

## VECTORS: GRAPHIC METHOD

**OBJECT:** To study the use of vectors for the representation of vector quantities; in particular to use graphical methods to study the equilibrium of a body under the action of concurrent forces.

**METHOD:** Concurrent forces in equilibrium will be studied as examples of vector quantities. The resultants of two or more forces will be determined by a graphic vector method. The results will be checked experimentally on the force table by balancing the resultant forces with their equilibrants.

**THEORY:** *Scalar quantities* are those that have magnitude only, but no direction. Examples are \$10.00 and a density of  $12\text{gm/cm}^3$ . *Vector quantities* have both magnitude and direction. Examples are a velocity 120mi/hr east and a force of 50lb. at an angle of  $30^\circ$  with the horizontal. Scalar quantities are added algebraically:  $\$10.00 + \$12.00 = \$22.00$ . But the addition of vector quantities must be done geometrically, as will be illustrated in this experiment. The *vector sum* of several vector quantities is that single vector quantity which is the equivalent of the original quantities.

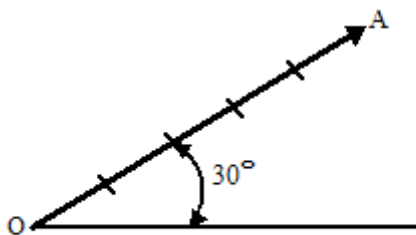


Fig. 1. Representation of a vector quantity by a vector.

A vector quantity is graphically represented by a line segment called a *vector*, such as that shown in Fig. 1. The length of the vector indicates the magnitude of the vector quantity. The direction of the quantity is represented by the orientation of the vector. The sense along the line is indicated by an arrowhead. The vector in Fig. 1 could, for example, represent a force of 100lb. acting at an angle of  $30^\circ$  above the horizontal.

The vector sum of a number of vector quantities is found by the process of geometric summation. In the present experiment this is done by purely graphical methods, but analytical procedures also can be used.

When the resultant of two vector quantities is to be obtained a *parallelogram method* can prove useful. As an illustration of the parallelogram method consider the two forces *a* and *b* represented in Fig. 2 by the vectors *OA* and *OB*. The

resultant *R* is found by constructing a parallelogram having the two vectors as sides. The *concurrent* diagonal of the parallelogram gives the vector that represents in magnitude and direction the desired resultant. The length of this vector is measured, and the application of the chosen scale shows the magnitude of the resultant. A protractor is used to measure the angle made by the resultant vector with some given axis, usually the positive direction of the X axis. If the figure is carefully drawn to a large scale the resultant can be determined with good accuracy.

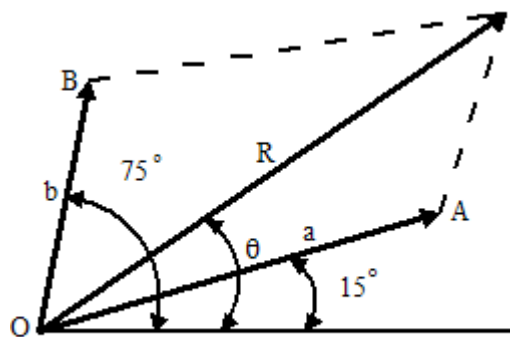


Fig. 2. Parallelogram method of finding the resultant of two vector quantities.

When the resultant of more than two vector quantities is desired a *polygon method* is used, as shown in Fig. 3. Draw the line *OA* to represent one of the vectors such as *a*. From the end of the vector *OA*, construct anyone of the other vectors, say *b*; from the end of this vector *AB* lay off the next vector *c*. The resultant *R* is the vector *OC* that closes the diagram, with its arrowhead touching that of the last vector.

**Equilibrium of Forces:** An object is in *equilibrium* if it has no acceleration, that is if its velocity is constant (in both magnitude and direction). A body at rest is an obvious example of equilibrium and will be used in this experiment.

Forces whose lines of action intersect at a common point are said to be *concurrent*. When several concurrent forces act upon a body it is possible that, in special cases, their vector sum, or resultant, is zero. In such cases the body is in equilibrium, insofar as its linear motion is concerned. If the resultant of several forces is not zero, the body acted upon is not in equilibrium, but it can be set into a condition of equilibrium by adding a single force of magnitude equal to the resultant but opposite in direction to it. This force is called the *equilibrant*. In Fig. 3 the equilibrant force *E* is represented by the vector *OD*. If the equilibrant turns out to have zero magnitude the vector diagram is a closed figure

and the system of forces is in equilibrium. Therefore the condition for a body to be in equilibrium, insofar as linear motion is concerned, is that the vector sum of all the forces acting upon it must be zero. This condition will be tested in the present experiment, as an example of the use of vectors.

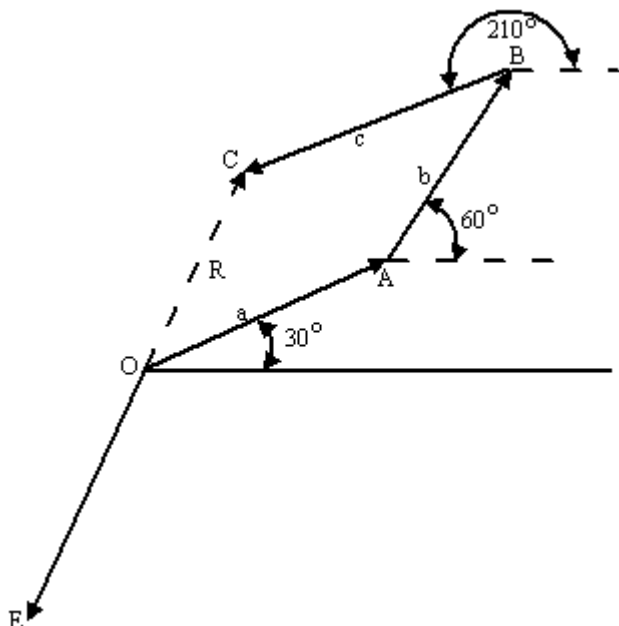


Fig. 3. Polygon method of finding the resultant of a number of vector quantities.

**APPARATUS:** Force table, Fig. 4; scale pans; set of slotted masses; graph paper; protractor; compass; ruler; level glass. The force table is adjusted to make its surface horizontal by the use of a level glass. The object on which the forces act is a ring at the center of the table. A pin within the ring prevents motion of the ring before equilibrium is established. Forces are applied by strings that pass over pulleys to scale pans. The direction of each force is read on the circular scale of the force table.

**PROCEDURE:** 1. A pair of forces may be selected or assigned to each group of students by the instructor. For example, a force  $F_1$  of 200gm. weight at  $30^\circ$  and another force  $F_2$  of 150gm weight at  $140^\circ$  might be used. The problem is to find the resultant and the equilibrant of these forces.

2. Using a sheet of cross-section paper, select a scale, for use with the forces  $F_1$  and  $F_2$ , big enough to fill up most of the sheet. By choosing a large scale, errors in measurement will be minimized. Draw vectors to represent  $F_1$  and  $F_2$  along the directions given. Complete the parallelogram as previously explained. Draw in the resultant vector  $R$  and measure its length carefully. Record the value of the chosen scale and use this length to determine the magnitude of the resultant force. Using a protractor determine the direction of the resultant.

3. Check your result on the force table. Use weights, including the weights of the holders, to represent each of the original forces and the equilibrant. Slide the strings around



Fig. 4. A force table.

the ring so that their lines will intersect if projected backward from the pulleys. Why is this desirable? When the ring appears to be in equilibrium, remove the pin, displace the ring somewhat, and tap the table. The ring should return to the central position. If it does not return to the center adjust the direction or the magnitude of the equilibrant force. Note the percentage difference between your experimental value of the equilibrant and that determined graphically. Record your data in a table, such as the sample shown below. Ask the instructor to check all your final results.

4. Select another group of forces or obtain them from the instructor and repeat the procedure outlined in Step 3.

5. Arrange any three forces on the force table so that they are in equilibrium. Draw a vector polygon to represent these forces. Do the vectors form a closed figure? Comment on the reasons why the diagram is or is not a closed figure.

6. After you have obtained a balanced condition for the equilibrium test try first adding and then subtracting a few 1gm masses in the equilibrant scale pan until you can observe that the balanced condition is destroyed. Record the average of this change of mass and calculate the percentage of this "error." Move the position of the equilibrant pulley until the equilibrium is just destroyed and note the limit of error in the direction of this force.

**QUESTIONS:** 1. Since the scale pans for the masses in this experiment were all alike could their weights be neglected for any or all of the observations? Explain.

2. Would there be any effect on the equilibrium observation in Step 3 if the force table had not been reasonably level? Explain.

3. Under the action of three forces the ring in a force table experiment comes to rest some distance away from the center of the table. Is the ring in equilibrium in this case? Why must it be centered in your experiment?

4. Three forces **A**, **B**, and **C** are to be held in equilibrium. Force **A** is *always* in the first quadrant. In what quadrants can force **C** be if force **B** is in the (a) first, (b) second, or (c) third quadrant? Justify your answers.

5. The forces on the force table act on a ring but are said to be concurrent. Explain. If the cords were attached rigidly to the ring would the forces necessarily be concurrent?

6. Indicate whether each of the following is a scalar or vector quantity: speed, velocity, mass, weight, work, torque, and volume.

7. A hammock is supported by two hooks at the same level. A man is seated in the hammock. Under what conditions will the pull on each hook be equal to the weight of the man?

8. Three forces of 12, 15 and 20lb. in a plane are in equilibrium. The 12lb force is directed horizontally to the right. By means of rough vector diagrams show what two configurations the other two forces could have.

9. By the aid of a vector diagram show why a tightly stretched horizontal cord could be broken by a comparatively small load fastened at the center of the cord.

10. Does it make any sense to calculate the percent difference in the direction of  $E$  and  $E'$  in the last part of step 6? Explain. If not, how could a meaningful comparison of direction be made?

DATA TABLE

Force	Trial 1		Trial 2		Trial 3	
	Magnitude	Direction	Magnitude	Direction	Magnitude	Direction
$F_1$						
$F_2$						
$R$ Resultant Graphical						
$E$ Equilibrant Graphical						
$E'$ Equilibrant Observed						