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SIMPLE HARMONIC MOTION

OBJECT: To study simple harmonic motion by measuring the force constant and the period of oscillation of a vibrating spring.

METHOD: A spring is hung vertically and increasing masses are attached to the lower end. The corresponding extensions of the spring are measured on a scale ruled on a mirror. From the data of weight and extension a graph is drawn and the best average of the force constant (force per unit extension) is obtained. A known mass *m* of weight *mg* is placed on the lower end of the spring, and the system is set into vertical oscillation. The period of the oscillations is measured and compared with its theoretical value.

THEORY: Simple harmonic motion is a periodic motion experienced by the bob of a swinging pendulum and in the oscillations of a mass attached to a spring. It is a to-and-fro, or vibrating, motion of objects stretched or bent from their normal positions and then released. Such an object moves back and forth along a fixed path, repeating over and over a fixed series of motions and returning to each position and velocity after a definite period of time. This type of motion is produced by varying forces and hence the object experiences varying accelerations.
Simple harmonic motion results when the restoring force (F) acting on the object is proportional to, and in the opposite direction to the displacement (y) of the object. This statement may be expressed symbolically as:

$$F = -ky \tag{1}$$

where k is a quantity called force constant of the spring; and Eq. (1) is the defining equation for k. The negative sign in Eq. (1) implies that the restoring force (F) is in the opposite direction to the displacement (y) of the object. In order to understand the physical meaning of Eq. (1) consider a spring hung vertically with a mass m attached at its lower end, Fig. 1. This mass m, of weight mg, stretches the spring to the equilibrium position O. If the mass is now pulled down from O to A' and set free, it will oscillate with a definite period between the limits A' and A. The distance $OA = OA'$ is called the amplitude of the oscillations and is the maximum displacement of the mass. It is assumed in simple harmonic motion that the amplitude remains constant, or that the total mechanical energy, kinetic plus potential, remains constant. This is only approximately correct for systems oscillating in the air because energy is dissipated to the air and also in the system itself.
However, this dissipation of energy is usually small enough to be neglected.

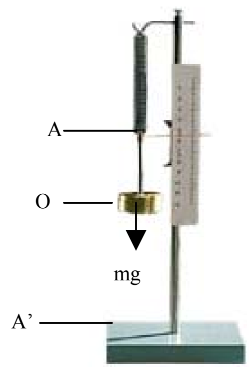

Fig. 1. Spring with weight mg in equilibrium at O, oscillating between limits A and A'.

Consider the motion of the mass m as it is oscillating between the limits A' and A, Fig. 1. At positions A' and A the mass is instantaneously at rest and the energy is all potential. At A the spring is less stretched than at O and the potential energy of the mass m at A is the work done in raising the mass from O to A, while at A' the spring is further extended and the potential energy at A' is the work done in stretching the spring from O to A'. At the equilibrium position O the oscillating mass has zero displacement but has its maximum velocity (v_m), and its maximum kinetic energy ($1/2mv_m^2$). As the mass moves from O to A, Fig. 1, it is being slowed-down, coming to rest at A, then speeding up in moving from A to O. Thus the mass is undergoing an acceleration in a downward direction while the displacement is in an upward direction. Similarly it can be seen that in the downward motion from O to A' the mass has an upward acceleration and a downward displacement. In both these cases the acceleration is in the opposite direction to the displacement. The acceleration, velocity and energy changes are summarized in Table I. When the oscillating mass has a displacement y, somewhere between O and A, the restoring force is $-ky$, the acceleration is $-ky/m$, and the energy is partly kinetic and partly potential.

TABLE I. CHARACTERISTICS OF OSCILLATING MASS

DISPLACEMENT	VELOCITY	ACCELERATION	KE	PE
-Max (OA')	O	+Max	O	Max
O	Max	O	Max	O
+Max (OA)	O	-Max	O	Max

The value of the force constant k, the restoring force per unit displacement, may be obtained by adding weights to the lower end and measuring the corresponding displacements. As long as the added weights do not produce any permanent change in the spring, that is, the spring returns to its original length when the weights are removed; it is found that the displacement is proportional to the added weights. This is a statement of *Hooke's law*, first given by Robert Hooke in 1676.

The potential energy of the oscillating mass m in Fig. 1, for any displacement y, can be obtained from the, work done in producing the displacement y. At the equilibrium position O there is no additional force; but for the displacement y the, additional force is ky. The additional force is proportional to the displacement y so that the average force required to extend, the spring through the displacement y is $ky/2$.Since this work is equal to the product of the average force and the displacement in the direction of the force, the work done for a displacement y is $ky^2/2$. Thus the energy at displacement y is given by

$$PE = \frac{ky^2}{2} \tag{2}$$

At the positions of maximum displacement $OA = OA' = y_{max}$ the potential energy is $ky^2_{max}/2$. Since the mass has no kinetic energy at these positions, the maximum potential energy is $ky^2_{max}/2$, and by the principle of conservation of mechanical energy this is the total energy and is constant. Thus the total energy of the oscillating system can be expressed as:

$$\text{Total Energy} = \frac{ky^2{}_{\max}}{2} \tag{3}$$

At a displacement y between O and A, Fig. 1, the potential energy is $ky^2/2$ and the kinetic energy must be equal to the difference of the total energy and the potential energy. Thus the kinetic energy at displacement y is given by:

$$KE = \frac{ky^2{}_{\max}}{2} - \frac{ky^2}{2} \tag{4}$$

When the system is set into oscillation the time of one complete vibration, called the period T, depends on the total mass oscillated and the force constant k. So long as the oscillations are not too large it can be shown that the period is independent of the amplitude of the oscillations and is given by:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{5}$$

This equation is derived from the expression for the acceleration a at the displacement y which is

$$a = F/m = ky/m \tag{6}$$

If *cgs* units are used in the quantities of Eq. (6) m is in grams, F is in dynes, and k in dynes/centimeter, then the energy will be in ergs. The corresponding quantities in the *mks* system of units are: m in kilograms, F in Newton's, k in Newton's per meter, and energy in joules. For the British Engineering system of units m is in slugs ($m = w/g$), F in pounds weight, k in pounds weight per foot, and energy in foot-pounds.

APPARATUS: A spring with mirror and support together with a weight holder and slotted weights are required. A timing device such as a stopwatch or stopclock is needed for determining the period of the oscillations.

PROCEDURE: Experimental: Set up the apparatus as shown in Fig. 2. The maximum load which can be placed on the weight holder depends on the, spring used, and may be supplied by the instructor. Otherwise a load not greater than that which causes an extension of about one-third of the original length should be used. Suppose this maximum load is 50gm exclusive of the weight holder. Record the zero reading with the weight holder present. Observe the scale readings when the pointer and its image in the mirror coincide. Add successive loads of 10gm and find the corresponding readings up to the maximum of 50gm. Tabulate the values of the load, scale readings and extension. Assuming a maximum load of 50gm for the spring, place 10gm on the weight holder and set the system

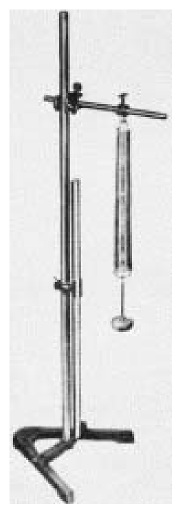

Fig. 2. Spring and weight holder attached to support and scale.

into oscillation by pulling the weight holder down a few centimeters. Measure the time for fifty complete oscillations and determine the period. Repeat this measurement at least once more. Place 30gm on the weight holder and determine the period then repeat this for 50gm.

Calculations: Plot a graph of load versus extension using the load as ordinate and extension as abscissa, and draw the best straight line through the points. Measure the slope of the straight line. By slope is meant the rate of rise of the line and is measured by dividing the vertical ordinate (load) by the horizontal abscissa (extension). The slope gives the force per unit extension and is the value of k used in the theory portion.

Calculate the periods for the three values of the masses oscillated using the value of k obtained above and Eq. (5).

Note that the mass oscillated is the sum of the masses of the weight holder and the added masses. Find the percent difference between the measured and calculated values of the period *T* by taking the difference in the two values, multiplying this by 100% and dividing by the measured period.

QUESTIONS: 1. If in Eq. (5) the mass *m* is given in grams and *k* in dynes per centimeter show that the units of the right-hand side of this equation are seconds.

2. In the theory portion it is stated that the restoring force is in the opposite direction to the displacement. State what the motion would be if the force were in the same direction as the displacement.

3. Using the value of *k* found in the experiment, find the maximum energy for a maximum displacement y_{max} of 5cm.

4. From the maximum energy of the system obtained in question 3, find the potential and kinetic energies when the displacement *y* is 2cm.

5. Repeat question 4 for a displacement of 4cm.

6. According to the theory how do the period and the maximum energy vary with the amplitude of the oscillations?

7. Would you expect that a fraction of the mass of the spring should be included in the equation for the period, Eq. (5)? Give reasons for the answer.