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# GRAPHICAL INTERPRETATION OF DATA

**INTRODUCTION**: Many experiments are performed to study the manner in which one property or quantity depends upon or varies with another property or quantity. For instance, how does the frictional force between two surfaces vary with the perpendicular force exerted by one surface on the other? Or, how does the length of a pendulum influence its period? Such variables may conveniently be displayed in the form of graphs that summarize the relationships that were studied in the experiment. This discussion will present in an elementary fashion some of the basic ideas connected with the drawing and interpretation of graphs and their use in developing physical laws that express analytically the relationships between the mutually varying quantities.

**SIGNIFICANCE OF GRAPHICAL PRESENTATION**: Physical laws and principles express relationships between physical quantities. These relationships may be expressed (1) in words, as is commonly done in a formal statement: (2) by means of symbols in an equation; or (3) by the pictorial representation called a *graph*. The choice of the means of expression is dictated by the use to be made of the information. If calculations are to be made, the equation form is ordinarily the more useful. The graph, however, presents to the trained observer a vivid picture of the way in which one quantity varies with another.

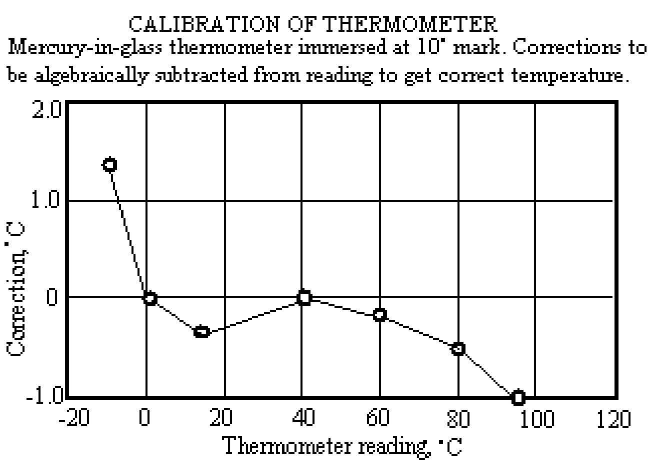

Fig. 1. Calibration chart.

A graph shows more clearly than a tabulation in adjacent columns the manner in which the dependent variable is related to the independent one. Graphs may also be used (1) to obtain pairs of values other than those plotted, (2) to smooth out inaccuracies in data, and (3) to indicate unsuspected trends and critical points.

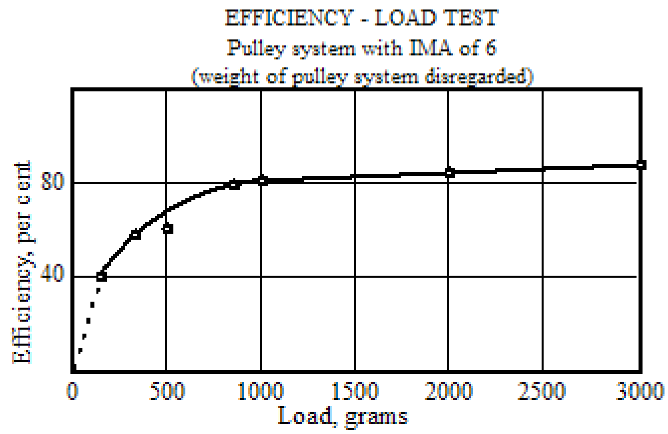

Fig. 2. Emperical data.

**TYPES OF GRAPHS**: Graphs may be divided into four general types. The first type, shown in Fig. 1, is a record of observed values at many points. Because the exact nature of the variation is unknown and may be irregular, the adjacent points are connected by straight lines. This type of graph is often used for statistical and calibration data.

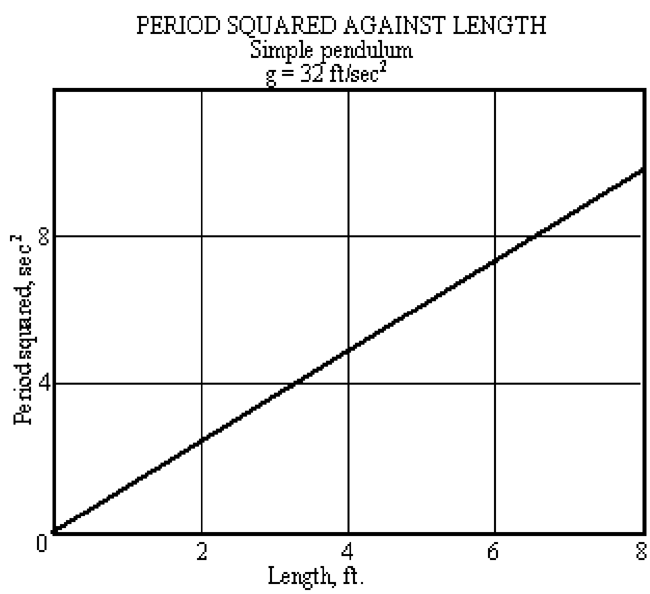

Fig. 3. Theoretical data.

The second type of graph is shown in Fig. 2; this is the type usually drawn to show the relation between two variables.

The observed points are plotted and a smooth curve is drawn that approximately fits the observed data. This type of curve represents an empirical relationship between the quantities plotted. (An empirical relationship is one that is derived from experiment alone, without a theoretical basis.)

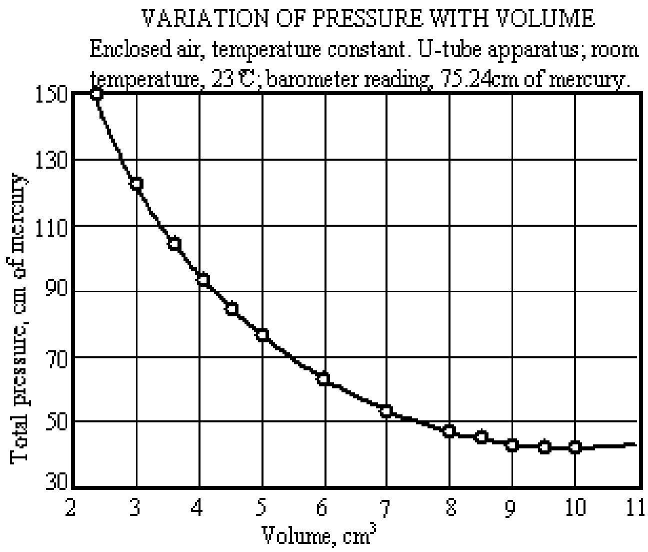

Fig. 4. Proper form for experimental data.

The tacit assumption is made that the points not on the curve are displaced because of experimental inaccuracy. The third type of graph, Fig. 3, represents theoretical relationships; the data are obtained by substituting arbitrary values in an equation expressing the theoretical relationship. Such a curve follows a mathematical form (straight line, sine curve, hyperbola) and the inclusion of specific points is meaningless.
In many cases a curve of this type is included with the second type to show the variation of the experimental data from the theoretical.

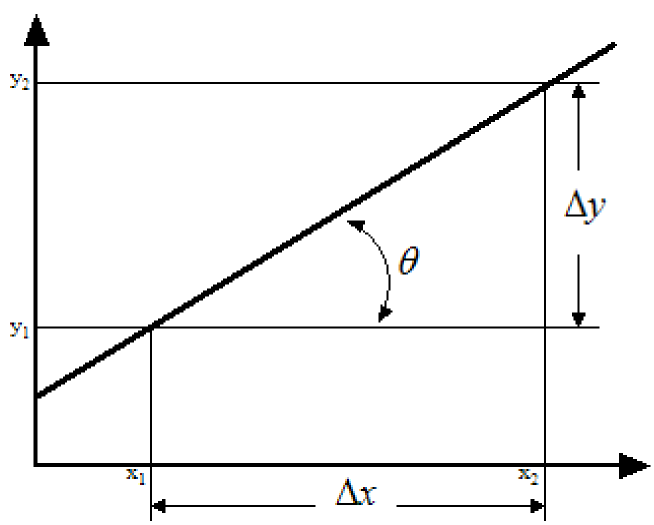

Fig. 5. Method of finding physical slope.

A fourth type of graph is a computational one prepared to be used in place of tabular data to pick off values for use in calculations. An example would be a curve showing the variation of pressure with altitude or the variation of density of a solid or liquid with temperature. Such graphs are always drawn to a very large scale on high-quality paper with many fine divisions. Because of the labor and, care involved in their preparation, the original is usually filed and a print used.

**INDEPENDENT AND DEPENDENT VARIABLES**: In any graph that shows the way in which two varying quantities are related to each other, one quantity is ordinarily considered as the *independent variable*, since it may be chosen at will. The other quantity is called the *dependent variable*, because it must change in terms of its relationship to the independent variable.
It is almost universally conventional to layoff the values of the independent variable along the horizontal or *X*-axis. The values of the dependent variable are then displayed along the vertical or *Y*-axis. The numbers along the *X*-axis are referred to as *abscissas*, while those along the *Y*-axis are called the *ordinates*. For example, in Fig. 2, showing the dependency of the efficiency on the load, the efficiency is seen to be 70% (ordinate) when the load is 500gm (abscissa).

**SHAPES OF CURVES**: The shape of a plotted curve may be irregular or it may be smooth. The majority of graphs are drawn on rectangular coordinate paper, but other types of coordinate drawings are used and will be discussed later.
When the curve is a straight line, it is easily drawn, easily identified, and, in many respects, more useful than any other form of curve. The variables are therefore often rearranged in such a form that the resulting curve is a straight line. The variation of the period of a simple pendulum with the length is not clearly shown by plotting the period against the length. The equation for the period T as a function of the length *l* is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

From this equation it is evident that T varies directly with $\sqrt{l}$: or $T^2$ is directly proportional to *l*. Hence plotting $T^2$ against *l* results in a straight line curve that passes through the origin.

VELOCITY - TIME RELATION FOR FREELY FALLING BODY

$$\text{Slope} = g = \frac{\Delta v}{\Delta t} = \frac{390 cm/\sec}{\frac{16}{40}\sec} = 980 cm/\sec^2$$

The famous Boyle's law inverse relationship of pressure and volume of a gas at constant temperature may be shown by plotting the pressure against the reciprocal of the volume.
If the curve is known to be a straight line, only enough points need to be plotted to locate the line. For a theoretical curve two points are sufficient. When experimental data are plotted, the error is minimized by plotting as many points as can readily be obtained.

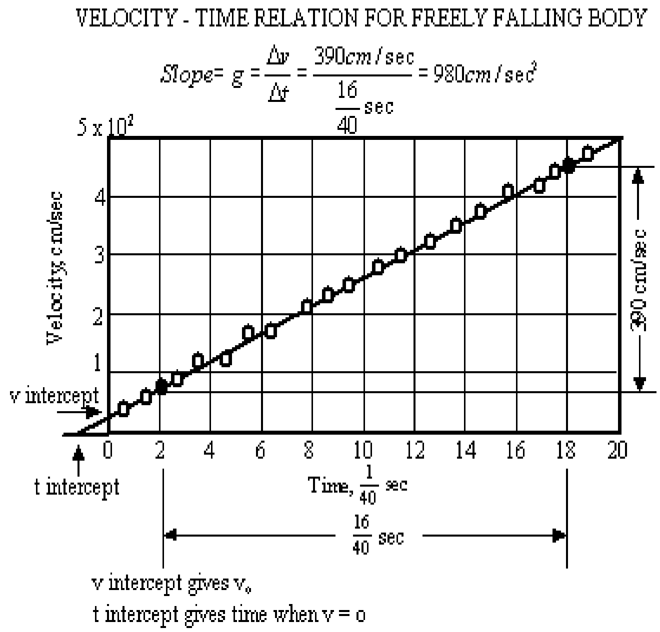


Fig. 6. Demonstration graph, showing interpretation by slope and intercepts.

**DRAWING THE CURVE**: Inasmuch as the curve has so great a significance the utmost care should be used in drawing it. Except in the rare case when adjacent points are connected by straight lines, curves are smoothed through an average of the points. The smooth continuous curve passes as closely as possible to all points but disregards those that are obviously far from the curve. As a rough approximation one may say that the best line will be the one that divides the points so that the sum of their displacements from the line will be a minimum.

The smoothing is done by laying a transparent scale or curve along the points and finding sections of the transparent guide that seem to match the direction of the points. As an aid in drawing smooth curves, it is often helpful to put one's eye nearly in the plane of the paper and to view the plotted data points at a glancing angle. It is not unusual to have to draw several trial curves before arriving at the final one. Many scientists make a practice of plotting the data points on the ruled side of fairly thin graph paper. Then they turn the paper over so that the points show through on the reverse side and do all the practice smoothing on that side lightly in pencil. When the final smooth curve has been arrived at, it is traced on the ruled side.

**SPECIFIC RULES FOR DRAWING GRAPHS ON RECTANGULAR-COORDINATE PAPER**: If a graph is to impart its full meaning, it must be constructed in accordance with standard rules so that it will have the same significance to every person who inspects it. Fairly rigid rules for the preparation of graphs have been adopted by representatives of the leading scientific and engineering societies. Some of the rules in present-day use are listed below for the guidance of the student in his presentation of laboratory data.

1. Plot the independent variable along the abscissa scale (*X* axis) and the dependent variable along the ordinate scale (*Y* axis). Draw heavy lines for these axes.

2. Choose a size of graph that bears some relation to the accuracy with which the plotted data are known. In general the curve should fill most of the sheet; if the spread from the lowest value to the highest value is small and the data are reliable to only two significant figures, do not spread out the graph to permit points to be read from the curve of three significant figures. Plot all data in the quadrant that is indicated by the signs of the numerical values. In many cases it is not necessary that the intersection of the two axes represent the zero values of both variables.

3. Choose scales for the main divisions on the graph paper that are easily subdivided. Values 2, 5, and 10 are best, but 4 is sometimes used; never use 3, 7, or 9, since these make it very difficult to read values from the graph. The divisions on the abscissa and ordinate scales need not be alike.

4. If the values are exceptionally large or small, use some multiplying device that permits using a maximum of two digits to indicate the value of the main division. The load may be plotted as "thousands of tons," the time in "microseconds." A multiplying factor such as x $10^{-2}$ or x $10^{-5}$ placed at the right of the largest value on the scale may be used. (See Fig. 6.)

5. In the white space *outside* the scale put on a clear complete label; letter everything on the graph so that it can be read from the lower left-hand corner. It is good form to write all labels in capital letters; some authorities capitalize only the first word and proper names; in either case a consistent style should be chosen. The scale label includes the name of the quantity plotted and its unit, separated by a comma. Abbreviate all units in standard form.

6. Locate each point in its proper place and encircle it. (In printed graphs the dots do not appear.) If several curves appear on the same sheet and the points might interfere, use squares and triangles to surround the dots of the second and third curve, respectively.

7. Draw the best smooth curve through the average of the points; ignore any points that are obviously erratic. Draw the curve up to but not through the circles. Indicate by dashes any extrapolated portions that extend materially past the terminal data points.

8. At an open space near the top of the paper state the title of the graph, that is, write a clear complete legend. Omit any unnecessary words, such as "Curve showing." After the main legend add any specific features needed to identify the number or the nature of the test equipment or conditions.

9. When more than one curve is plotted on the same sheet, differentiate them clearly. If possible label them by (a) and (b) and explain in the sub-legends. If they are differentiated by different symbols, such as circles or squares, or by dotted and dashed lines, make a neat key in a clear space on the graph sheet. Most authorities prefer not to write along the curve.

10. In graphs for technical reports (not for publication) it is good practice to put the initials of the experimenter and the date in the lower right-hand corner.

11. For reports of student experiments select standard quality conventional graph paper of 8 ½ by 11-inch size with convenient rulings, such as 20 per inch or 10 per centimeter; do not use quadrille (cross-ruled) paper.

12. It is often very helpful in smoothing out a curve to hold the paper so that its plane is nearly in the plane of the eye.

Looking at the curve in this way at a glancing angle makes it possible to notice irregularities that might otherwise result in an irregular graph.

**ANALYSIS AND INTERPRETATION OF GRAPHS**: One of the great advantages of graphical analysis is the simplicity with which new information can be obtained directly from the graph by observing its shape, slope, and intercepts.

**Shape**: The shape of a graph indicates immediately whether one variable increases or decreases as the other one increases and it also distinguishes between intervals of slow and rapid variation. In the case of a straight-line graph one can easily determine the form of the variation. Even in this case a careful distinction must be made between *linear variation and direct proportion*. Linear variation means simply a first-degree relationship and is indicated by any straight-line graph. A direct proportion, on the other hand, means that the variables are directly proportional and therefore simultaneously zero, which occurs only for straight-line graphs that *pass through the origin of coordinates.*

**Slope**: In discussing the slope of a graph we must distinguish carefully between physical slope and geometric slope. In the case of a linear graph the *physical slope* may be found by drawing a large triangle, as shown in Fig. 5, and dividing $\Delta y$ by $\Delta$x using for each the scales and units that have been chosen for those axes. The result is independent of the choice of scales and may express a significant fact about the relationship between the plotted variables. For example, the slope of a graph of velocity against time gives the acceleration. The units of the acceleration will appear from the calculation. In a typical case the physical scope $m$ of a graph such as that in Fig. 6 is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4.7 - 0.8)\times 10^2\, cm/\sec}{\frac{18}{40} - \frac{2}{40}\sec}$$

$$= \frac{3.9\times 10^2\, cm/\sec}{\frac{16}{40}\sec} = 9.8\times 10^2\, cm/\sec^2$$

In contrast to the physical slope the geometric slope is defined to be tan 8, where 9 is the angle between the line and the *X*-axis. This slope depends upon the inclination of the line and hence depends upon the choice of scales. In graphical analysis, one is always concerned with the physical rather than the *geometric* slope.
In the case of curved-line graphs the slope will vary from point to point. Its value at any point is defined as the slope of the tangent line at that point.

**Intercepts**: Significant information is sometimes revealed by the intersection of a graph with a coordinate axis. Thus the intercept of a velocity-time graph with the velocity axis gives the value of the velocity when the time was zero, that is, when the experimenter chose to begin counting time. Similarly, the intercept on the time axis gives the value of time when the velocity was zero; a negative intercept on the time axis indicates the length of time that the body was in motion before the experimenter began to count time.

**GRAPHS ON OTHER THAN RECTANGULAR COORDINATE PAPER**: Many variations can be represented more easily by graphs that are drawn on paper not ruled off in rectangular coordinate lines. For example, polar coordinate paper can clearly show how the luminous flux from a lamp source changes from point to point in a vertical plane. A power law is more easily visualized if the data are plotted on logarithmic paper. Some relationships are better shown on semi-logarithmic paper. These forms of graphical analysis are described in more advanced treatments.

**GRAPHICAL EXERCISES**: 1. The following data were obtained by observing a falling body:

| Speed, ft/sec ........... | 2.50 | 5.68 | 9.00 | 12.20 | 15.00 |
|---|---|---|---|---|---|
| Time, sec ................ | 0 | 0.100 | 0.200 | 0.300 | 0.400 |

Plot a curve showing the relationship between speed and time use time as the independent variable. What does the shape of the graph show? Calculate the slope of the graph. When did this body start to fall (when was its speed zero)?

2. The following data were obtained by observing the temperature of a can of oil as it cooled:

| Time, min ........... | 0 | 8 | 16 | 24 | 32 | 40 |
|---|---|---|---|---|---|---|
| Temperature, °F ...... | 200 | 121 | 77 | 52 | 38 | 30 |

Plot a curve showing the relationship of temperature and time. What does the shape of this graph show? Calculate the slope at 8 min and at 28 min. Comment on them.

3. When the molar volume (volume occupied by one molecular weight) of hydrogen gas was measured carefully for two ranges of pressure, the temperature being held constant in all cases at 0°C, the following measurements resulted:

Range 1

| Pressure (p) (atmospheres) | Molar Volume (V) ($cm^3$ per mole) |
|---|---|
| 0.5 | $44.82 \times 10^3$ |
| 1.0 | 22.41 |
| 1.5 | 14.94 |
| 2.0 | 11.21 |
| 2.5 | 8.97 |
| 3.0 | 7.48 |

(a) Study these data and see if you note any systematic relationship between the variables.

(b) Plot graphs of (*V*) vs. (*P*) for both ranges of pressure. Use two sheets of graph paper.

(c) Calculate and tabulate values for the product (*PV*).

(d) How does the product (*PV*) behave in the low range of pressure? What law is distilled from such data?

(e) How does the product (*PV*) behave for the high-pressure range? Does this law hold for low pressures?

(f) Plot the product (*PV*) vs. (*P*) for the high range of pressure. Comment on the significance of this result.

Range 2

| Pressure (P) (atmospheres) | Molar Volume (V) ($cm^3$ per mole) |
|---|---|
| 200 | 127.1 |
| 300 | 90.1 |
| 400 | 71.6 |
| 500 | 60.6 |
| 600 | 53.2 |

**QUESTIONS**:

1. A meter stick is placed alongside a yardstick and the lengths in centimeters of various numbers of inches are observed. What is the significance of the shape and slope of the curve of reading, in centimeters, plotted against length, in inches? What would one expect the value of this slope to be?
2. The heating effect of an electric current in a rheostat is found to vary directly with the square of the current. What type of graph is obtained when the heat is plotted as a function of current? How could the variables be adjusted so that a linear relation would be obtained?
3. The current in a variable resistor to which a given voltage is applied is found to vary inversely with the resistance. What is the shape of the current-resistance curve? How could these variables be changed in order that a straight-line graph could be obtained?
4. A curve is plotted to show the distance traveled, starting from rest, by a marble rolling down an inclined plane, as a function of time. (a) What is the shape of this curve? (b) What is the significance of the physical slope of this curve? (c) This slope at various points is then plotted as a function of time. What would you expect this curve to look like and what is the significance of the slope of the second curve? (d) What sort of curve would you expect to get if the slope at various points of the second curve were plotted against time?
5. A power equation is of the form $y = ax^b$, but the constants *a* and *b* are unknown. Experimental values of y and x are plotted with logo y as ordinates and log *x* as abscissas, and a straight line is obtained. How may the values of *a* and *b* be determined from this graph?