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TORQUE: THE MODEL BALANCE

OBJECT: To study the concept of torque and the condition for a body to be in rotational equilibrium; to find the center of gravity of a body.

METHOD: The "unknown" weights of a pair of scale pans on a model balance, Fig. 3, are determined by the use of the "law of moments" (the second condition for equilibrium) from the measured lever arms of the balance and a known weight. The center of gravity of the balance-arm system is located by a method of balancing. Its weight is then determined by supporting the system from some place other than the center of gravity, restoring equilibrium by the use of known weights, and writing the equation for the torques acting upon the system. A brief study of the method of "double weighing," which eliminates certain balance errors, is made by interchanging the positions of the known and unknown weights and the proper adjustment of the former.

THEORY: It is well known that a given force does not always produce the same effect of rotation on an object, as for example, when the force is applied at different places or in different directions. This turning effect is known as *torque or moment of force*.

Torque, or moment of force, is that which produces or tends to produce a change in the rotary velocity of a body. Its measure is given by the *product of the acting force and its lever arm*. Lever arm is the *perpendicular* distance from the axis of rotation to the *line of action* of the force.

The defining equation for torque is

$$L = fr \qquad (1)$$

where L is the torque developed by the force f which acts through a lever arm r.

The absolute metric unit of torque is the centimeter-dyne. (Note the inversion of this manner of writing these units, as compared with the unit of *work*, the dyne centimeter.) It is probably more common to use the gravitational units of torque, namely the centimeter-gram in the metric system and the pound-foot in the British system.

As an illustration of the concept of torque, Fig. 1 illustrates a weight W being lifted by a crowbar with a force which is supposed to be of constant magnitude, but acting at different places or in different directions. From the definition of torque it is apparent that F_1 will produce the maximum torque, F_2 about half as much, and F_3 and F_4 no torque at all. The torque produced by F_5 is given by the product of F_5 by the distance OB, Fig. 1(b). Note that the lever arm of a force is not always measured along the body. The importance of obtaining the correct lever arm of a force in any given problem cannot be too strongly emphasized, as experience has shown that this is the chief cause of most of the difficulty which students in elementary physics have when working with torques.

Equilibrium of a Body: A body is in equilibrium when it has no linear or rotary acceleration, i.e., both its linear and rotary velocities are constant. The conditions for the equilibrium of a rigid body are: (1) the vector sum of all the forces acting on the body must be zero; (2) the algebraic sum of the torques of all the coplanar forces about any axis must be zero. Condition (2) is sometimes known as the principle (or law) of moments." The algebraic sign of a torque is perfectly arbitrary, but for convenience the usual convention is to regard torques which tend to produce counter-clockwise rotation as positive, and those tending to produce clockwise rotation as negative. Thus if this condition for equilibrium is applied in the above example, Fig. 1, the algebraic sum of all the torques about the point O can be placed equal to zero,

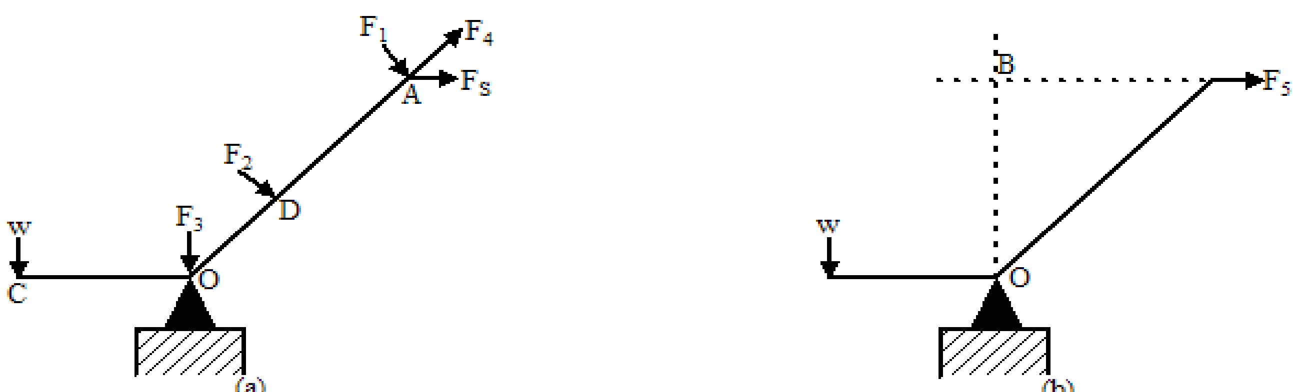

Fig. 1. Unequal torques due to the same force acting at different places or in different directions.

Fig. 2. Illustrating the method of "double weighing."

whence

$$W \times \overline{CO} - F_1 \times \overline{AO} - F_2 \times \overline{DO} - F_5 \times \overline{BO} - F_3 \times 0 - F_4 \times 0 = 0 \qquad (2)$$

It is important to note that, while in this example the equation for the moments of force about the *actual* axis of rotation was written, this is not the only point for which a similar equation might have been written. If a body is in equilibrium, the algebraic sum of the torques about *any axis whatever* is equal to zero. In most cases it is convenient to choose as the point of reference the actual axis of rotation; this will reduce to zero the torque due to the thrust of the pivot, for usually this force is unknown.

Center of Gravity: In the preceding example, any consideration of the effect of the weight of the crowbar itself has been omitted. This effect mayor may not be negligible, depending on the relative magnitude of the forces involved. It is convenient, in many cases, to treat the entire weight of a body as if it were concentrated at a single point, called the center of gravity.
The center of gravity of a body may be defined as that point at which all the weight of the body acts as if it were concentrated, in so far as the force action due to the weight is concerned.
For uniform bodies, the center of gravity can be found from geometrical considerations. For non-uniform substances, other methods must be used, one of which will be illustrated in this experiment.
The solution of most problems is simplified if the *entire* weight of the body is assumed to be concentrated at the center of gravity, rather than by attempting to divide the total weight into proportional parts on the sides of the axis of rotation.

Method of Double Weighing: The errors of the common balance due to unequal lever arms of the scale pans may be eliminated by a "double weighing" method. This consists of weighing the object in the usual manner and then interchanging the positions of the known and unknown weights. Let the weight of the unknown body be designated by X, that of the standard masses in the first weighing by W_1 and the weight of the known masses in the second weighing be called W_2. Now although neither the arms of the balance nor the weights of the pans are equal, the torques exactly balance each other in all weighings and hence they may be ignored in all cases. Designate by l_1 the length of the left-hand lever arm and by l_2 the length of the right-hand lever arm. The conditions for the two weighings are illustrated in Fig. 2. Equating the torques in the two cases gives

$$Xl_1 = W_1 l_2 \qquad (3)$$

$$W_2 l_2 = X l_2 \qquad (4)$$

Dividing corresponding sides of Eqs. (3) and (4)

$$\frac{Xl_1}{W_2 l_1} = \frac{W_1 l_2}{X l_2} \qquad (5)$$

From which

$$X^2 = W_1 W_2 \text{ or } X = \sqrt{W_1 W_2} \qquad (6)$$

Thus it is seen that the unknown weight is obtained in terms of the two standard weights and nothing else, so that the major balance errors are completely eliminated.

APPARATUS: The principal piece of apparatus (Fig. 3) is called a "model balance." It consists of a meter stick which can be slipped through a metal frame containing two knife-edges, one fixed and one vertically adjustable.

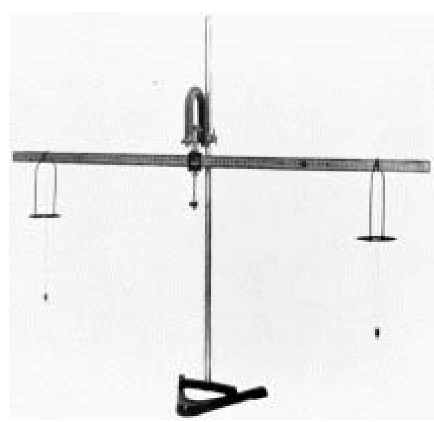
Fig. 3. The Model Balance

The meter stick has another knife-edge, permanently inserted at the 75cm point, for the determination of the weight of the stick. Scale pans, provided with plumb bobs, furnish a method for applying known forces and measuring their lever arms. A sliding collar is provided in the metal

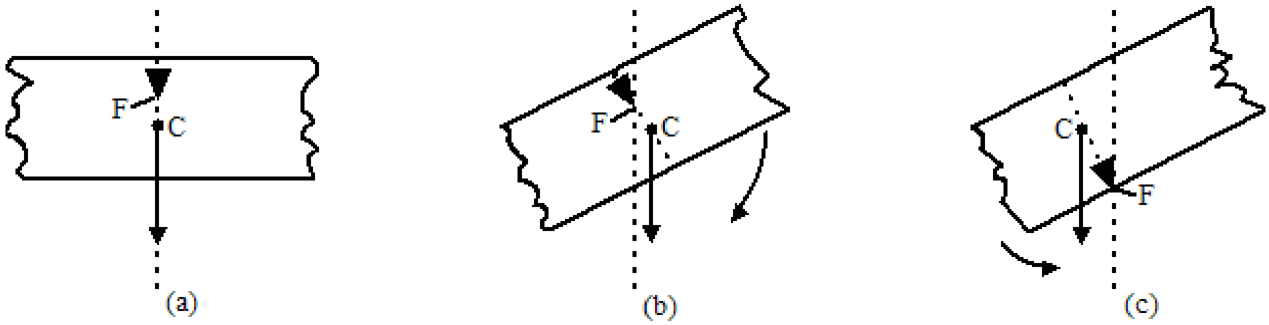

Fig. 4. Showing how the direction of rotation indicates whether the center of gravity is above or below the axis of rotation.

frame so that the center of gravity of the meter-stick system may be raised or lowered.

As auxiliary apparatus there are needed a vernier caliper, an "unknown" mass (about 200g), a set of weights (1 - 500g), a beam balance and a horizontally-mounted meter stick.

PROCEDURE: 1. As an application of the torque concept, the weights of the scale pans will be determined without actually weighing them on scales.

Arrange the metal frame on the meter stick so that the knife-edges are about 2cm apart and the sliding collar is near the bottom.

Support the beam (without the pans) from the upper knife-edge. Adjust the bar, by slipping it through the frame, until it balances in a horizontal position. Hang the pans from the bar, well out toward the ends, and slide one of them along until the beam is again balanced. It is unnecessary here to attempt to adjust the position of the pans to much less than ½mm. (Why?) Record the position of the fulcrum and of each scale pan. It is well in this and future cases in this experiment to make a simple sketch in the data form to show the various values observed. Using suitable symbols to designate the unknown weights of the pans, write down the equation of torques. Add 50g to one of the pans and, without altering the other, slide the weighted pan toward the fulcrum until balance is restored. Record values, and write the equation of torques for this setting. Solve the two equations for the two unknown weights. Check by weighing the pans on the trip scales and note the percentage difference between the values obtained by weighing and those got ten by applying the method of torques.

2. The center of gravity of the beam will next be located. The manipulation here is based on the fact that, if a body be supported at its center of gravity, it will be in equilibrium if there are no forces acting on it other than that due to its weight. Remove the pans and support the bar from the *lower* knife-edge, which should be about 3cm lower than the upper knife edge. Balance the beam in a horizontal position. Final delicate adjustment can sometimes be made by rotating the collar around a vertical axis. It is now evident from Fig. 4 (a) that the center of gravity C of the beam system is in the same vertical line as the fulcrum F. Now push one side of the stick downward. If the beam then tends to return to the horizontal, a little study of Fig. 4 (b) will show that the center of gravity is below the fulcrum. On the other hand, it is seen from Fig. 4 (c) that if the beam tends to turn still further downward, the center of gravity must be *above* the fulcrum.

By vertically adjusting the small sliding collar on the lower part of the frame, the center of gravity can be made, with repeated trials, to *coincide* with the fulcrum. When this is done, the beam will show little tendency to move out of any position in which it is placed; it is said to be in neutral equilibrium.

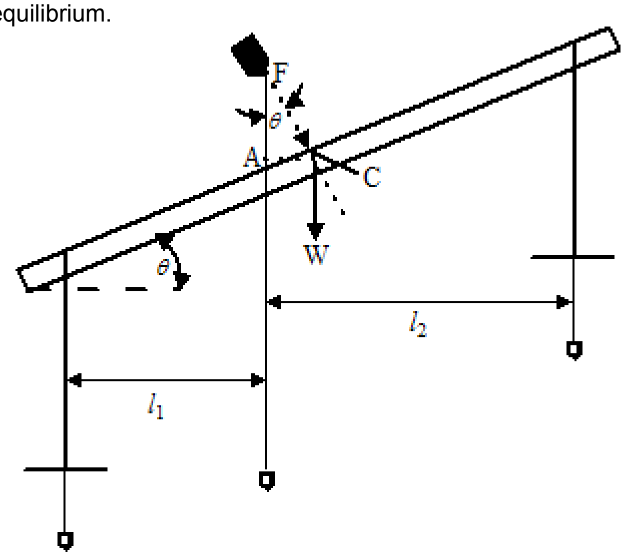

Fig. 5. Obtaining the weight of the meter-stick system by the principle of moments.

3. The weight of the meter-stick system may be obtained by the use of the principle of moments as follows: With the knife edges exactly as in part 2, support the system from the upper knife edge, place the pans in the notches near the ends of the bar and so adjust the weights in the pans that the meter stick makes an angle of about 20° to 30° with the horizontal. From Fig. 5 it is clear that the weight *W* of the meter-stick system causes a torque about F equal to *W x AC*. By using the weights of the pans, measuring the respective lever arms and writing the equation of moments about F, the unknown weight *W* may be obtained. Since the distance *AC* is small, it may best be obtained in terms of *CF* and the angle θ. (A little consideration will show why θ is the same as the angle the stick makes with the horizontal.) The angle θ may be measured either with a protractor or in terms of distances measured with the auxiliary meter stick. A vernier caliper should be used to measure *CF*, after the stick is removed from its stand.

4. A check on the weight thus obtained may be made by

supporting the bar upon the knife-edge at the 75cm mark and producing a balance with the beam horizontal by applying a suitable load near the end of the beam, as in Fig. 6. Leave the iron frame on the meter stick exactly as used in part 3. Write the equation of torques and determine the weight of the bar and frame. Check by weighing and note the percentage difference.

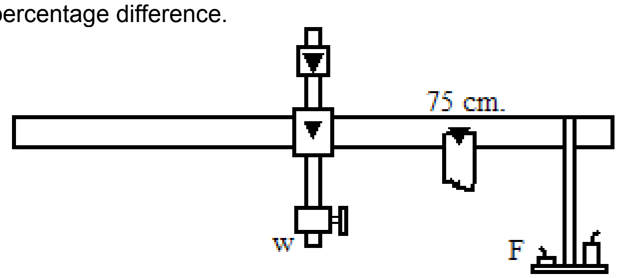

Fig. 6. "Weighing" the meter-stick system by the law of moments.

5. *Method of Double Weighing*: Hang pans of materially different weights in the notches near the ends of the bar. Slip the bar through the frame until a balance is obtained. This gives a model balance whose scale pans are unequal in weight and whose lever arms are also unequal. However their respective torques exactly balance each other, and hence are ignored in the following computations.
Place an object of unknown weight on one of the pans, and balance it by placing a known weight on the other. Write the equation of torques, calling the lever arms assumed unknown) l_1 and l_2. Now place the unknown weight on the other scale pan, and balance again with a known weight. Write another equation of torques, hand l_1 and l_2 remaining the same as before (but unknown). The solution of these two equations will furnish a value for the unknown weight in terms of the two known weights and nothing else, as given in Eq. (6).

QUESTIONS: 1. Show how the center of gravity of a thin, uniform sheet of metal might be located by using only a plumb bob.

2. In order to assist a horse to pull a wagon out of a rut, at what place on the wheel could a force be applied most effectively? Why?

3. Discuss the relative accuracy which should be obtained in determining the weight of the meter-stick system in parts 3 and 4 of this experiment.

4. Where is the center of gravity of a doughnut? Explain reasoning.

5. The center of gravity of a 50g meter stick is located at 51.0cm and the stick is supported at 70.0cm. Where must an 80g weight be hung to secure equilibrium?

6. Referring to Fig. 5 for the meaning of the symbols, the following data were obtained in part 3 of atypical experiment performed as in the above. The angle θ was 30°, l_1 was 34.3cm, l_2 was 35cm, *CF* was 4cm, the pan at the left weighed 60g and the one at the right weighed 50g. Calculate the weight of the meter-stick system from these data.

7. Explain how the position of the center of gravity of a balance arm above or below the knife-edge affects the sensitivity of the balance.