MEASUREMENT OF LENGTH
WITH A RULER AND VERNIER CALIPER

OBJECT: To study the use of the ruler and the vernier caliper in the measurement of length; to measure the dimensions of a rectangular block and a cylindrical cup and to compute their volumes from these observations.

METHOD: A ruler is first used to measure the dimensions of a rectangular block in both metric and British units. The volume of the block is then calculated from these two sets of observations. Next, a vernier caliper is studied to determine the least count of the instrument, in both metric and British units. The vernier caliper then is used to measure the dimensions of the block previously studied, in both metric and British units. The volumes calculated from these observations are compared with those obtained from the ruler measurements and the accuracies of the instruments are contrasted.

THEORY: Most of the instruments used by physicists and engineers depend directly or indirectly upon the observation of lengths, one of the fundamental concepts in science. The experimental sciences owe much of their progress to the precision with which quantitative measurements of lengths can be made.

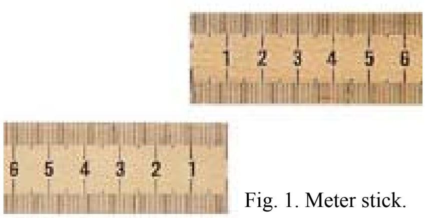

Fig. 1. Meter stick.

Measurements with Rulers: The most widely used ruler in the physics laboratory is the meter stick, Fig. 1. The scale shown is the metric one, with the smallest rulings in millimeters. This enables readings to be estimated, on a sharp edge or line, to a fraction of a millimeter. The opposite side of this ruler is graduated in inches and eighths of an inch. This allows for estimations of a few sixteenths of an inch, depending upon the skill of the observer and the nature of the object being measured.
The best method for using a thick ruler to measure the length of a block, for example, is to place the ruler *edgewise* over the block, as shown in Fig. 2. In order to eliminate "zero error' the ruler should not be placed with the zero end at the edge of the block. Rather the ruler should be placed so that a whole number like 1 or 10 coincides with the edge of the block. Also in order to eliminate an error called "parallax", the eye of the observer should be in the position shown as "right' in Fig. 3 (a), not the one shown as "wrong" in Fig.3 (b).

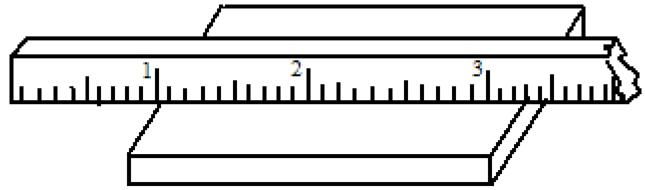

Fig. 2. Placing a ruler to measure length.

Usually it is desirable to estimate fractions of the smallest scale divisions in order to secure the most accurate measurement available. For example, in Fig. 2 the reading should be recorded as 2.36.

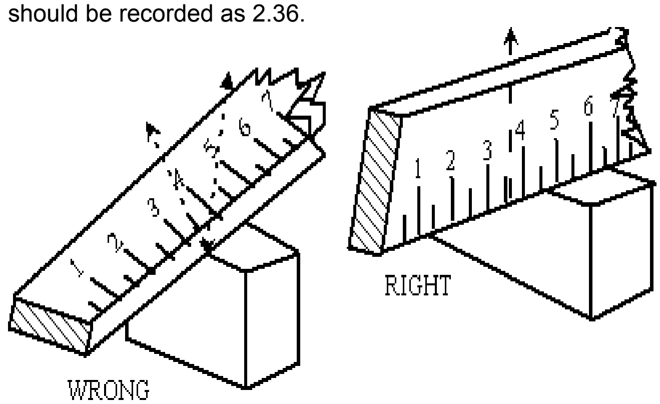

Fig. 3. Precautions in the use of rulers.

The Vernier Principle: More accurate measurements of fractional parts of the smallest divisions on a scale can be made by the use of a *vernier scale*, invented BV Pierre Vernier in 1631. A caliper provided with a vernier scale is known as a *vernier caliper*. (Note the fact that one should not refer to this device as "a vernier")
A vernier scale is an auxiliary scale that slides along the main scale. The graduations on the vernier scale are different in length from those on the main scale. In the most familiar type of vernier caliper *n* divisions on the main scale correspond in length to *n* - 1 divisions on the vernier scale. For example, the vernier scale of Fig. 4 (a) has 10 divisions that correspond in length to 9 divisions on the main scale. Each vernier division is therefore shorter than a main-scale division by 1/10 of a main-scale division. In Fig. 4 (a) the

zero mark of the vernier scale coincides with the zero mark of the main scale. The first vernier division is 1/10 main scale division short of a mark on the main scale, the second division is 2/10 short of the next mark on the main scale, and so on until the tenth vernier division is 10/10, or a whole division, short of a mark on the main scale. It therefore coincides with a mark on the main scale.

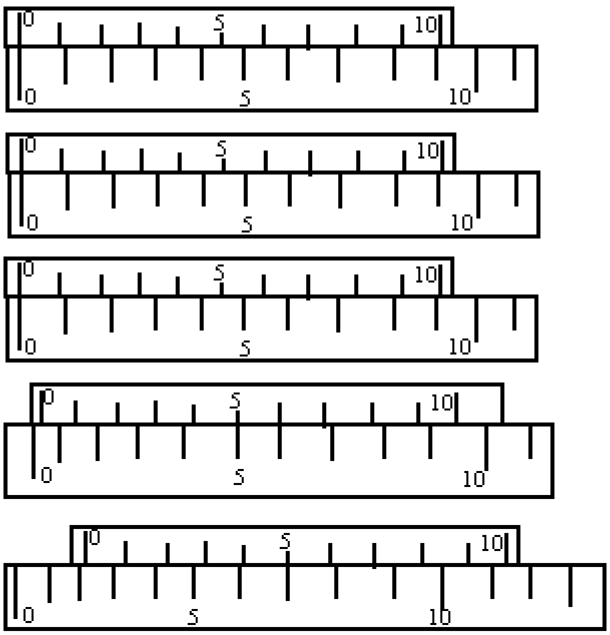

Fig. 4. Main and vernier scales.
(a) Note that 10 vernier divisions equal 9 main-scale divisions.
(b) Note that the first vernier division coincides with a main-scale division.
(c) Note that the second vernier division coincides with a main-scale division.
(d) The distance from the zero of the main scale to the zero of the vernier scale is 0.6 of a scale division.
(e) Reading of 2.4 main-scale divisions.

If the vernier scale were moved to the right until the first vernier division coincided with the first scale division. as in Fig. 4 (b), it is clear that the distance between the two zero divisions would be one-tenth of the smallest scale division. Similarly the distance between the zero divisions in Fig. 4 (c) is two-tenths of a small scale division and in Fig. 4 (d) this distance is six-tenths of a small scale division. Note the fact that the value of the main scale division that coincides with a vernier division is immaterial. It is the number of the vernier scale division that tells the fraction of a main-scale division that the zero of the vernier scale has moved beyond any main-scale mark. For example in Fig. 4 (e) the zero is to the right of the second mark on the main scale and the fourth mark of the vernier scale coincides with a main-scale mark. The reading is 2.0 divisions (obtained from the main scale up to the vernier zero) and 0.4 division (obtained from the vernier coincidence), or 2.4 divisions.
Instruments are manufactured with many different vernier-scale to main-scale ratios. The essential principle of all vernier scales is, however, the same, and the student who masters the fundamental idea can easily learn by himself to read any special type.
In brief, the general principle of the vernier scale is that a certain number *n* of divisions on the vernier scale is equal in length to a different number (usually one less) of main-scale divisions. In symbols

$$nV = (n-1)S \qquad (1)$$

where *n* is the number of divisions on the vernier scale, V is the length of one division on the vernier scale, and *S* is the length of the smallest main-scale division.
The term *least count* is applied to the smallest value that can be read directly by use of a vernier scale. It is equal to the difference between a main-scale and a vernier division. It can be expressed by rearranging Eq. (1), thus

$$\text{Least count} = S - V = \frac{1}{n}S \qquad (2)$$

Fig. 5. A metric-scale vernier caliper.

Vernier Calipers: A wide variety of vernier calipers are available for various kinds of measurements. A familiar metric-scale vernier caliper is shown in Fig. 5. Often this instrument has also British scales. Some vernier calipers are provided with devices to measure internal depths and both outside and inside diameters of cylindrical objects. In Fig. 6 the jaws *c* and *d* are arranged to measure an outside diameter, jaws *e* and *f* to measure an inside diameter, and the blade *g* to measure an internal depth. The knurled wheel *W* is used for convenient adjustment of the movable jaw and the latch *L* to lock the jaw in position.

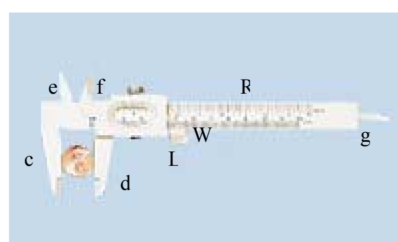

Fig. 6. Measuring a variety of dimensions with the vernier caliper.

When you have occasion to use a new type of vernier scale, first determine the least count of the instrument. In order to make a measurement with the instrument, read the number of divisions on the main scale before the zero of the vernier scale and note which vernier division coincides with a mark of the main scale. Multiply the number of the coinciding vernier mark by the least count to obtain the fractional part of a main-scale division to be added to the main-scale reading.

APPARATUS: Metric ruler; vernier caliper; metal cylinder; cylindrical cup; trip scales and masses.

PROCEDURE: 1. Use the metric-scale ruler to measure the dimensions of a metal cylinder. Be sure to observe the precautions mentioned above. Make about four observations of each dimension. From the average values of these dimensions calculate the volume of the cylinder $(v = \pi r^2 h)$.

2. Check the vernier caliper for "zero error" by noting the reading when the jaws are closed. Decide whether to add or subtract this error to later readings of this instrument.

3. Repeat the observations of Step 1 with the vernier caliper. Calculate the volume of the cylinder from these measurements and compare this volume with that determined from measurements with the ruler. Note the percentage difference between your two values of the volume. Suggest possible sources for the difference in these values.

4. Determine the least count of the British-scale vernier caliper. Measure, as in Step 3, the dimensions of the cylinder and compute its volume. From this value and the metric value, calculate the ratio between the cubic inch and the cubic centimeter. Note the percentage difference from your observed ratio to the standard value.

5. Use the metric-scale vernier caliper to measure the inside diameter and the depth of a cylindrical cup, Fig. 7. Compute the volume of the cup. Measure the mass of the empty cup and its mass when filled with water. Use these data to obtain the volume of the cup and compare this value with that found from the measurements with the vernier caliper.

Fig. 7. Cylindrical cup.

QUESTIONS: 1. A metric ruler is to be used to measure a block that is approximately 10cm long. If the measurement must be accurate to 0.5% what must the smallest division on the scale be?

2. In measuring the volume of a cylindrical body, which would produce a larger error in the results, a 1% error in the diameter or a 1% error in the length? Give reasons.

3. State whether each of the following would produce systematic or random errors in this experiment and explain why: (a) the corners of the block are slightly rounded; (b) opposite sides of the block are not quite parallel; (c) the rule was not placed on edge; (d) the end of the rule was used.

4. In Table I partial information is given about five different verniers. Copy the table and fill in the blank spaces.

TABLE I

Length of a Main Division	Least Count	Number of Vernier Divisions
in	1/128 in	16
2 1/2 mm	mm	10
1/2 deg	min	6
1/4 in	0.01 in	
1/4 deg	1 min	

5. On some verniers n vernier divisions are equal in length to (2*n* - 1) main divisions. Show that the least count is one-*n*th of a main division. What advantage has such a vernier over one constructed in the usual way?

6. Which measurement in this experiment was the least precise? How could its precision be improved? For this purpose would it be desirable to use some instrument designed to read to thousandths of a millimeter?

CENCO CATALOG NOS. OF NEEDED EQUIPMENT

Metric Ruler	No. 73115
Vernier Caliper	No. 72678
Metal Cylinder	No. 76570
Cylindrical Cup	No. 76525
Balance, Trip Scale	No. 03470
Metric Weight Set	No. 09125