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TORSION PENDULUM

OBJECT: To determine the rotational inertia of a cylindrical disk by means of a torsion pendulum.

METHOD: A uniform wire about a meter in length is firmly clamped at its upper end and has attached at the lower end a circular disk. By rotating the disk about an axis which runs lengthwise through the wire and then releasing the disk, the system is set into angular simple harmonic motion, the period of which depends on the material and dimensions of the wire as well as the rotational inertia of the disk. A cylindrical ring is then placed symmetrically over the disk and the system set into angular simple harmonic motion. By measuring the period and using the computed value of the rotational inertia of the ring, the rotational inertia of the disk is determined. This value is compared with the value calculated for the rotational inertia of the disk.

THEORY: Consider along uniform wire rigidly clamped at its upper end A and supporting, at its lower end, a circular disk whose rotational inertia about an axis through the wire is *I* (Fig. 1). If the disk is rotated through some angle, a restoring torque is set up in the wire tending to bring the disk back to its equilibrium position. When the disk is released, this torque gives the disk an angular acceleration. If a torque *L* acts on a body having a rotational inertia *I*, it gives the body an angular acceleration of a where

$$L = Ia \qquad (1)$$

Eq. (1) indicates that the angular acceleration which a body receives is directly proportional to the torque acting on it and inversely proportional to its rotational inertia. This expression relating torque and angular acceleration is analogous to Newton's second law of motion, *f* = *ma*, which deals with force and linear acceleration.

The restoring torque set up in the wire when its lower end is twisted through some angle θ (Fig. 1) depends upon the torsion constant of the wire as well as the angle of twist. The torsion constant L_o depends upon the dimensions of the wire and upon the material of which it is made, i.e., upon the shear modulus of the material; this torsion constant L_o of the wire is defined as the torque required to twist the lower end of the wire through unit angle (one radian). Consider the motion of the disk at some instant when the angle of twist is II radians and the restoring torque is $L_o\theta$. The relationship between the restoring torque $L_o\theta$ and the angular acceleration α of the disk at this instant is, from Eq. (1),

$$L_o\theta = -Ia$$

or

$$a = -L_o\theta / I \qquad (2)$$

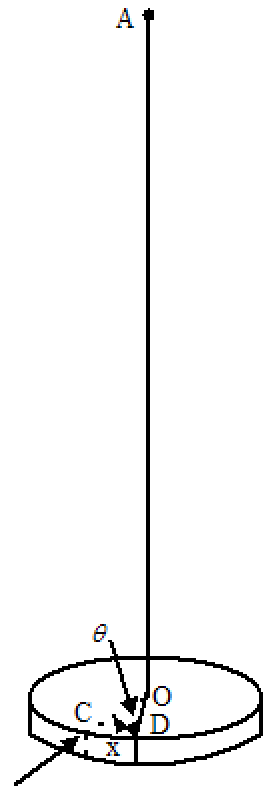

Fig. 1. Disk hanging on end of wire which is twisted through angle θ.

where *I* is the rotational inertia of the disk about an axis passing through the wire. The negative sign is used because *a* is positive in the direction of increasing θ, which is opposite to the direction of the restoring torque $L_o\theta$. Since L_o and *I* are constant for a given apparatus, it follows that the angular acceleration for an angular displacement θ is proportional to θ and in the opposite direction. This is the condition necessary for the disk to execute angular simple harmonic motion.

Simple harmonic motion (S.H.M.) is defined as motion in which the acceleration is directly proportional to the displacement and oppositely directed. Clearly this definition

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may be applied to either translational or rotational motion. A S.H.M. is characterized by its amplitude and either its frequency or period. Amplitude is the maximum displacement of the body executing S.H.M. from its position of rest. Frequency n is the number of complete vibrations per second, while the period T is the time for one complete vibration. From these definitions of frequency and period it follows that

$$n = 1/T \tag{3}$$

It is to be noted that the period is the time for one complete vibration and is therefore the interval between two successive transits in the same direction through any reference point.

There exists a significant relationship between S.H.M. and uniform circular motion from which the period of the S.H.M. may be obtained. When the motion of a point traveling with uniform speed in a circle is projected upon a diameter of the circle, the motion of the projection may be shown to satisfy the definition of S.H.M. Suppose a particle, shown at A in Fig. 2, is moving with uniform speed v) in a circular path of

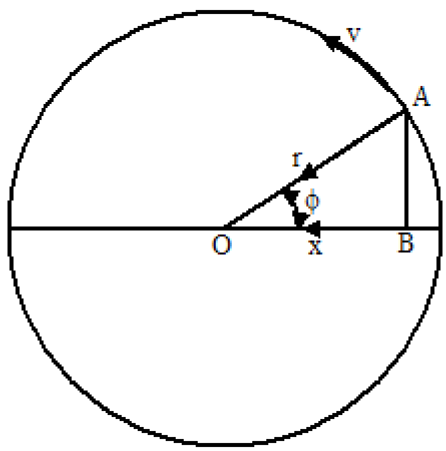

Fig. 2. The projection on a diameter of a point moving with uniform speed in a circle moves with linear simple harmonic motion.

radius r. The period of rotation T of A is $2\pi r/v$. As A revolves about the center O, the projection B on the diameter of the circle oscillates to and fro with the same period. The amplitude of the vibration is the radius r of the circle. The particle A has a centripetal acceleration given by

$$a_c = v^2/r = 4\pi^2 r/T^2 \tag{4}$$

The component of this acceleration upon the line OB is

$$a = -a_c \cos\phi = -a_c x/r = -4\pi^2 x/T^2 \tag{5}$$

The negative sign indicates that the acceleration a is oppositely directed to the displacement x which is measured outward from the center O. Since T is a constant, it follows that the expression given in Eq. (3) satisfies the definition for S.H.M. The period of the S.H.M. is given by

$$T = 2\pi\sqrt{-x/a} \tag{6}$$

where x is the linear displacement from the equilibrium position and a is the linear acceleration of the particle for this displacement.

This expression may be readily transformed into the corresponding expression for the period of an angular simple harmonic motion.

Consider the point Don the periphery of the disk (Fig. 1). This point vibrates in the arc of a circle of radius r, the radius of the disk. When the angular displacement of D from its equilibrium position C is θ, the displacement x of D from C is $r\theta$ and the linear acceleration a of D is r_a where a is the angular acceleration of the point D. Substituting these values for x and a in Eq. (6), it follows that the period of the angular simple harmonic motion is given by

$$T = 2\pi\sqrt{-x/a} = 2\pi\sqrt{-r\theta/r_a} = 2\pi\sqrt{-\theta/a} \tag{7}$$

where θ is the angular displacement (in radians) from the equilibrium position and a is the angular acceleration (in radians per sec^2) for this displacement. From Eq. (2)

$$-\theta/a = I/L_o \tag{2a}$$

Hence the period of the S.H.M. is given by

$$T = 2\pi\sqrt{I/L_o} \tag{8}$$

If a circular ring is placed symmetrically over the disk and the system set into S.H.M., the period of the vibrations is given by an expression similar to Eq. (8) where I represents the total rotational inertia of disk and ring about an axis through the wire.

The rotational inertia of a circular disk of mass M and radius r about an axis through its center and perpendicular to its plane (Fig. 3a) is given by

$$I = \tfrac{1}{2}Mr^2 \tag{9}$$

The rotational inertia of a ring of mass M whose inner and outer radii are respectively r_1 and r_2 about an axis through its center and perpendicular to its plane (Fig. 3b) is given by

$$I = \tfrac{1}{2}M\left(r_1^2 + r_2^2\right) \tag{10}$$

APPARATUS: The apparatus consists of the stand used in the experiment on Young's modulus, a circular brass disk with an index line on its side, a brass ring and a long wire (Fig. 4). A timing device such as a stopwatch, a pair of calipers and a balance for determining the mass of the disk and ring are required. A telescope with across hair in the eyepiece is useful for observing the period of the vibrations.

PROCEDURE: Firmly fasten the wire in the clamp at the top of the stand and attach the other end of the wire to the small clamp which has screw threads on it. Securely screw the lower clamp into the disk and arrange the index line on the edge of the disk in a convenient position for making observations.

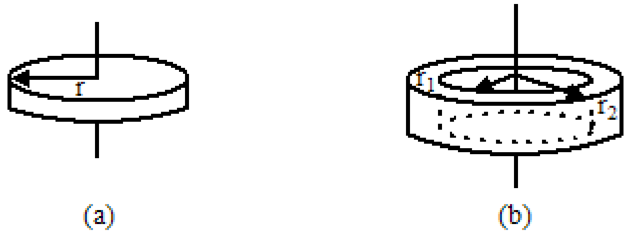

Fig. 3. (a) Solid disk, (b) Ring having rectangular cross section, showing axes through centers of mass and perpendicular to faces.

This may be done by loosening the upper clamp and suitably turning the wire. If a telescope is available, set it at a suitable distance from the apparatus and focus it on the index line; otherwise set a vertical pointer near, but not touching, the index line. By these means the central or undisturbed position of the disk is indicated.

Fig. 4. Apparatus for determining the torsion constant of a wire and the rotational inertia of a ring. The ring and disk are shown mounted on the apparatus which is conventionally used for the determination of Young's modulus.

Rotate the disk through some small angle and set the wire and disk in torsional vibration. Determine the time of fifty complete vibrations. Start timing when the disk passes the central position moving, say, to the right. Be careful to call the first count, when the timing starts, "zero." It is a common error to call this "one." Repeat the observation until consistent results are obtained for the period of oscillation.

Place the ring concentrically over the disk. Set the system in motion as above and determine the period of the torsional oscillations. Detach the ring and disk from the wire and measure the diameter of the disk and the inner and outer diameters of the ring. Determine the masses of the disk and ring.

Compute the rotational inertia of the ring using Eq. (10). From the measured periods of oscillation of the disk alone and of the ring and disk, together with the computed value of the rotational inertia of the ring, calculate the torsion constant L_o of the wire and the rotational inertia of the disk. Compute the rotational inertia of the disk using Eq. (9) and give the percentage difference of the two values obtained for this quantity.

QUESTIONS: 1. On which of the following factors does the torsion constant L_o depend, and in what ways: (a) the diameter of the wire; (b) its length; (c) the material of which the wire is made; (d) the modulus of rigidity of the wire?

2. Why is the rotational inertia of the ring and disk as used together in this experiment equal to the sum of their separate rotational inertias?

3. Does the period of vibration depend on the amplitude? Discuss for small and very large amplitudes.

4. In observing the time required for a rather slowly oscillating torsion pendulum to make a number of oscillations, would it be more accurate to record times when the disk passed through its equilibrium position or when it passed through one of its extreme positions? Explain.