_1 + m_2 + m_o + x} \qquad (10)$$

Clearly, the effect of friction is to diminish the force effective in producing acceleration. Comparison of Eqs. (9) and (10) shows the percentage error in the acceleration due to friction to be

$$\frac{a' - a}{a'} = \frac{x}{m_1 - m_2 + x} \qquad (11)$$

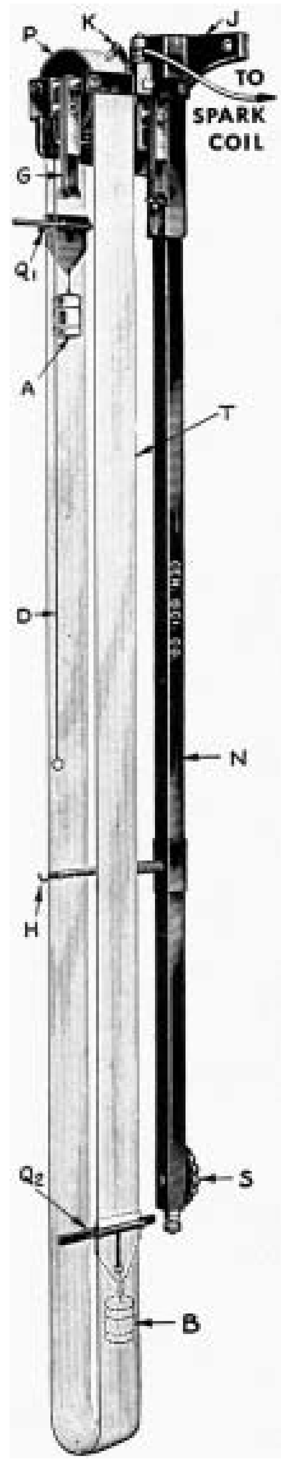

Fig. 2. Atwood's Machine, electrical recording type

APPARATUS: The type of Atwood machine employed in this experiment is illustrated in Fig. 2. A wide aluminum pulley P of known rotational inertia is mounted in cone bearings on a cast iron bracket J attached to the wall by means of screws. A strip of paraffin-coated paper tape T passes over the pulley and supports the masses A and B which are attached to the tape by means of special clips Q_1 and Q_2 to which weight hangers are permanently attached. The weight hanger attached to Q_1 has a compartment for lead shot used in making the correction for friction. The mass of Q_1 and its weight hanger is equal to that of Q_2 and its hanger, thereby making it convenient to counterpoise the system. One of the clips Q_1 is provided with a cradle for carrying the accelerating weights which are made in the form of cylindrical metal riders provided in magnitude S of 10, 20, 30 and 40 grams. The bracket J supports a column N provided with a leveling screw S. Upon the column slides a receiving hook H for picking off the rider as it passes. The position of the hook can be adjusted along the column, and it can be turned out of the way when desired. Attached to the bracket are spring buffers G which engage a bumper arm on the clip Q_2 and reduce the shock of impact. A friction brake (not shown in the illustration) can be released by pulling a cord D. An insulated spark terminal K may be clamped at various positions along the pulley.

As the system moves under the influence of the pull of gravity, the tape passes over the pulley and is punctured periodically by a series of sparks that are caused to pass at regular intervals between the pulley P and the spark terminal K. The sparks melt the paraffin coating, leaving a sequence of dots to mark the passage of the system. The distance between successive dots, therefore, indicates the distance through which the system moves in the corresponding interval of time. The sparks are generated by a spark coil, one of the secondary terminals being connected to the spark terminal K, the other being grounded.

The spark control system, represented diagrammatically in Fig. 3, contains two specially designed units: the spark timer

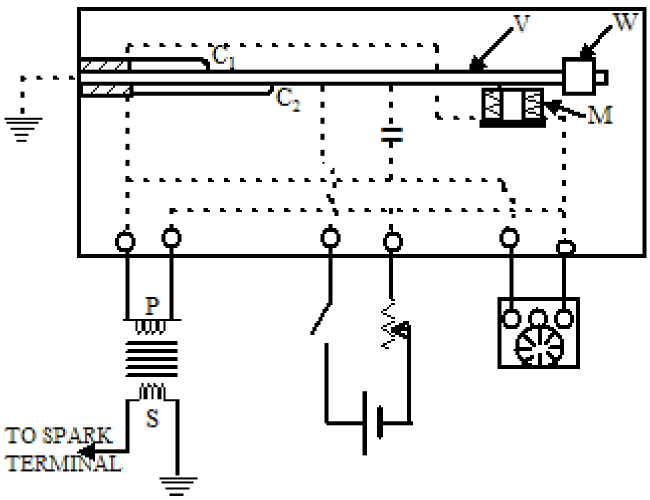

Fig. 3. Spark Control System

(Fig. 4) which regulates the spark frequency, and the impulse counter (Fig. 5) which indicates the number of sparks occurring in a given time. The spark timer is an electrically driven vibrating bar V, the frequency of which can be varied by altering the amount and position of the loading weight W. Large changes in frequency can be made by interchanging vibrator bars. The vibration is maintained by an electromagnet M, the current to which is interrupted periodically by means of a pair of contacts C_1. The vibration of the bar operates a second and independent set of contacts C_2 which makes and breaks the current in the primary of the induction coil. The impulse counter is essentially a magnetic escapement by means of which a sweep hand is caused to move one division of a dial at every magnetic impulse. The mechanism is set in operation by depressing a button which closes the circuit to the counting magnet. The button may be locked down by rotating it slightly. The spark interval is determined by observing the number of impulses registered in a given time on a dial.

Additional apparatus consists of a 6-volt battery, an induction coil, a 20ohm rheostat, a switch, a stopwatch, a meter stick, a vernier caliper, a platform balance and a set of weights. Required weights consist of two of each of the following: 5, 10, 20, 50 and 100gm.

PROCEDURE:

Experimental: Connect the electric circuit as shown in Fig. 3, grounding the bracket, the spark timer and one of the secondary terminals of the induction coil. *The vibrator on the spark coil must be screwed down tightly. Have the circuit inspected by the instructor before closing the switch.*

Adjust the frequency of the timer so as to give approximately 6 sparks per second, and determine the exact value of the time interval by observing the number of impulses registered in 1 minute. Make three observations and take the average value. Place the spark terminal K near the edge of the tape so that several records may be taken on one strip.

Determine the (equal) masses of the clips Q_1 and Q_2 with their attached weight hangers (unless this value is stamped on them). Call this mass *k*.

Make a loop of tape (2in width) such as that illustrated in Fig. 2 and attach the clips Q_1 and Q_2 as shown. The uncoated side of the paper should be next to the pulley. Set the receiving hook H near the lower end of the column. By means of slotted weights add a mass of 170gm to each weight hanger.

Fig. 4. Spark Timer

Determine the frictional correction by adding shot to the hanger attached to Q_1 until the system moves with constant velocity when given a start. Remove Q_1, weigh it carefully, and record the value of *x*. *Do not remove the shot from the hanger*.

Set the system in its starting position and place the 10gm rider in the cradle. The accelerating force is now 10gm and the total suspended mass $m_1 + m_2$ is $2(170 + k) + x + 10$. The value of $m_1 + m_2$ is to be maintained constant throughout the experiment.

Start the sparks and release the brake. Apply the brake again just before the bumper bar on Q_2 strikes the buffer G. As soon as the fall is completed, shut off the sparks. Move the spark terminal K about a centimeter and prepare for a second fall. Replace the 10gm rider with the 20gm one and remove 5gm from each side, thereby keeping the sum $m_1 + m_2$ constant. Make a record as before. In this way make a total of four records on one tape following the schedule shown in Table I. *Check the sum $m_1 + m_2$ before each run.*

Before removing the tape, adjust it as it was at the instant the rider was caught by the receiving hook H and make a mark across the tape at the spark terminal. On the part of the record below this mark, the dots show gradually increasing spacing and form a record from which the acceleration can be determined. Above the mark the dots are equally spaced, indicating the terminal velocity of the system.

Using the vernier caliper, measure and record the diameter of the pulley; also record the value of the rotational inertia *I* which is stamped on it.

Analysis of Data:

Required Analysis: Remove the tape and stretch it on the table. A typical record is illustrated diagrammatically in Fig. 6 which represents two traces.

Fig. 5. Impulse Counter

The distance *s* measured from the initial dot to the pick-off mark is the distance over which the accelerating force has acted on the system. The distance between the uniformly spaced dots in the upper half of the

TABLE I

Obs.	Mass added to each side (gm)	Mass of Rider (gm)	Total Mass (gm)
A	170	10	2 (170 + k) + x + 10
B	165	20	2 (165 + k) + x + 20
C	160	30	2(160 + k) + x + 30
D	155	40	2 (155 + k) + x + 40

record is a measure of the final velocity v_f acquired over the distance *s*. The distance between any two successive dots divided by the time interval between sparks gives the average velocity over the particular interval concerned. If the spark interval is taken as an arbitrary unit of time, the velocity in centimeters per interval is equal numerically to the distance in centimeters between the successive dots. The accelerations are to be determined from two such measurements of velocity, one (v_1) near the beginning of each record and another (v_2) near the end of the accelerated part of each record, just before the pick-off. Using a vernier caliper, determine the extreme values of v_1 and v_2 for all the records. *Be sure that the measurement of v_2 does not extend beyond the pick-off mark*. Enter these values in columns 2 and 3 of Table II. In each case count the number of time intervals during which the velocity increased from v_1 to v_2, remembering that the average velocity is the actual velocity *at the middle of the time interval*. Enter these values of the time in column 4. Dividing the difference between corresponding terms of columns 2 and 3 by the number of time intervals from column 4 gives the acceleration in *centimeters per interval per interval*. Enter these experimentally determined values of acceleration in column 5. Express the experimentally determined values of the acceleration in cm/sec^2 and record in column 6. Taking into account the equivalent mass m_o of the pulley, compute the values of a predicted by Eq. (9) and enter in column 7.

Plot a curve of the experimentally determined acceleration versus the accelerating force (column 6 vs. column 1). Determine the inertia of the system from the slope of the acceleration-force curve and compare with the known value.

Compute the percentage effect of friction, using Eq. (II).

Optional Analyses: 1. From the data of one fall, compute the linear kinetic energy acquired by the masses m_1 and m_2. In making this computation the final linear velocity of the system is obtained from the uniform distances in the portion of the record after the pick-off. Using as many of these intervals as possible, compute the average value of v_f Also compute the angular kinetic energy of the pulley from its known rotational inertia (stamped on it) and from its angular velocity determined from v_f and the radius of the pulley. Compare the total kinetic energy acquired by the system with the work done by the force of gravity.

2. Select any record and, from the final velocity v_f and the corresponding distance *s* over which the system was accelerated, compute the acceleration, using the appropriate equation of uniformly accelerated motion. Compare with the value obtained from Eq. (9).

3. Starting with the initial dot of one record, label the dots 0, 1, 2, 3, etc. Measure the distances s_1, s_2, s_3, etc. of the successive dots from the one marked 0. In each case count the number of intervals elapsed. Plot a curve of the distance versus the square of the time (in intervals). From the slope of

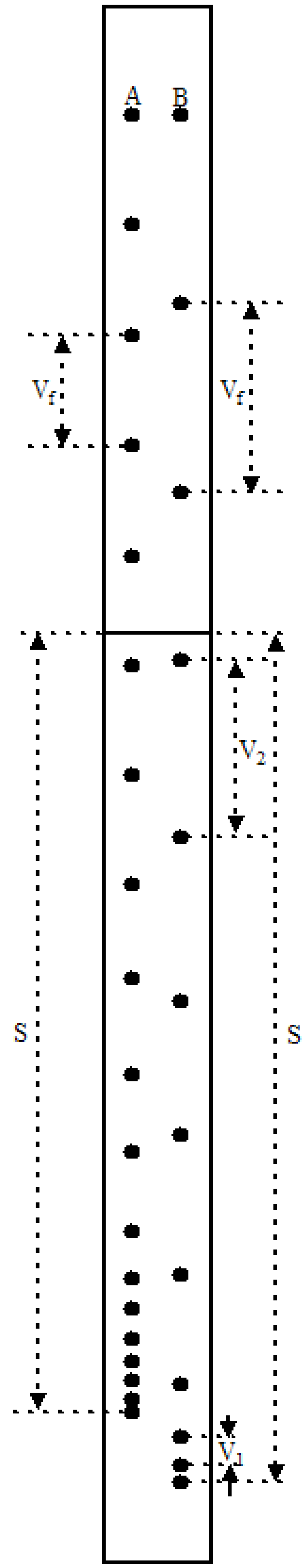

Fig. 6. Typical Spark Record

the curve determine the acceleration. Express the result in cm/sec^2 and compare with the corresponding value in column 6.

QUESTIONS: 1. Is the acceleration-force curve consistent with Eq. (2)? Explain.

2. Discuss the discrepancy between the experimental and computed values of the acceleration. Is it influenced by friction? Explain.

3. What is the purpose of looping the tape as shown in Fig. 2?

4. From the last fall, compute the tension in the tape before and after the brake was released.

5. Discuss the effect of pulley friction upon the tension in the tape.

TABLE II

1	2	3	4	5	6	7
Accelerating Force $m_1 - m_2$ (gm - wt)	First Velocity v_1 (cm/int)	Second Velocity v_2 (cm/int)	Time t (int)	Acceleration (experimental) a (cm/int^2)	Acceleration (experimental) a (cm/sec^2)	Acceleration (computed) a (cm/sec^2)