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# MECHANICAL ADVANTAGE, EFFICIENCY, POWER: THE WINDLASS

**OBJECT**: To study the concepts of mechanical advantage, efficiency, and power of a simple machine by the use of a windlass (wheel and axle).

**METHOD**: The Ideal mechanical advantage of a windlass (Fig. 1) is determined from the measured ratio of the diameter of the large wheel of the windlass, to which the input force is applied, to that of the axle which supports the opposing load. The actual mechanical advantage is measured from the ratio of the load lifted to the applied force necessary to raise the load at uniform speed. The efficiency is calculated from the ratio of the ideal to the actual mechanical advantage. To determine the power he can develop, the observer turns the wheel (Fig. 2) against the friction caused by a belt dragging in the grooved wheel and attached to weights so that the force against which the observer is turning may be measured. The power is expressed in terms of these forces, the circumference of the pulley and the time required to make an observed number of revolutions.

**THEORY**: With many machines it is possible to obtain a very large output force from a small applied force and to do work in a more convenient and advantageous manner. However it must be remembered that no machine can violate the fundamental law of the conservation of energy. If the machine makes it possible to produce a large opposing force with a comparatively small applied force, it will always be found necessary to move the applied force a correspondingly larger distance than the opposing force moves. Hence it is always necessary to keep clearly in mind the distinction between such important terms as force and work, work and power, and efficiency as opposed to mechanical advantage.

**Work**: Work is done when motion is produced against a resisting force. The measure of the work *W* which is done is given by the product of the applied force *F* and the displacement *s* produced in the direction of that force. In symbols

$$W = Fs \tag{1}$$

In the British system the most used unit of work is the *foot-pound*, which is the work done by a force of one pound that produces a displacement of one foot in the direction of the force. The cgs unit of work is the *erg;* this is the work done when a force of one dyne produces a displacement of one centimeter in the direction of the force. The unit of work in the mks system is the *joule* or *Newton-meter*; (one joule = $10^7$ ergs.)

**Efficiency**: The efficiency of a machine is the ratio of the useful work output of the machine to its total work input. This ratio is usually expressed as a percentage, in which case the ratio is multiplied by 100%.

$$\text{percentage efficiency} = \frac{\text{useful work output}}{\text{total work input}} \times 100\% \tag{2}$$

Notice that this definition is a ratio of *work*, and not of *force*. The limiting values of efficiency are from 0 to 1 or from 0 to 100 percent.

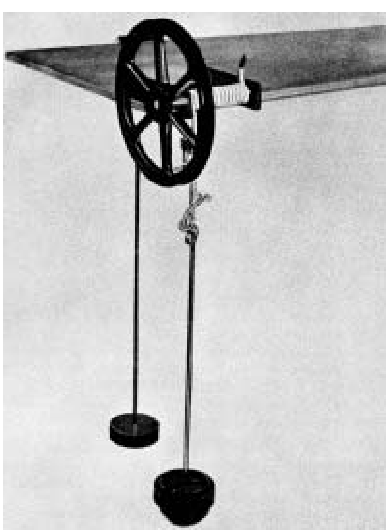

Fig. 1. The Windlass.

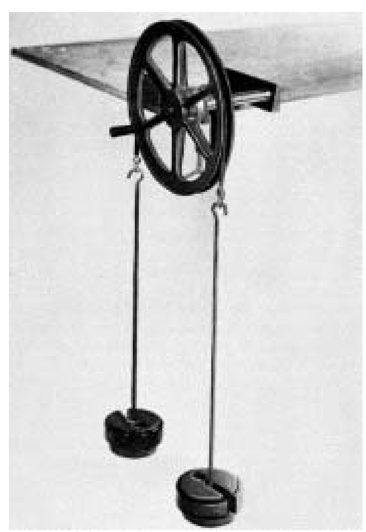

Fig. 2. Windlass Dynamometer, arranged for the measurement of "manpower."

**Mechanical Advantage**: The *mechanical advantage* of a machine should not be confused with its *efficiency*. Mechanical advantage is essentially a force ratio, while efficiency is a *work* ratio. It is helpful to consider both the actual and the *ideal* mechanical advantages of a machine. Actual mechanical advantage (AMA) is defined as the ratio of the opposing or resisting force $F_o$ to the applied force $F_a$. Symbolically

$$AMA = F_o / F_a \tag{3}$$

The ideal, or theoretical, mechanical advantage (IMA) of a machine would be the value of this ratio only in the ideal case where the machine was frictionless. For such an ideal case the work output of the machine would equal the work input or

$$F_o s_o = F_a s_a \tag{4}$$

where $s_o$ represents the displacement of the opposing force and $s_a$ the displacement of the applied force. From this equation it follows that

$$\frac{F_o}{F_a} = \frac{s_a}{s_o} \tag{5}$$

in the *ideal* case. The ratio $s_o / s_a$, the distance moved by the applied force to that moved by the opposing force, is called the *ideal mechanical advantage*

$$IMA = s_o / s_a \tag{6}$$

For any actual machine the work output is necessarily less than the work input and hence the actual mechanical advantage is always less than the ideal advantage. In the case of simple machines the ideal mechanical advantage may be expressed in terms of the geometrical constants of the machine.
It is evident that if each of the distances in Eq. (6) be divided by the time required to travel that distance, a ratio of the *speed* of the applied force to the speed of the opposing force would be obtained. Consequently the ideal mechanical advantage is also equal to the *speed ratio* of the applied to the opposing forces.
Mechanical advantage should not be confused with efficiency. The limiting values of mechanical advantage range from zero to an infinitely large amount. Note that an advantage of 1 is very common and quite desirable in many cases and that an advantage of less than 1 is not a "disadvantage" but quite helpful in certain machines where a large speed ratio is desired instead of a large force ratio.

**Relation Between the Mechanical Advantages and Efficiency**: From the definition of efficiency

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}} = \frac{F_o s_o}{F_a s_a} \times \frac{s_o}{s_a} \tag{7a}$$

This may be written

$$\text{Efficiency} = \frac{F_o}{F_a} \div \frac{s_a}{s_o} = \frac{AMA}{IMA} \tag{7b}$$

This ratio is usually multiplied by 100% and thus expressed as a percentage.

**The Mechanical Advantage of the Windlass**: The ideal mechanical advantage of a windlass, or wheel and axle, from Eq. (6) is evidently

$$IMA = \frac{s_a}{s_o} = \frac{\pi d}{\pi D} = \frac{d}{D} \tag{8}$$

where $d$ is the diameter of the large wheel (or twice the length of the crank arm) and $D$ is the diameter of the axle.
It will be seen that the IMA may be made as large as desired by increasing d or decreasing $D$.

**Power**: The power $P$ developed by a machine is defined as the time rate at which it does work; i.e., it is numerically the work $W$ done per unit of time $t$. Symbolically

$$P = \frac{W}{t} = \frac{Fs}{t} = Fv \tag{9}$$

Some of the common units of power are the horsepower (h. p.) and the watt. One horsepower is the power developed by a machine which does work at the rate of 550ft-lb./sec. (Note that 1hp is *not* 550ft-lb.) One watt is the power developed by a machine which does work at the rate of 1joule/sec. One horsepower is the equivalent of 746watts.
Since the concepts of work and power are so frequently misused by the average person, the student should scrupulously observe the fundamental distinction between these terms.

**The Windlass Dynamometer**: A common method for measuring the average power developed by a machine is to use a brake, known as a *dynamometer* or *Prony brake*, whereon the energy output of the machine may be dissipated as heat. In the smaller machines, this brake may conveniently take the form of a strap passing halfway around the pulley and supporting two weights as shown in Fig. 2. The net force against which the pulley turns is given by the difference between the force $F_1$ opposing the motion and that force $F_2$ which tends to assist in turning the wheel. The distance which the pulley travels against this force can easily be obtained from the circumference of the pulley and the number of revolutions which it makes. The working equation may be obtained by substituting in Eq. (9) the observed forces $F_1$ and $F_2$, the diameter $D$ of the wheel, and the number of revolutions $N$ made in the time $t$. This gives for the average power

$$P = \frac{W}{t} = \frac{Fs}{t} = \frac{(F_1 + F_2)\pi DN}{t} \tag{10}$$

If the power is to be expressed in horsepower, the forces must be in pounds, the diameter in feet, the time in seconds and the result divided by 550. If the power is to be in watts, the forces must be in dynes (gram weight multiplied by

980cm/sec$^2$), the diameter in centimeters, the time in seconds and the result divided by $10^7$.

**APPARATUS**: The windlass, Fig. 1, consists of a large grooved wheel which turns with little friction in a heavy frame rigidly clamped to the table. The opposing force is attached to a rope which passes through a hole in the axle and winds around it as the wheel is turned. The applied force is furnished by suitable weights on a holder fastened to a cord which passes around the grooved wheel.

For the measurement of "manpower" the apparatus is arranged as in Fig. 2 as a windlass dynamometer, or Prony brake. The wheel is turned by hand with a handle near the edge. The frictional opposing force is supplied by a belt in the grooved wheel, the ends of the belt being attached to known weights. The belt is made of two portions of different materials, one part being leather and the other copper. This arrangement serves properly to balance the loads at the various speeds at which the brake is turned.

An alternative plan for applying the forces to the dynamometer is to have the apparatus arranged as in Fig. 3. Here the ends of the belt are attached to 15kg spring balances whose tension can be varied by vertically adjusting the horizontal rod which supports them.

Additional apparatus required includes a meter stick, a vernier caliper, a set of slotted weights up to 500gm, a set of slotted kilogram weights, a 50gm weight holder, a 1kg weight holder, and a stop watch or clock.

**PROCEDURE**:

**Mechanical Advantages and Efficiency of the Windlass**: Arrange the apparatus as in Fig.1. Measure the diameter of the axle with a vernier caliper and the diameter of the wheel with a meter stick. Make proper allowances for the thickness of the ropes and the depth of the groove. Calculate the ideal mechanical advantage from the ratio of the diameters.

Next attach a 10kg load to the axle by means of the rope and hanger and then add sufficient load to another hanger attached to the cord passing around the wheel in order to raise the heavy load at a uniform rate, after it is started by hand. From the ratio of these two forces determine the actual mechanical advantage. Finally calculate the efficiency from the ratio of the actual to the ideal mechanical advantages. Repeat with a series of loads ranging downward to zero. Plot a curve showing the variation of the efficiency with the load.

**Measurement of Manpower**: Arrange the Prony brake as shown in Fig. 2. For one observation $F_1$ may be about 6kg, while F. may be about 1½kg. For the alternative arrangement (Fig. 3) both forces must be read while the wheel is turning. Turn the pulley as fast as possible, keeping the forces constant. A second observer should determine with a stop clock the time required to turn the wheel through a suitable number of counted revolutions (25 to 50). Record the necessary data and calculate the average power, both in British and metric units. Make all necessary original observations in both systems of units; i.e., measure *D* in both centimeters and in feet and the forces in both grams weight and pounds. Having calculated from the original independent data the power in horsepower and in watts, check the results by 746 and comparing with the observed

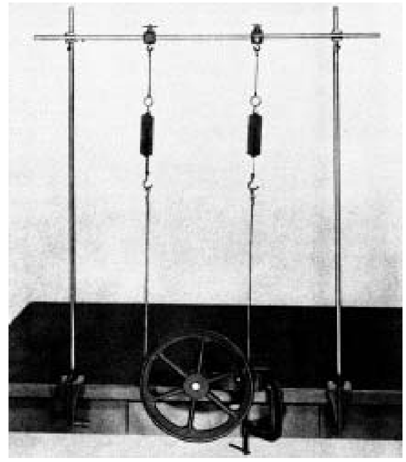

Fig. 3. An alternative arrangement of the Windlass Dynamometer.

power in watts.

Such other observations may be taken as time will permit. For example, each observer may test his own maximum horsepower or the wheel may be turned more slowly at a rate which could be kept up for a considerable time.

**QUESTIONS**: 1. Why is it not correct to define 1ft-lb. as the amount of work required to move a 1lb. body 1ft?

2. Discuss the relative values of the errors introduced into this experiment on the measurement of "human horsepower" by reason of the friction in (a) the axle bearings; (b) the strap brake.

3. Prove that efficiency = power output/power input. Does efficiency = force output/force input?

4. Mr. H. G. Wells, in describing a character in one of his books, makes the statement, "He was a man of much power, but of little energy." Does this statement characterize correctly a type of individual?

5. The human dynamometer used in an experiment on "manpower" had a diameter of 1.3ft. It was loaded on one side with a force of 22lb. and on the other with 4.0lb. If 90 revolutions are counted in 45sec, what is the horsepower output? The wattage output?

6. A strap brake is attached to the pulley of a motor. The pulley has a radius of 20cm and exerts a net force of 50gm-weight on the balances attached to it. In order for the motor to make 1500 revolutions in three minutes, 3082 joules of work must be supplied to it. Calculate the wattage output and the efficiency of the motor.

7. How can a given wheel and axle be used to produce first a high and then a low speed ratio (ideal mechanical advantage)? What effect does this change have on the efficiency?

8. Show how one could measure his average human power by running up a flight of stairs. State the data that should be measured and the calculations that must be made for this determination.