CENCO Physics

Selective Experiments In Physics

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ROTATIONAL INERTIA: ANGULAR MOTION

OBJECT: To study angular motion and the concept of rotational inertia; in particular, to determine the effect of a constant torque upon a disk free to rotate, to measure the resulting angular acceleration, and to determine the rotational inertia of the disk.

METHOD: A constant torque is applied to a metal disk which is free to rotate. The disk is found to move with uniform angular acceleration. This acceleration is measured by a spark-recording method in which sparks at regular and measurable time intervals make their traces upon a coated paper, containing a polar-coordinate scale, fastened to the side of the disk. The angular acceleration is determined from the slope of the angular velocity-time graph, the velocities being calculated from the observed angular distances between spark traces and the time between sparks. The torque is the product of the measurable force applied to the edge of the disk and the radius of the disk. The rotational inertia is then determined from the ratio of the torque to the angular acceleration. Finally this "observed" value of the rotational inertia is compared with the "theoretical" value calculated from the geometrical constants and the mass of the disk.

THEORY:

Angular Speed: The angular speed of a body is defined as its time rate of change of angular displacement, or the ratio of the angular distance which it has traversed to the time required to travel that distance. The defining equation for average angular speed $\bar{\omega}$ is

$$\bar{\omega} = \theta / t \tag{1}$$

where θ is the angular distance traversed and *t* is the time required for the body to travel that distance. *Instantaneous* angular speed ω, is the limit of the above ratio as tile time is made vanishingly small. In symbols

$$\omega = L_{im \to 0}\left(\frac{\Delta\theta}{\Delta t}\right) \tag{2}$$

where $\Delta\theta$ is the small increment of angular distance traversed in the corresponding element of time Δt. The absolute units of angle and time in both metric and British systems are the radian and second, respectively, and hence the absolute unit of angular speed is the *radian per second*. There is a distinction between angular *speed* and angular *velocity* which is similar to that between linear speed and linear velocity. The *direction* of an angular velocity is specified as the direction of its axis of spin; the sense of the direction is related to the *sense* of the rotation as the direction of advance of an ordinary right-handed screw is related to its direction of rotation.

Angular Acceleration: Whenever a body has its angular velocity changed it has an angular acceleration. This is defined as the time rate of change of angular velocity, or the ratio of the change in angular velocity to the time required to produce the change. In symbols, average angular acceleration a is defined by the equation

$$\bar{a} = \frac{\omega_t - \omega_o}{t} \tag{3}$$

where ω_t, is the final angular velocity, ω_o is the initial angular velocity and *t* is the time required to change the velocity. The *instantaneous* value of the angular acceleration 4 is the limit of this ratio as the time is made vanishingly small. Symbolically

$$a = L_{im\Delta t \to 0}\frac{\Delta\omega}{\Delta t} \tag{3a}$$

where $\Delta\omega$ is the change in angular velocity taking place in the small increment of time Δt. If a curve of angular velocity is plotted against time, as in Fig. 1, a straight line is obtained when the angular acceleration is constant, i.e., when the angular velocity changes uniformly. It is seen from Eq. (3a) that the angular acceleration is the slope of such a curve; this fact will be utilized in the present experiment. The absolute British and metric units of angular acceleration are identical, namely the *radian per second per second*.

Rotational Inertia: *Rotational inertia (also called moment of inertia) is that property of a body which causes it to oppose any tendency to change its state of rest or uniform angular velocity*. It will be observed that this property is analogous in rotary motion to *inertia* in linear motion, as inertia is that property of matter by virtue of which the body opposes any tendency to change its state of rest or uniform *linear* velocity. The measure of the rotational inertia *I* of a body is the ratio of the torque *L* acting upon it to the angular acceleration a produced by that torque. In symbols the defining equation is

$$I = L / a \tag{4}$$

When Eq. (4) is written in the form

$$L = Ia \qquad (4a)$$

its analogy to the familiar equation for linear motion involving force, inertia and acceleration- namely, $f = ma$ - is apparent. It may easily be shown that Eq. (4a) is merely another form for stating Newton's second law of motion, as applied to rotary acceleration. The absolute metric unit of rotational inertia is the centimeter-dyne per radian per second per second; this is the equivalent of the gm-cm^2.

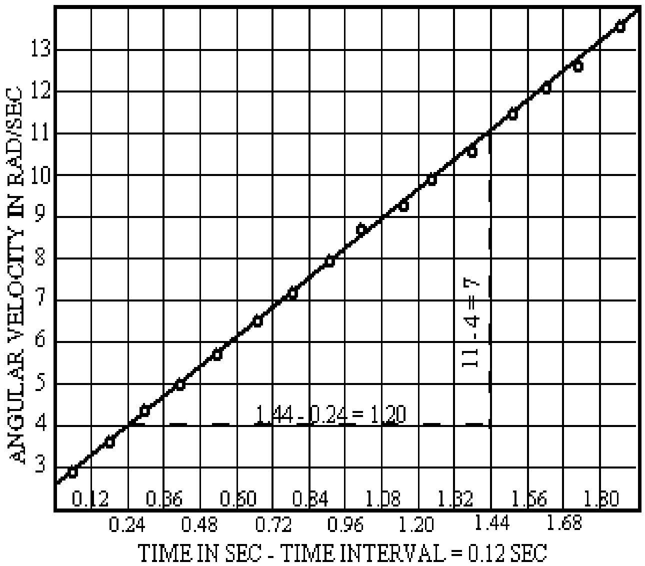

Fig. 1. Angular Velocity vs. Time for disk rotating under influence of constant torque. (Angular acceleration = slope of curve = $\frac{11-4}{1.44-0.24}$ = 5.85 radians/sec^2.)

The Determination of Rotational Inertia: The rotational inertia of a simple geometrical solid can be calculated by special equations derived by the use of integral calculus. For example, the rotational inertia of a uniform cylinder of radius *r* and mass *M* rotating about its longitudinal axis is given by

$$I = \frac{1}{2} Mr^2 \qquad (5)$$

The rotational inertia of an object about any axis may be obtained experimentally, no matter how irregular or non-homogeneous the body may be, by applying a known torque to the body and measuring the resulting angular acceleration. From Eq. (4),

$$I = L/a = fr/a \qquad (6)$$

where *f* is the applied force, and *r* is its lever arm.

In this experiment the rotational inertia of a circular disk is measured by applying a known torque (due to a known weight fastened to a cord wrapped around the rim of the disk), and measuring the resulting angular acceleration by the use of a technique described later.

The Accelerating Force: It might be thought at first glance that the accelerating force on the disk is equal to the weight *mg* of the object attached to the cord. This is true when the disk is at rest. But when the object is moving downward with an acceleration *a*, its inertia *m* gives rise to a force which is opposite in direction to the weight (Fig. 2). Since the object has an acceleration downward, the tension *f* in the cord is less than the weight *mg*. To determine the tension it is only necessary to apply Newton's third law of motion, equating the acting and reacting forces. Hence

$$mg = f + ma \text{ or } f = m(g - a) \qquad (7)$$

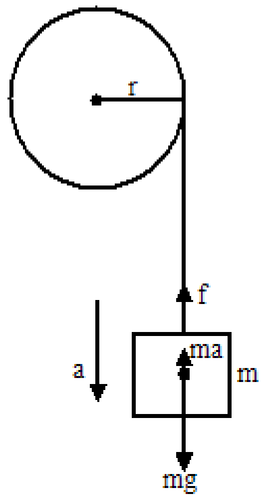

Fig. 2. The various forces which result in the tension f in the cord.

The linear acceleration of the descending object is the same as that of the rim of the disk to which the cord is attached. Since the linear acceleration of a particle equals its angular acceleration multiplied by its radius of rotation, or

$$a = ar \qquad (8)$$

Eq. (7) may be written as

$$f = m(g - ar) \qquad (9)$$

and hence Eq. (6) becomes

$$I = \frac{m(g - ar)r}{a} \qquad (10)$$

This is the working equation of this experiment since it makes it possible to determine *I* in terms of the known value of g and the measurable quantities *m, a* and *r.*

APPARATUS: The chief piece of apparatus is the rotational inertia disk and assembly (Fig. 3). The disk is mounted on a horizontal axis in precision pivot bearings, so as to turn with negligible friction. It is made in three cylindrical steps, of simple geometric form, so that the rotational inertia of each part can be readily calculated. Upon the plane face of the large disk there is fastened with bits of Scotch tape a sheet of coated paper, with polar-coordinate rulings in degrees. A spark point, mounted on a simple slide with insulating supports, is arranged to move across the face of the disk as it rotates. High-potential sparks, passing at regular intervals from the point to the disk, puncture the paper, and the heat thus developed melts a bit of the paraffin coating and causes a clearly recognizable spot. The location of these marks upon the printed scale gives the angles traversed during successive equal time intervals. The wheel bearings are supported by a frame so designed as to permit the thread which accelerates the disk to clear the edge of the table. The accelerating weight is attached by a light silk cord wound around the rim of the disk and fastened to a pin on its periphery.

The spark timing device, or spark timer (Fig. 4), consists of an electrically-maintained vibrating steel bar provided with electric contacts for making and breaking a circuit at equal intervals, the length of one interval being the full period of the bar. Two sets of contacts are provided, one of which is used in maintaining the vibration by opening and closing the circuit through an electromagnet; the other set, independent

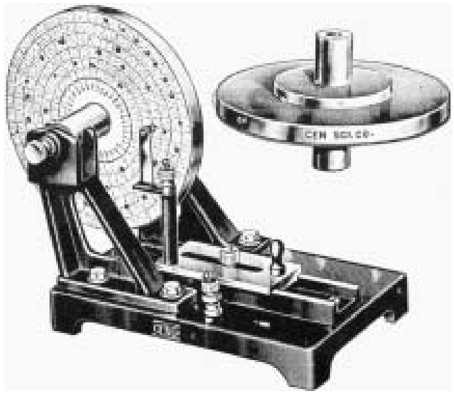

Fig. 3. Rotational Inertia Apparatus. The insert shows the three-step disk. The wheel is shown covered with the coated-paper chart with polar-coordinate rulings.

of and insulated on one side from the first, opens and closes the primary of a spark coil for producing the timed sparks. The secondary terminals of the spark coil are connected to the two binding posts on the rotational inertia apparatus. Electrical connections on the spark timer are as shown in Fig. 5. A 6-volt storage battery is connected to the two center binding posts V. The primary of a spark coil is connected to the "spark-coil" binding posts S. One of the terminals on the secondary of the spark coil is connected to the insulated

Fig. 4. Spark Timer

sparking point near the disk; the other secondary terminal is connected to the grounded frame of the rotational inertia apparatus. The impulse counter, used to measure the frequency of the vibrations, is connected to the binding posts I.

The impulse counter, Fig. 6, counts the electrical impulses which produce the sparks. Each impulse causes a sweep hand to move one division on the dial, one complete revolution of the pointer representing 60 impulses. A small pointer records the number of whole revolutions. A push-button key must be depressed to complete the circuit through the counter. By rotating the push-button it may be locked down in the operating position. A stopwatch or clock is used to measure the time of a suitable number of impulses registered on the counter and hence to determine the time interval between sparks.

As additional auxiliary apparatus there are needed a vernier caliper, a set of slotted iron weights of 1gm to 200gm, a 50gm weight holder, a spool of silk thread, several coated charts, a roll of Scotch tape, scales capable of weighing the disk, a half-meter stick with caliper jaws, 6-volt storage battery, a double-pole, double-throw switch, stop watch or clock, two C-clamps, scissors, and an outside caliper.

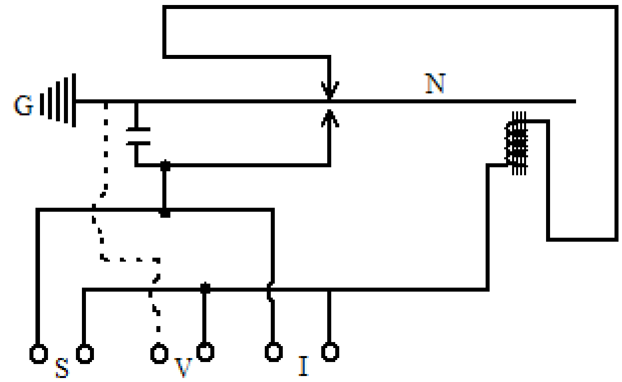

Fig. 5. Schematic diagram of the electrical circuit of the Spark Timer showing terminals for connection of Spark Coil S, Battery V, and Impulse Counter.

PROCEDURE:

Experimental: For a preliminary trial fasten a *used* sensitized chart to the disk by means of a few pieces of Scotch tape touching the edge of the chart and bending over the periphery of the wheel. Replace the disk on its supports and tighten the knurled screw. Wrap the silk cord several times around the rim, fastening one end to the pin provided for that purpose and the other end to the accelerating weight. A fall of the latter of one and one half to two meters is desirable. This may be obtained by mounting the

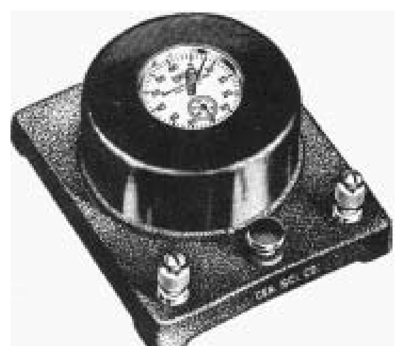

Fig. 6. Impulse Counter which accurately indicates the frequency of the vibrating bar of the Spark Timer.

rotational inertia apparatus upon a high table or shelf. The spark point should be adjusted so that it is about 1/32 inch from the chart. Connect the spark timer, induction coil and impulse counter as indicated in Fig. 5. The frequency of the vibrator should be kept at about 8 per second. The vibrating bar and stationary contacts should be adjusted so as to be in alignment. The stationary contacts should be arranged so that when the bar is at rest there will be a gap of ¼ to ½ millimeter between the contacts of each set. The contacts should be secured in this position by means of the lock nuts. To increase the amplitude of vibration, the electromagnet adjusting screw is turned so as to bring the electromagnet closer to the bar; to reduce the amplitude it is moved away

from the bar. *The make and break contacts on the spark coil should be carefully closed so that the breaker cannot vibrate. Be sure that the frame of the spark timer, the frame of the rotational inertia apparatus and one side of the induction coil secondary are well grounded.* ***The spark timer should be firmly clamped near its center to the table.***
A part of the force applied to the disk is used in overcoming friction. To minimize the error which this would cause a small weight should first be attached to the cord and adjusted until it is sufficiently large to neutralize the friction. The proper force will have been applied when, after the wheel has been given a slight initial rotation, the weight descends thereafter with constant speed. Next increase this weight by about 20 per cent to allow for its own "loss of weight" when it is falling with an acceleration, as it will be when the accelerating weight is applied. Attach a 100gm mass to the cord, in addition to that found to be necessary to overcome the friction. When all is in readiness and the sparks are coming regularly, release the wheel, without giving it an initial velocity, and begin slowly to move the spark point in toward the axis. Just before the descending object reaches the floor, open the switch which stops the sparks. Examine the trace carefully to see that all the sparks have registered. Afresh chart may be applied to the disk when the technique of running the apparatus has been mastered.
Attach another fresh chart and repeat the manipulation, using an accelerating weight of 200gm, in addition to that required to overcome friction.
The impulse counter should be left in the circuit while the spark timer is in operation, if possible, so that the time interval may be determined without any changes in the electrical circuit. Also, the contact and electromagnet adjustments on the timer should be left unchanged until the time interval has been determined. Measure with a stopwatch or clock the time corresponding to a suitably large number of impulses indicated on the counter. A time of at least one or two minutes should be used. By dividing this time, in seconds, by the number of impulses during the time, the time interval T between consecutive sparks is obtained.
Measure the mass of the disk (if its value is not stamped upon it). With the vernier caliper determine the respective radii of each of the three steps of the disk, R_1, R_2 and R_3 (Fig. 7). Measure also the thicknesses of each step, i.e., the axial length of each cylinder, l_1, l_2 and l_3. Some of these measurements may be made best with a vernier caliper, others with an outside caliper and a meter stick, or a meter stick provided with caliper jaws may be used. In recording these data it would be well to make a large sketch, similar to Fig. 7, and to indicate upon it the observed dimensions. Note and record the angular positions of consecutive spark prints. These may be recorded in degrees. By subtraction obtain the differences between consecutive readings. These differences are the angular distances passed over during succeeding equal time intervals T. By dividing each of these distances by the time interval T, the average angular velocities for these intervals are obtained. The interval T should be in seconds, so that the angular velocities will be in degrees/second.

Computations and Interpretation of Data: Plot a curve similar to Fig. 1 showing the relation of average angular velocity to time, using angular velocity as ordinates and time as abscissas.

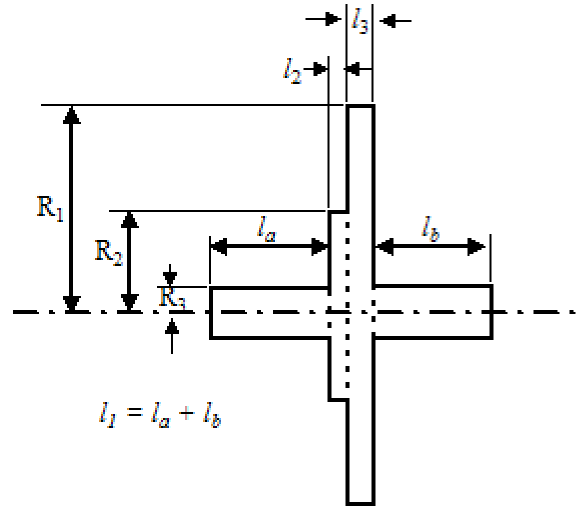

Fig. 7. Dimensions of the three-step rotational inertia wheel.

Locate the points for each average angular velocity at the *mid-point (not the end*) of the corresponding time interval. Since the curve is a straight line, indicating uniform acceleration, the *average* angular velocity is identical with the *actual* angular velocity at the middle of the interval. From the slope of the angular velocity-time curve, determine the angular acceleration. (Choose points as far apart as conveniently possible when determining the slope.) Plot a second curve and determine the angular acceleration for the second case, when the 200gm accelerating weight was used.
All of the above may be done by using the angles in degrees, to avoid unnecessary calculations. The final values of the angular accelerations must, however, be expressed in radians per second per second.
Having measured the values of a, substitute these and the other necessary data in Eq. (10) and calculate the experimental values of I. Note the percentage difference between the two values, as this gives one measure of the experimental uncertainty.

Computed Value of I: The disk is made of three steps, of different radii. The total rotational inertia: I is the sum of the separate rotational inertias of the three cylinders. In symbols

$$I = \frac{1}{2}\left(M_1 R_1^{\ 2} + M_2 R_2^{\ 2} + M_3 R_3^{\ 2}\right) \qquad (11)$$

where M_1 = mass of large disk
M_2 = mass of smaller disk
M_3 = mass of shafts
R_1 = radius of large disk
R_2 = radius of smaller disk
R_3 = radius of shafts

The masses of the three parts may be calculated by multiplying the density and respective volumes of the parts. The density may be obtained by dividing the total mass by the total volume.
A simpler method for computing I is as follows. Refer to Fig.

7 for the meaning of the symbols. Since

$$M_1 = M\frac{V_1}{V}$$
$$M_2 = M\frac{V_2}{V}$$
$$M_3 = M\frac{V_3}{V}$$

where V is the total volume and V_1, V_2 and V_3 are the respective partial volumes, it is possible to write Eq. (11) as

$$I = \frac{1}{2}\frac{M}{V}\left(V_1R_1^{\ 2} + V_2R_2^{\ 2} + V_3R_3^{\ 2}\right) \qquad (12)$$

Substituting in Eq. (12) for each volume its equal in terms of its respective $\pi R^2 l$,

$$I = \frac{1}{2}M\frac{R_1^{\ 4}l_1 + R_2^{\ 4}l_2 + R_3^{\ 4}l_3}{R_1^{\ 2}l_1 + R_2^{\ 2}l_2 + R_3^{\ 2}l_3} \qquad (13)$$

Note the percentage difference between the calculated value of I from Eq. (13) and the average of the two experimental values.

Optional Analysis: 1. Plot a curve showing the relation of the total angular distance traversed to the times required to move those distances. Discuss the shape of the curve.

2. By choosing corresponding values of total angular distances traversed and the appropriate average angular velocities, plot an angular velocity vs. angular distance curve. Explain its shape.

3. Select some point on the curve of angular distance vs. time and draw a tangent to the curve. From the slope of the tangent determine the angular velocity at that instant and compare it with the computed value.

QUESTIONS: 1. The first spark point does not occur when the angular velocity is zero. What effect will this have upon the observed value of *a*? Explain.

2. If data were taken for angular distances traversed after the accelerating force had been removed and the corresponding angular velocities plotted against time, what sort of curve would be expected? Why?

3. From the units of torque and angular acceleration, show that the absolute metric unit of rotational inertia is the gram (centimeter)^2.

4. Explain how, by a simple experiment involving rotational inertia, it would be possible to determine which of two identical-appearing eggs was hard cooked and which was uncooked.

5. Derive a symbolic expression for the linear acceleration of a sphere which starts from rest and rolls down a plane inclined an angle θ to the horizontal. (*Answer*: $a=5/7g\sin\theta$. The value of I for a sphere may be assumed, namely $I=2/5mr^2$.)

6. What portion of the total kinetic energy of a rolling solid disk is energy of rotation?