TORSION PENDULUM

OBJECT: To study the relation between the period of a torsion vibration and the factors that determine the period; to measure the rotational inertia of various objects and the coefficients of rigidity of steel and brass.

METHOD: A cylindrical rod about a meter in length is clamped firmly at its upper end and has a body, the rotational inertia of which may be calculated, attached at the lower end. By rotating the body at the lower end and then releasing it the rod may be set in vibration, the period of which depends on the diameter, length and material of the rod as well as the rotational inertia of the suspended body. Knowing the constants of the rod it is possible to measure the rotational inertia of various bodies attached to it.

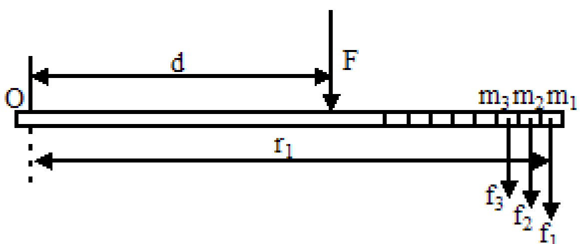

Fig. 1. Rod free to turn about a horizontal axis at O is acted on by a force F.

THEORY: In order to obtain the period of oscillation of a torsion pendulum, it is first necessary to find the relationship between the torque acting on a body and the angular acceleration which the torque produces. Consider along bar acted on by a force *F* at a distance *d* from an axis O about which the bar is free to turn (Fig. 1). The force *F* has a moment $F \times d$ about O and causes the rod to start rotating about the axis with an angular acceleration *a*. Suppose the rod is made up of a large number of masses m_1, m_2 etc. at distances r_1, r_2 etc., respectively, from the axis. The mass m_l has a linear acceleration of ar_1 centimeters per second squared. The force f_1 producing this acceleration is by Newton's second law of motion

$$f_1 = m_1 a r_1 \qquad (1)$$

The force f_1 is produced by the force *F* acting on the bar and is transmitted to m_l by the rigidity of the bar. Similarly $f_2 = m_2 a r_2$, etc. Now the sum of the torques f_1, f_2, etc. about the axis through O must be equal to the torque of *F* about O or

$$F \times d = f_1 r_1 + f_2 r_2 + f_3 r_3 +$$
$$= m_1 a r_1^2 + m_2 a r_2^2 + m_3 a r_3^2 + \ldots$$
$$= a\left(m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots\right)$$
$$= aI \qquad (2)$$

where *I* is called the rotational inertia of the rod about the axis through O. Thus

$$I = \sum mr^2 \qquad (3)$$

The symbol L is often used for the torque $F \times d$. Hence Eq. (2) may be written

$$L = Ia \qquad (4)$$

This relationship between torque and angular acceleration is similar to the relationship *F* = *ma* between force and linear acceleration.

Theorem: If *I* is the rotational inertia of a body about any axis and I_o the rotational inertia of a body about a parallel axis through the center of mass of the body

$$I = I_o + Mh^2$$

where M is the mass of the body and *h* is the distance between the two axes.

In Fig. 2 let *I* be the rotational inertia of the body about an axis through A perpendicular to the plane of the figure, and

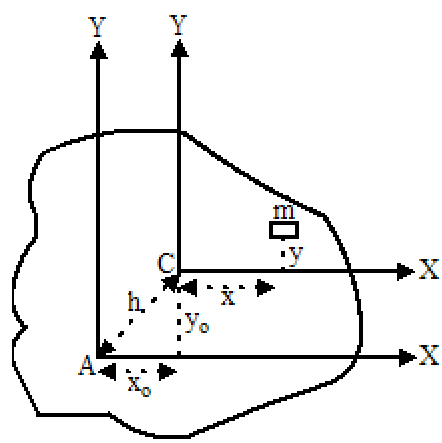

Fig. 2. Diagram illustrating method for calculating rotational inertia of object about axis through A in terms of rotational inertia through center of mass C.

I_o that about a parallel axis through the center of mass C. The coordinates of C with reference to the origin at A are x_o and y_o

$$AC = h = \sqrt{x_o^{\,2} + y_o^{\,2}} \tag{5}$$

Let *m* be an element of mass at a distance of x and y, respectively, from C and hence at a distance of ($x + x_o$) and ($y + y_o$), respectively, from A

$$\begin{aligned} I &= \sum m\left\{(x + x_o)^2 + (y + y_o)^2\right\} \\ &= \sum m\left\{\left(x^2 + y^2\right) + \left(x_o^{\,2} + y_o^{\,2}\right) + 2xx_o + 2yy_o\right\} \end{aligned} \tag{6}$$

In this case x_o and y_o are constants and *x* and *y* distances measured from the center of mass of the body. From the definition of center of mass it follows that $\sum mx = 0$ and $\sum my = 0$. (Note that both *x* and *y* may assume negative as well as positive values.) Hence

$$I = \sum m\left(x^2 + y^2\right) + \sum m\left(x_o^{\,2} + y_o^{\,2}\right) \tag{7}$$

$$I = I_o + Mh^2 \tag{8}$$

Rotational Inertia of Various Objects: 1. Circular ring of negligible thickness: Consider a ring of mass *M* and a mean radius *r*. If the thickness of the ring is negligibly small, all the elements of mass are at the same distance from the center. Hence the moment of inertia of such a ring about an axis through its center and perpendicular to the plane of the ring is

$$I = Mr^2 \tag{9}$$

2. The rotational inertia of a circular disk of radius a and mass M about an axis through its center and perpendicular to its plane is given by

$$I = \frac{Ma^2}{2} \tag{10}$$

3. The rotational inertia of a right circular cylinder of radius *a* and mass *M* about an axis through its center and perpendicular to its base is given by

$$I = \frac{Ma^2}{2} \tag{11}$$

4. The rotational inertia of a ring of mass *M* and having inner and outer radii *a* and *b*, respectively, about an axis through its center and perpendicular to its plane is given by

$$I = \frac{M(a^2 + b^2)}{2} \tag{12}$$

Period of Vibration of a Torsion Pendulum: Consider a long uniform rod rigidly clamped at its upper end A and supporting at its lower end a disk, whose rotational inertia about an axis through the rod is 1 (Fig. 3). If L_o is the torsion constant of the rod then L_o is the torque required to twist the lower end of the rod through unit angle (1 radian). If the disk is rotated through an angle θ and then released, the restoring torque is $L_o\theta$. This torque causes the disk to swing back to its undisturbed position with an angular acceleration. If a is the angular acceleration of the disk when the displacement is θ, then from Eq. (4)

$$L_o\theta = -Ia$$

or

$$a = -\frac{L_o\theta}{I} \tag{13}$$

(The negative sign is used because α is positive in the direction of increasing θ which is opposite to the direction of the restoring torque $L_o\theta$.) Thus the angular acceleration for an angular displacement θ is proportional to θ and in the opposite direction. This is the condition required for the disk to execute simple harmonic motion (S. H. M.).

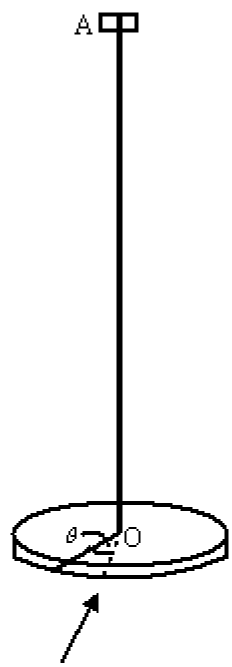

Fig. 3. Disc attached to cylindrical rod rotated through angle and then released thus setting the system in simple harmonic motion.

In the case of linear displacements a body executes simple harmonic motion whenever the acceleration due to a displacement *x* is proportional to and in the opposite direction to the displacement *x*. For the simple harmonic motion of a spring the acceleration a for a displacement *x* is given by

$$a = \frac{-Cx}{M} \tag{14}$$

where C is the force required for unit extension of the spring and M is the mass of the vibrating body. The period T of the simple harmonic vibration of the spring may be shown to be

$$T = 2\pi\sqrt{\frac{M}{C}} \qquad (15)$$

Thus for the angular displacements and accelerations given by Eq. (13) the period of the simple harmonic vibration set up is

$$T = 2\pi\sqrt{\frac{I}{L_o}} \qquad (16)$$

APPARATUS: The apparatus consists essentially of three steel rods and one brass rod together with a circular disk, a ring and two cylindrical inertia masses. A wall bracket is needed for clamping the rods in a vertical position. A timing device such as a stopwatch, a pair of micrometer calipers and a meter stick are also required. A telescope with across hair in the eyepiece is useful as an aid in observing the period of the vibrations.

PROCEDURE:

Required Experimental:

Part A. Firmly clamp the steel rod of smallest diameter (5/64") in the wall bracket and attach the disk firmly to the lower end so that the index line on the edge of the disk is toward the front (Fig. 4). This mark serves as an index for timing. If a telescope is available set it at a suitable distance from the apparatus and focus it on the index line; otherwise set a vertical pointer near but not touching the index line. By these means the central or undisturbed position of the disk is indicated.

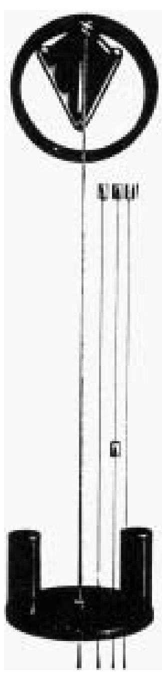

Fig. 4. Torsion Pendulum with disk, cylindrical masses, ring and various rods.

Rotate the disk through some small angle and set the rod and disk in torsional vibration. Determine the time of fifty complete vibrations. Start timing when the disk passes the central position moving, say, to the right. Be careful to call the first count, when the timing starts, "zero." It is a common error to call this "one." Repeat the observation until consistent results are obtained for the period of oscillation.

Part B. Place the ring concentrically on the disk and measure the period of oscillation.

Part C. Remove the ring and place the two inertia cylinders on the disk within the white circles at the end of a diameter marked on the disk. The edges of the cylinders should just reach to the edge of the disk. Measure the period of oscillation.

Measure in centimeters the diameters of the disk and cylinders and also the inner and outer diameters of the ring. Measure the diameter of the rod in at least four places by means of a micrometer caliper calibrated in millimeters. Measure in centimeters the length of that part of the rod between the holders- that is, the part actually producing the torsional vibrations. The masses of the disk, the ring and the cylinders may be supplied by the instructor. If not they should be determined by means of a suitable balance.

Optional Experimental: In the following experiments only the disk is used:

Part D. Remove the rod from the wall bracket and replace it by the steel rod of next larger diameter (7/64"). Attach the disk to the lower end of the rod and measure the period of oscillation.

Part E. Clamp this rod at its center holder and with the disk attached at the lower end measure the period of oscillation.

Part F. Replace this rod with the steel rod of the largest diameter (10/64") and with the disk attached at the lower end measure the period of oscillation.

Part G. Replace the steel rod with the brass rod and with the disk attached at the lower end measure the period of oscillation.

Measure the diameter of each of the rods in at least four places.

Required Calculations:

1. Calculate the rotational inertias of the disk, of the ring and of the two cylindrical masses using Eqs. (10), (11) and (12). Using Eq. (8) calculate the rotational inertias of the cylinders about an axis through the center of the rod and disk.
2. From Eq. (16), together with the data of Part A and the rotational inertia of the disk, calculate the torsion constant L_o for the steel rod.
3. Using the value of L_o found in the previous calculation and the data of **Part B**, calculate the rotational inertia of the ring and disk together. Compare this value with the sum of the rotational inertias of the disk and ring as computed above in 1. State the percentage difference in the two values obtained.
4. Using the value of L_o and the data of **Part C** calculate the rotational inertia of the combination of disk and two

cylindrical masses. Compare this value with the sum of the computed ones. State the percentage difference in the two values obtained.

Optional Calculations:

5. From the data of **Parts D, E** and **F** calculate L_o for each of the steel rods. Similarly calculate L_o for the brass rod using the data of **Part G**.

6. Using the value of L_o for anyone of the rods of steel, calculate the coefficient of torsional modulus *n* using the equation

$$n = \frac{2lL_o}{\pi r^4}$$

where *l* is the length of the rod in centimeters and *r* its radius in centimeters. This equation is derived in the experiment on modulus of rigidity.
Express the result to the proper number of significant figures multiplied by 10 raised to the proper power.

7. Similarly calculate the torsional modulus *n* for the brass rod.

8. Compare the values of L_o found in this experiment with those found in the experiment on modulus of rigidity in which the same rods are used.

QUESTIONS: 1. If an error of one per cent is made in the determination of the period, what percentage error does this make in the determination of L_o; of *n*?

2. If an error of one percent is made in the determination of the diameter of the torsion rods, what error would this make in the determination of *n*?

3. Does the period of the vibrations depend on the amplitude of the vibrations?

4. Show that both sides of Eq. (16) have the same dimensions.