CENCO Physics Selective Experiments In Physics

• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

INCLINED LINEAR-AIR-TRACK ATWOOD MACHINE

OBJECT: To study Newton's second law of motion by comparing the experimental values of forces and accelerations of objects on an inclined linear air track Atwood Machine with similar values computed from other measurements.

METHOD: An inclined linear air track is used as a modified Atwood Machine. Data are taken for conditions of zero acceleration of the object or glider on the track for both its upward and downward motion. From these data the force component of the weight of the glider parallel to the track is obtained. This value is compared to that required by the geometry of the system.
The glider then is accelerated by an applied unbalanced force. The magnitude of the resulting acceleration is computed by applying Newton's second law of motion. This acceleration is then experimentally determined from distance and time measurements of the glider traveling along the track. From a comparison of the two values, Newton's second law is verified and the magnitude of the frictional force opposing the motion of the glider is determined.

THEORY: When two objects of unequal masses, m_2 greater than m_l, are tied to the ends of a cord and hung over a fixed pulley of negligible mass (an Atwood machine, Fig. 1), the

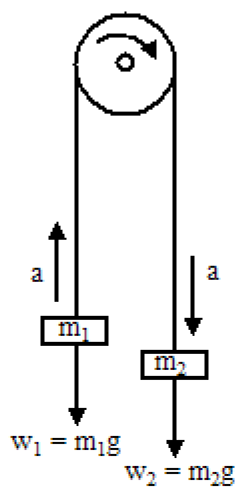

Fig. 1. Operating principle of Atwood machine in conventional setup.

difference of the two weights acting on the system is $m_2 g - m_1 g$, where g is the acceleration due to gravity. When this differential force is greater than the retarding frictional force f acting on the system, the remaining unbalanced force F will produce an acceleration of the system. Applying Newton's second law of motion, $F = ma$, one may write

$$(m_2 g - m_1 g) - f = (m_2 + m_1)|a| \quad (1)$$

The symbol $|a|$ refers to the magnitude of the acceleration which must be the same for both masses. The directions of the accelerations, however, are different. It is understood that the symbol a from here on refers only to magnitude. This apparatus, an Atwood machine, has historically been used in laboratory experiments for the study of accelerated motion. It gives good results since the relatively small frictional force of a good pulley is quite constant. A variation of the Atwood machine, permitting the study of additional physical phenomena, might be designed as shown in Fig. 2 where the object of mass m slides over the inclined plane.

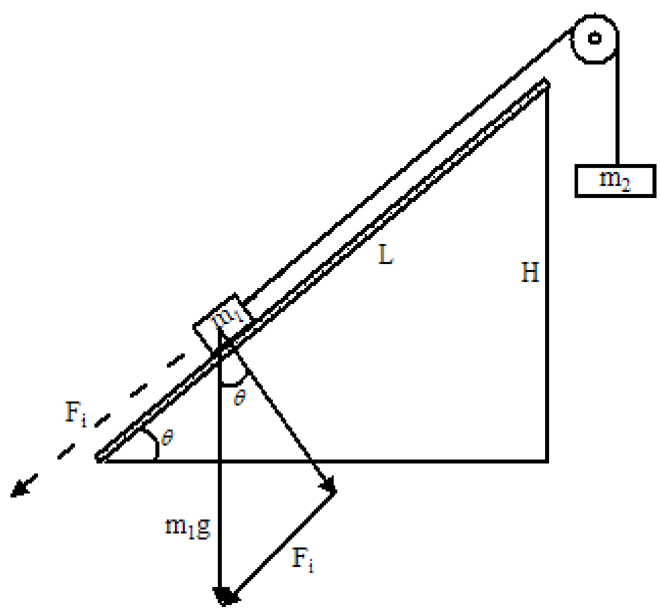

Fig. 2. Operating principle of Atwood machine used in inclined-plane experiments.

Two modifications of Eq. (1) are now required to apply it to this modified form of the Atwood machine. First of all, only the component F_i of the weight of the sliding object, which acts parallel to the incline, instead of the entire weight, is involved in the cord tension. Thus F_i must replace $m_1 g$ in Eq. (1). Secondly, a second term f' must be supplied to the equation to compensate for the retarding force of friction of the object sliding over the inclined surface. Hence Eq. (1), so modified, becomes

$$(m_2 g - F_i) - (f - f') = (m_2 + m_1)a \qquad (2)$$

One might assume that this equation, applied to the inclined plane Atwood machine, would provide an even more instructive experiment than the form used in Fig. 1. However no satisfactory design of the type shown in Fig. 2 has been available until recently. The chief hindrance is the fact that the usual solid inclines have a variable frictional force. The sliding frictional force is so dependent on microscopic surface irregularities that even an inclined plane which is highly polished will have different frictional forces from one position to another along the incline. A not-so-ordinary lubricant, however, has been found to reduce the friction to an almost-zero value. This lubricant is nothing more than a layer of compressed air located between the sliding surfaces. In effect, this layer of air acts like a cushion to make the upper of the sliding surfaces float instead of rubbing on the lower one. Thus the only resistance offered to the relative motion of the sliding surfaces is that produced by the air, and that resistance is so very small as to be negligible. One apparatus which makes use of this lubricant is the *air track*.

The linear air track is constructed of a closed-end hollow tube with a plane bearing surface through which many uniformly-spaced small holes have been drilled (Fig. 3).

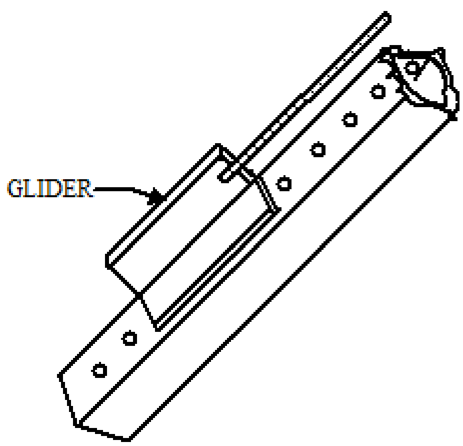

Fig. 3. Detail of the linear air track with glider.

When compressed air is supplied to the interior of the chamber, the sliding unit or glider is lifted slightly from the surface to ride on a thin cushion of air. One can readily ascertain that there is negligible friction between the air track and glider by placing the glider on the track and note how a minute elevation of one end which will cause the glider to "float" down the incline. Only a fraction of a millimeter elevation above the horizontal position per meter length of track is required. Thus the frictional force of the glider moving over the track is so small that it may be disregarded. It is possible, therefore, to use a modified Atwood machine in conjunction with an air track for laboratory experimentation. Figure 4 shows the linear air track used to provide a modified Atwood machine with the track elevated on angle θ above the horizontal position. Since f' approaches the value zero, when an inclined air track is used, Eq. (2) reduces to

$$(m_2 g - F_i) - f = (m_2 + m_1)a \qquad (3)$$

When the value of m_2 is adjusted so that the glider moves up the incline with zero acceleration, that is, it maintains the same initial upward velocity given to it, Eq. (3) simplifies to

$$(m_2 g - F_i) - f = 0 \qquad (4)$$

Similarly when the mass m_2 is reduced to such value m'_2, less than F_i that the glider will move down the incline, maintaining the same initial downward velocity given to it, then

$$(F_i - m'_2 g) - f = 0 \qquad (5)$$

Subtracting Eq. (4) from Eq. (5) eliminates f, the frictional force term associated with the pulley, and gives

$$F = (m'_2 g + m_2 g)/2 \qquad (6)$$

Thus the value of F_i may be obtained experimentally by finding the required values of m_2 and m'_2 for zero acceleration of the glider up and down the air track respectively.

The geometry of the diagram of Fig. 2 shows that F_i can be determined also from the measured dimensions of the height H and length L of the air track.

$$F_i = m_1 g \sin\theta = m_1 g (H/L) \qquad (7)$$

These two values of F_i theoretically should be equal. Suppose the system supports the mass m_2 which would give a constant velocity up the incline. If now an additional object of mass Δm is added to m_2, its extra weight will be the unbalanced force available to produce an acceleration of the system. Applying Newton's second law of motion to solve for the magnitude of the acceleration,

$$\Delta mg = (m_2 + \Delta m + m_1)a \quad \text{or} \quad a = (\Delta mg)/(m_2 + \Delta m + m_1) \qquad (8)$$

The acceleration can be experimentally determined also by measuring the time t required for the glider, starting from rest, to move a distance s along the incline. Since

$$s = v_o t + \frac{1}{2}at^2 \text{ and } v_o = 0$$

$$a = 2s/t^2 \qquad (9)$$

The values obtained for the glider acceleration under an accelerating force Δmg by the methods of Eq. (8) and Eq. (9) should be theoretically the same.

APPARATUS: Linear air track and glider with a very light low friction pulley attached to one end; compressed air source; light pail and supply of sand or metal pellets; accurate platform balance, meter stick, and stop-watch.

PROCEDURE: Time readings, entered as *t* in Eq. (9), are of short duration. To obtain a satisfactory degree of accuracy, therefore, it is essential that the time readings be made carefully. Make repeated time measurements, as the glider moves over a specified distance *s*, with a stopwatch having tenth-second graduations. You may use a spark timer and spark sensitive paper with the apparatus to get even better distance and time measurements.

In each of the following tests load the glider to a weight only slightly less than the air pressure can support. Adjust the air pressure so that the glider just floats smoothly along the track.

(A) *Testing the fractional force with the linear air track*. Place the air track in as nearly a horizontal position as judged by sight. Place the glider on the track, apply the air pressure, and note the motion of the glider. It will almost certainly accelerate along the track. Elevate the lower end of the track until the glider retains a small given initial velocity. How many thicknesses of paper must now be placed under one end of the track to again show an acceleration? From this information the student can draw a reasoned estimate of the magnitude of the frictional force opposing the motion of the glider.

(B) *Comparing the experimental value of F_i with that predicted from the geometry of the incline.*

(1) Set up the apparatus at approximate the elevation angle, $\theta = 40°$, illustrated in Fig. 4. With the glider near the bottom of the incline, adjust the hanging load, by adding or removing sand or metal pellets from the hanging pail, until the glider maintains its given initial low velocity as it moves up the length of the track. Record the value of this load weighed to an accuracy of 0.1gm.

(2) Repeat the above for a comparable motion of the glider down the length of the track.

(3) Weigh the loaded glider and also measure the length *L* and height *H* of the track.

Compute F_1 using, first, the dynamic experimental data and, second, the geometry of the incline. Compare these two values.

(C) *Comparing the value of the glider acceleration (obtained by applying Newton's second law of motion) with the value obtained by time and distances measurements.*

Using the experimental arrangement of B (1) above, add an additional load to the hanging pail in order to accelerate the system. A convenient additional load is a mass from a box of balance weights. Select a mass Δm such that the glider, starting from rest, requires approximately 3.5 seconds to move a maximum possible distance up the air track. A series of time readings should be taken for this displacement of the glider.

Using Eq. (8) solve for the acceleration of the glider.

Find the value of this acceleration by using the measured data for time and path lengths in Eq. (9).

Compare the above two values of acceleration and draw conclusions.

Fig. 4. Operational setup for the inclined-plane experiments.

PROBLEMS: 1. What evidence was obtained during the experimentation to show that the frictional force of the pulley was sufficiently constant to permit the use of Eq. (1)?

2. From your data compute the value of *f*, the frictional retarding force of the pulley which was used in this experiment. Assume that the rotational inertia of the pulley is negligible.

3. The force required to initiate sliding motion between two solid surfaces is greater than that required to continue the motion. State two reasons why one would expect this statement to be true.

4. Using the value of the acceleration obtained by Eq. (8), compute the tension in the cord attached to the pail.

5. Assuming that the frictional force of a glider on the incline is a constant but not equal to zero, would Eq. (6) still apply?

6. "Air cushion" machines called *hovercraft* are now commercially used in transportation. Is the only cost required to propel these machines from one point to another based on the small power expended to keep them in forward motion? Explain.

7. It has been stated that apiece of equipment so heavy that to transport it on a track would damage the roadbed, could be transported over the same road in a "Hovercraft" without damaging the roadbed. Is this statement true? Give reasons for the answer.