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THE SIMPLE PENDULUM

OBJECT: To investigate the motion of a simple pendulum, and to make an experimental determination of the acceleration due to gravity.

METHOD: The period of a simple pendulum is measured for each of several lengths, and curves of period versus length and square of period versus length are plotted. From the average value of the ratio of the length to the square of the period, the acceleration due to gravity is computed.

THEORY: A simple pendulum is defined, ideally, as a particle suspended by a weightless string. Practically it consists of a small body, usually a sphere, suspended by a string whose mass is negligible in comparison with that of the sphere and whose length is very much greater than the radius of the sphere. Under these conditions, the mass of the system may be considered as concentrated at a point- namely, the center of the sphere- and the problem may be handled by considering the translational motion of the suspended body, commonly called the "bob," along a circular arc.

A compound pendulum consists of a body of any shape or size vibrating about a horizontal axis under the influence of the force of gravity. Thus a ring hung on a peg, or a bar supported at one point, is a compound pendulum. In this case, the mass may not be considered as concentrated at a point, and the motion is one of rotation rather than translation. The mathematical formulation for the compound pendulum is somewhat more complicated than in the case of the simple pendulum. Consider the diagram of a simple pendulum shown in Fig. 1. In its equilibrium position the bob is at the point A vertically below the point of support O. In this position the downward pull of gravity *w* is counteracted by the upward pull *p* of the cord. When the bob is displaced, to some point B, the weight *w* may be resolved into two components, one *n* normal to the arc AB which is counteracted by the pull *p* of the string, and a force *f* tangent to the arc which tends to restore the pendulum to its equilibrium position. The greater the displacement, the greater is this component *f* and the less the force pin the string, as can be seen by comparing positions B and C. Thus the bob is subjected to a translational force *f* which increases with the displacement and always tends to reduce the displacement.

When the pendulum is released from a given displacement, it moves with increasing velocity toward its equilibrium position, acquiring thereby a momentum which carries it through the neutral position and produces a negative displacement. It should be noted here that the choice of positive and negative directions is purely arbitrary: it is convenient, although not necessary, to call displacements to the right positive and those to the left negative. Neglecting the negative displacement will be equal exactly to the initial positive displacement. When the point B' is reached, the restoring force causes a reversal of the motion and the bob returns to B. This to-and-fro motion of a pendulum is called *vibratory*, or *oscillatory*, motion.

It is interesting to note the energy changes that occur during the oscillation of the pendulum. Potential energy is defined as the energy which a body possesses because of its position, and kinetic energy is that due to its motion. When the bob is displaced (say from A to C) it is lifted against the force of gravity ($w = mg$) through a distance *h*. The increase

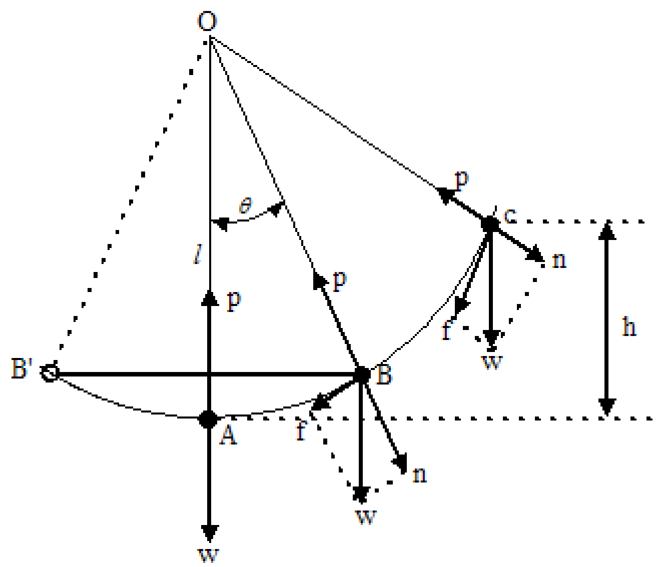

Fig. 1. Diagrammatic Analysis of the Simple Pendulum.

in potential energy is equal to the work done *mgh*. As the body falls from C it loses potential energy and acquires kinetic energy. A body of mass m traveling with a velocity *v* has a kinetic energy equal to ½ mv^2. The kinetic energy at A is equal to the potential energy at B provided there has been no loss due to friction. Stated differently, at any point in its path the sum of the potential and kinetic energies is a constant if the frictional forces are negligibly small. Three fundamental quantities are involved in the motion of the pendulum.

1. The *length* of the pendulum is measured from the point of suspension to the center of the spherical bob.
2. The *period* is defined as the time required for the pendulum to execute its complete motion; i.e., the time

between successive transits through any point *in the same direction*.

3. The *amplitude* of the motion is defined as the maximum displacement from the equilibrium position; it may be described in terms of the angular displacement θ, or in terms of the linear displacement *s* along the arc.

A fundamental characteristic of the pendulum is its *tautochronous* property; i.e., the period is independent of the amplitude, provided the amplitude is not too great. This tautochronous property was first observed by Galileo in the sixteenth century.

The complete formulation of the mathematical relationships between the length *l*, the period *T,* and the amplitude θ is quite complicated, but when certain limitations are introduced a simple approximation results which is satisfactory in many practical cases. One way of stating the limitation on the amplitude is to require that θ shall be so small that the chord BB' shall be equal approximately to the arc BAB'. Under this restriction, it can be shown that, neglecting friction,

Fig. 2. Simple Pendulum.

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

It is to be noted that in this equation neither the amplitude θ nor the mass *m* of the bob appears. Thus the bob may be of any material and of any size subject to the condition that its radius *r* be small in comparison with *l*. Moreover, the period is the same for all amplitudes up to the value of θ set by the above approximation. For example, if in Fig. 1 θ represents this maximum value, the time required for the bob to travel from B to B' and back is the same as the time required for a very small vibration about the point A. It is this tautochronous property that makes the pendulum useful as a timing device.

By making measurements of *l* and *T*, the relationship expressed by Eq. (1) can be used to determine the acceleration *g* due to gravity.

APPARATUS: The essential apparatus employed in this experiment consists of a simple pendulum, which is composed of a metal ball suspended by a light cord from a rigid support. The only auxiliary apparatus required consists of a length measuring instrument, usually a meter stick, and a time measuring device which may be a stopwatch or merely an ordinary timepiece with a second hand. A vernier caliper for measuring the diameter of the ball is desirable, although not absolutely necessary. A sheet of rectangular coordinate paper is needed for graphing the data.

TABLE I

1	2	3	4	5
Length	Time of 50 Vib. t	Period T	Square of Period T^2	Ratio l/T^2

PROCEDURE:

Experimental: 1. Make a simple pendulum of a ball and string and suspend it from a suitable support as shown in Fig. 2. The support should be sufficiently rigid that no appreciable movement will be imparted to it by the vibration of the pendulum. Measure the diameter of the bob, using a vernier caliper if one is available; if not, a reasonably good measurement can be made with two blocks and a meter stick. Make the initial length of the pendulum 120cm, taking into account the radius of the bob. Start the pendulum vibrating through a small arc, not greater than 5 degrees between extreme displacements. Determine the time required for 50 vibrations; in doing so count each passage of the bob, *in the same direction*, through the midpoint beginning with the count of "zero." Enter the data in columns 1 and 2 of Table I. Take a series of six such observations, shortening the length each time by 20cm. **Caution**: The *angular* displacement must be kept within the limit specified; if the linear displacement is held constant the angular displacement will, of course, increase.

2. For some convenient length compare the periods when the arc is less than 5 degrees and when it is over 30 degrees.

Analysis of Data: From the data in column 2 compute the corresponding periods and enter in column 3. Square these periods and enter in column 4. Compute the ratios of *1* to T^2 and enter in column 5.

On the same sheet of Cartesian coordinate paper plot two curves: (1) length as abscissa and period as ordinate - column 1 vs. column 3; (2) length as abscissa and square of period as ordinate- column 1 vs. column 4.

To calculate g, square Eq. (1) which gives

$$T^2 = 4\pi^2 \cdot \frac{l}{g} \qquad (2)$$

Solving for *g* yields

$$g = 4\pi^2 \cdot \frac{l}{T^2} \qquad (3)$$

Take the average of column 5, substitute in Eq. (3) and compute the value of *g*. Compare with the generally accepted value.

QUESTIONS: 1. Compare the shapes of the two curves.

2. Explain how curve 2 confirms the relationship expressed by Eq. (1).

3. How is the period influenced by the amplitude for small amplitudes? for large amplitudes?

4. By what factor is the period of a simple pendulum altered when its length is doubled?

5. Explain how the simple pendulum could be used to compare the values of g in two different localities, e.g. at sea level and on a mountain top.

6. What experimental errors influence the determination of g in this experiment?

7. Discuss the effects of a yielding of the support upon the results of this experiment.

8. A so-called "seconds pendulum" is one that passes through its equilibrium position once a second. (a) What is the period of such a pendulum? (b) By referring to the graph, determine the length of a seconds pendulum.

9. Discuss the energy transformations that occur during one complete vibration of the pendulum.