• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

# Uniformly Accelerated Linear Motion

**OBJECT**: To study the motion of a falling body; in particular, to measure its change of velocity and its acceleration.

**METHOD**: Because of the large value of the acceleration of a falling body, the spaces traversed during even a few seconds are too large for laboratory dimensions and hence much smaller time intervals must be used if measurements are to be made involving reasonable distances. In the present experiment this is accomplished by using as the falling body a vibrating tuning fork of known frequency. A stylus attached to one prong of the fork records the vibrations as a sinuous line on a suitably coated strip of paper (Fig. 1). From this record of the vibrations, measurements of the distances fallen during successive equal time intervals make it possible to plot graphs showing the relationship of distances to time and velocity to time. From these curves the characteristics of the motion are studied and acceleration determined.

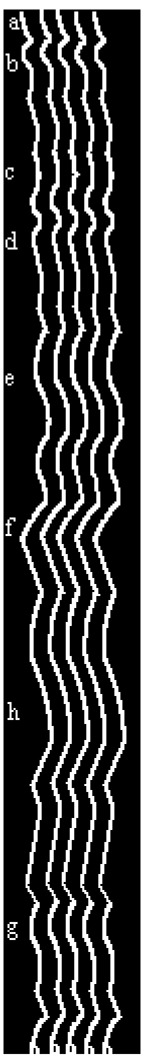


Fig. 1. Traces made by falling tuning fork.

**THEORY**: The *average speed* $\bar{v}$ of a body is the quotient of the distance *s* which it traverses and the time *t* required to travel that distance. In symbols

$$\bar{v} = s/t \qquad (1)$$

The *instantaneous* speed *v* of an object is defined as the limit of this ratio as the time is made vanishingly small. Symbolically

$$v = \Delta s/\Delta t \qquad (2)$$

where $\Delta s$ represents a small increment of distance traversed in the corresponding increment of time $\Delta t$.

In Fig. 2 curve (a) shows the distance-time relationship for a freely falling body. On any such curve Eq. (2) states that the instantaneous speed is given by the slope of a tangent drawn to the curve at the point for the instant in question. If the speed were constant the slope would be constant and the curve would be a straight line. For a freely falling body this is evidently not true, as the speed, and hence the slope of the curve, is continually increasing.

When the velocity of a body varies, the motion is said to be *accelerated*. Acceleration is defined as the time rate of change of velocity; in symbols

$$\bar{a} = \frac{v_t - v_o}{t} \qquad (3)$$

where $\bar{a}$ represents the average acceleration of a body which changes its velocity from $v_o$ to $v_t$ in the time *t*. Since acceleration has the dimensions of a velocity divided by a time, the absolute unit in the metric system will be the centimeter per second per second and in the British system the foot per second per second; usually written, cm/sec$^2$ and ft/sec$^2$.

If a body moves in a straight line, making equal changes of velocity in equal intervals of time, its acceleration must be constant, and it is said to be moving with *uniformly accelerated* motion. This is the type of motion produced when a constant force acts upon a body which is free to move. The most common example of this is the motion of a freely falling body. This acceleration *g* is called the "acceleration due to gravity" and has a value of approximately 980cm/sec$^2$ or 32.2ft/sec$^2$.

The relationships between the three quantities velocity, distance, and time, in uniformly accelerated motion are readily deduced from the above definitions. Eq. (3) yields directly

$$v_t = v_o = at \qquad (4)$$

which expresses the dependence of $v_t$ upon *t* in terms of the constants $v_o$ and *a*. It is the equation of a straight line, the slope of which is equal to the acceleration.

Since for *uniformly accelerated* motion the average velocity during an interval *t* is the arithmetical mean of the terminal velocities, in view of Eq. (1),

$$s = \bar{v}t = \frac{v_t = v_o}{2}\,t \qquad (5)$$

Substitution of $v_t$ from (4) yields

$$s = v_o t + 1/2\,at^2 \qquad (6)$$

When $v_o$ = 0, Eq. (5) shows that the distance-time curve is a parabola. The slope of the curve at any point (slope of the tangent) is the velocity at the corresponding instant.

A velocity-time curve for a freely falling body is plotted as curve (b) in Fig. 2. The time interval *T* is the interval of a chosen number of vibrations of the fork. The time *T* for the case shown is 0.0368 sec. Since the graph is a straight line

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it may be concluded that the velocity changes at a uniform rate. The slope of this curve $\Delta v / \Delta t$ is the acceleration. Since the slope is constant, the acceleration is constant. Hence the *average* velocity during the time interval is identical with the *instantaneous* velocity at the middle of that time interval.

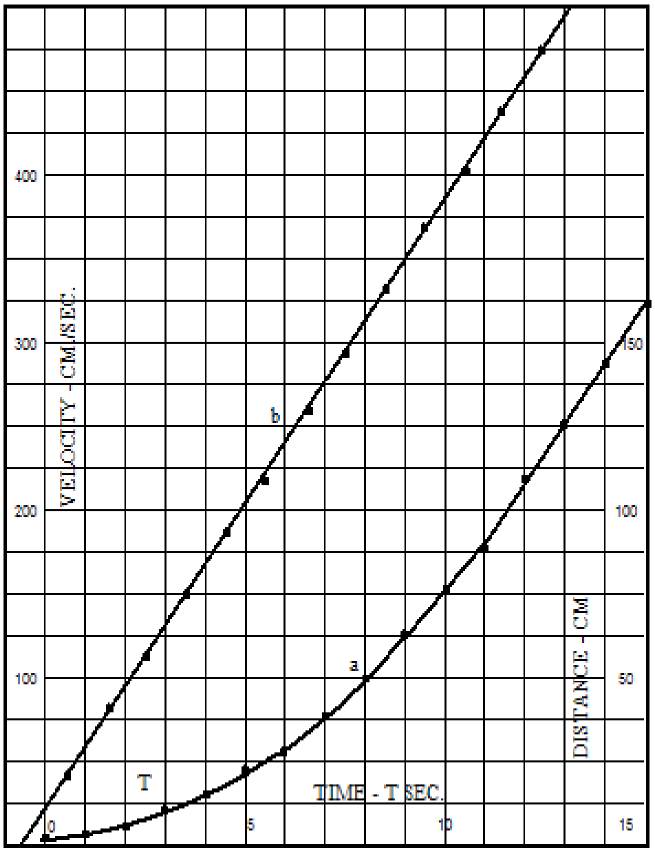


Fig. 2. Curves showing the relationship between (a) Distance and Time, and (b) Velocity and Time, for a freely falling body.

In the present experiment the value of *g* will be determined from the slope of such a velocity-time curve, as plotted from the experimental data.

The principal points in the preceding discussion may be summarized as follows:

(a) The average speed of a body is obtained by dividing the distance which it traverses by the time required to travel that distance.

(b) The instantaneous velocity of an object is the limit approached by the ratio $\Delta s / \Delta t$ as $\Delta t$ approaches zero. This velocity is also equal to the slope of the tangent to the distance-time curve at the point in question.

(c) The acceleration of an object is the time rate of change of its velocity, or $a = \Delta v / \Delta t$. It is also the slope of the tangent to the velocity-time curve at the instant considered.

(d) For a constant acceleration, the velocity-time curve is a straight line and the *average* velocity of the body is also the actual (or instantaneous) velocity at the midpoint of the time interval used.

Let $s_1$ be the distance fallen in the first time interval *t* and $s_2$ that fallen in the second equal interval. Then

$$s_1 = v_1 t + 1/2\, at^2 \tag{7}$$

$$s_2 = v_2 t + 1/2 at^2 \tag{8}$$

Subtracting Eq. (7) from Eq. (8) gives

$$s_2 - s_1 = v_2 t - v_1 t = (v_2 - v_1) t \tag{9}$$

Dividing both sides of Eq. (9) by $t^2$

$$\frac{s_2 - s_1}{t^2} = \frac{v_2 - v_1}{t} \tag{10}$$

But the right-hand member of Eq. (10) is the acceleration *a*, hence

$$a = \frac{s_2 - s_1}{t^2} \tag{11}$$

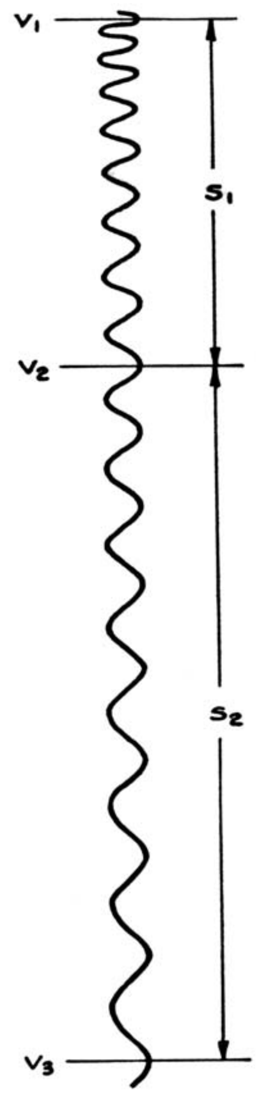


Fig. 3. Analytical method of measuring the acceleration.

From this equation the acceleration is determined, using the measured values of $s_1$, $s_2$ and $t$.
There are other more elaborate methods of averaging the increments of distance, represented in Eq. (11) by $s_2$-$s_1$, to obtain a more accurate determination of the acceleration. However the simple method represented by Eq. (11) gives values as accurate as are justified by the nature of this apparatus.
There is necessarily a certain amount of friction in the apparatus between the stylus and the recording surface and between the frame of the falling tuning fork and the support rods. Hence the observed value of the acceleration is less than the standard value for a freely falling body.

**APPARATUS**: The apparatus is shown in Fig. 4. The tuning fork F with stylus S attached to one prong is mounted in carriage C. When the carriage is released from its uppermost position by withdrawing the catch H, it falls between the vertical guides G and G', and is caught in the dashpots D and D'. The friction between the carriage and the guides is small when the apparatus has been properly leveled. The tuning fork is electrically driven so that constant amplitude of vibration is maintained throughout the time of fall. Consequently the waves which are traced by the stylus on the coated surface of plate P are of the same amplitude, which makes easy the location of the crests of the waves and therefore the accurate determination of the distances corresponding to the equal time intervals.
The strip of heavy plate glass P is held in grooves at its lower and upper ends. The plate can be shifted horizontally so that a number of tracings can be obtained.
The surface of the plate on which the records are to be traced may be prepared in several ways. It may be coated with carbon from smoke of a gum camphor flame; another method is to apply a suspension of whiting or lamp black in alcohol with a camels' hairbrush. Another satisfactory coating is Bon Ami applied with a moist cloth or sponge.
The most satisfactory method obtaining a permanent record of the traces is the use of two specially prepared paper strips, held against the glass plate by means of two clamps.
When the records have been made on a strip it can be torn off at the clamps and removed for the measurements. The additional strip permits a second student or group of students to use the apparatus without the interruption of having to prepare the surface of the plate. The strips with the tracings constitute permanent records which should be filed with the report of the experiment.
The auxiliary apparatus required includes a four to six volt storage battery, meter stick, double-pole, single-throw switch, and small celluloid triangle.

**PROCEDURE:**
**Experimental:** With the carriage in the position shown in Fig. 4, attach the plumb bob R and adjust the leveling screws in the base E until the bob hangs over the center of the bolt head between the dash pots D and D'. While the glass plate is in position and before its surface has been prepared, adjust the stylus S so that it presses lightly on plate P. The pressure of the stylus on the plate should be sufficient to produce a good trace; if too much pressure is applied, errors due to friction will be increased.
Connect the battery to the binding posts B and B' through the switch. Still using the unprepared plate, set the tuning fork into vibration by closing the switch and giving the fork an initial impulse by squeezing the prongs together with the fingers and then allowing them to spring apart. The small screw T, which periodically interrupts the electrical circuit and thus maintains the fork in vibration, should be so adjusted that the sparking at the contact is a minimum. Use a battery of such voltage that an amplitude of vibration of several millimeters is obtained. *Do not keep the switch closed when the fork is not vibrating* as this tends to overheat the coil. Make a few preliminary trail falls, releasing the fork frame by pulling the catch H away from the carriage C. As soon as the fork has fallen, open the electrical circuit. Tilt the plate backward before bringing the carriage up again. See that the dash pots D and D' are kept tightened.
Having made the necessary preliminary trial falls with the uncoated surface, prepare the glass plate to receive the trace in the manner indicated by the apparatus to be used or as directed by the instructor. Make a suitable number of falls until two good traces for each student have been obtained.

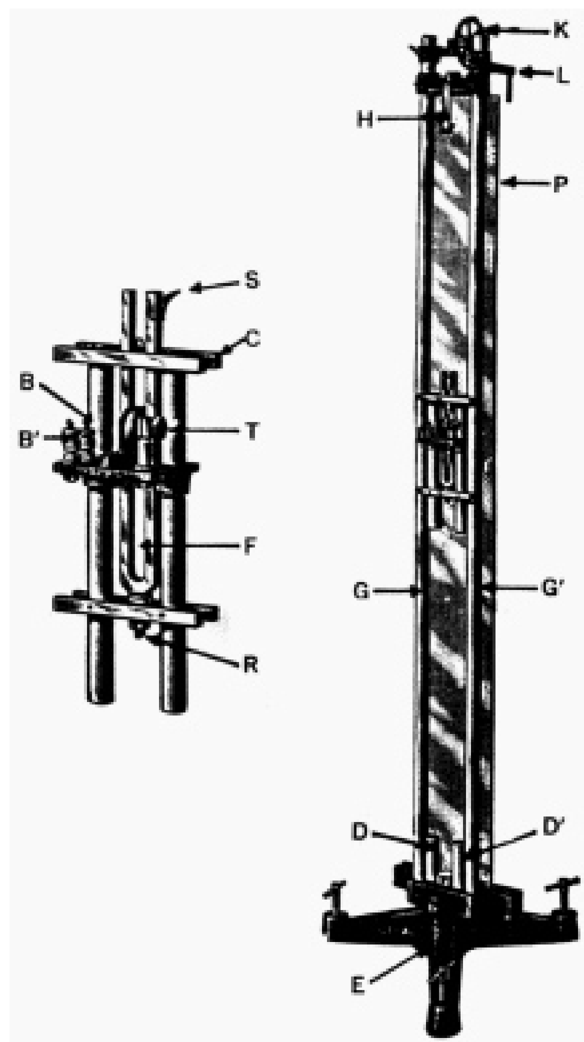

Fig. 4. The Acceleration Apparatus

**Tabulation of Data:**

**I. Graphical Method:** Remove the paper (or the coated plate if that method is used) and stretch it out, coated side up, on a flat surface in good light. Beginning near the top, at a place where the traces are distinct, pin-mark as finely as accurately as possible the crest of each fourth wave. This can best be done by placing a small transparent right-angled triangle across the crest and scribing with a needlepoint a fine line bisecting the crest and extending about 1/2cm above it.

Place a meter stick *edgewise* on the trace in such a manner that the graduations are directly touching the trace. Leave the meter stick stationary and read the respective distances from the first pin mark to the various following crests in turn. Estimate carefully each reading to fractions of millimeters. Call these successive total distances $S_1$, $S_2$, $S_3$, etc. Now subtract each position reading from the one immediately following it. This difference gives the distances $s_1$, $s_2$ and $s_3$, etc., fallen during successive equal time intervals. By dividing these distances by the time interval, the average velocity for that interval may be calculated. These values of average velocities are also the interval considered. The time interval T is obtained by dividing the number of vibrations used in marking off the traces by the frequency of the fork.

Table I shows a typical form for recording the data.

TABLE I

| Ordinal No. of Time Interval | S (cm) Total Distance Traversed | s (cm) Distance Traversed in One Time Interval | $\bar{v}$(cm/sec) Av. Velocity During One Time Interval (T = 0.0368 sec) |
|---|---|---|---|
| 0 | 0 | ...... | ...... |
| .... | ...... | 1.56 | 42.4 |
| 1 | 1.56 | ...... | ...... |
| .... | ...... | 2.78 | 75.6 |
| 2 | 4.34 | ...... | ...... |
| .... | ...... | 4.18 | 114.0 |
| 3 | 8.52 | ...... | ...... |

**II. Analytical Method:** If the simple analytical method described above is to be used, the measurements will be made merely by dividing the whole trace into only two equal time intervals and measuring the two distances fallen during those two equal periods. The acceleration is then obtained by substituting these values in Eq. (11).

**Interpretation of Data**:

***Required Analysis***: Plot a curve showing the relation of average velocity to time, using velocity as ordinates and time as abscissas, and plotting the points in the first quadrant. Locate each average velocity at the *mid-point* of the corresponding time interval, since for uniform acceleration the average velocity is identical with the actual velocity at the middle of the interval. Place the zero of abscissas (time intervals) somewhat to the right of the left-hand edge of the graph paper, since at "zero" time (the first marked wave crest) the body already had a small initial velocity. From the slope of this velocity-time curve determine the acceleration of the falling body. Calculate the percentage difference between this value and the standard value, *g*. State reasons for any differences found.

On the same graph sheet plot a curve to show the total distance *S* fallen against the time. The times are simply the product of the ordinal numbers by the sparking interval *T*.

Thoroughly interpret the graphs in the report of the experiment. This interpretation should include conclusions to be drawn from the shapes of the curves, their slopes, and their intercepts. Careful explanations of the reasons for all conclusions should be given.

***Optional Analyses***: 1. Compute the value of acceleration by applying the method of equal intervals to the last column of the data as in Table I.

2. By taking corresponding values of distances and velocities for particular times from curves as in (a) and (b) of Fig. 2, plot a velocity-distance curve. Explain its shape.

3. Select some point on the distance-time curve and draw a tangent to the curve. From the slope of the tangent determine the velocity at that instant and compare it with the computed value.

4. Using the data for any two points on the record, compute the initial velocity by the use of Eq. (5). Compare this value with the initial velocity determined from the intercept of the velocity-time curve.

**QUESTIONS**: 1. If by some suitable mechanism the falling body had been given an initial downward push instead of being just released, would the resulting observed value of the acceleration have been different? Explain.

2. Classify the following as to whether they would introduce *systematic* or *random* errors in this experiment: (a) friction, (b) estimations of fractional parts of millimeters on the scale, (c) incorrect calibration of tuning fork, (d) error in location of exact center of crests of traces.

3. Neglecting friction, which of the following statements properly characterizes the motion of a heavy object thrown violently downward from a tall building: (a) uniform speed, (b) uniform deceleration, (c) constant acceleration, (d) uniformly increasing acceleration, or (e) a non-uniformly changing acceleration?

4. How would the observed value of the acceleration be affected if a tuning fork in a heavier frame had been used?

5. What would be the appearance of the velocity-time curve if the effect of friction could not be neglected?