CENCO Physics Selective Experiments In Physics
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THE LADDER

OBJECT: To study the equilibrium of a body acted upon by non-concurrent forces in a plane as illustrated by a ladder resting against a smooth wall.

METHOD: A model ladder carrying a load is placed in a rectangular frame. By means of spiral springs, vertical and horizontal forces are applied to it until it is "freed" or isolated from its constraints. These forces are non-concurrent- that is, they do not act along lines passing through the same point. From the magnitude and location of the forces a test is made for equilibrium.

THEORY: For the static equilibrium of a rigid body under the action of forces in a plane, two conditions must be satisfied. First, the vector sum of all of the forces acting on the body must be equal to zero. From this it follows that if each of the forces is resolved into its rectangular components, the sum of the X-components of all the forces must be zero and the sum of the Y-components of all the forces must likewise be zero. Expressed mathematically,

$$\sum F_x = 0, \qquad \sum F_y = 0 \tag{1}$$

Secondly, the sum of the moments of all the forces about any axis perpendicular to the plane of the forces must be zero

$$\sum F \times l = 0 \tag{2}$$

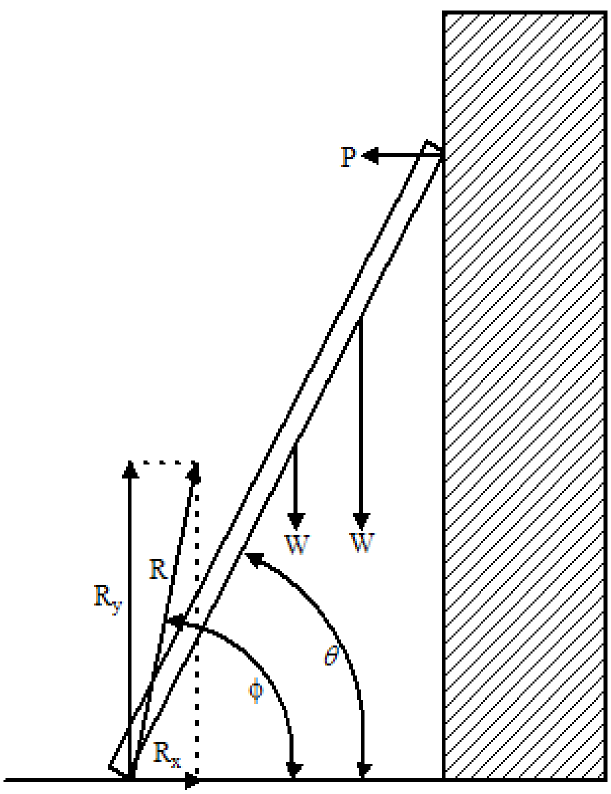

Fig. 1. Ladder resting on "rough" ground and leaning against a "smooth" wall.

In the case of a ladder resting on rough ground and leaning against a smooth vertical wall (Fig. 1), four forces act upon it, namely: the weight of the ladder *w*, the load *W*, the force at the wall *P*, and the reaction at the ground *R*. Since there is no friction between the ladder and the wall- that is, the wall is *smooth*- the force at the wall is normal to its surface- in this case horizontal. On the other hand, there is friction between the ladder and the ground- that is, the ground is rough- and the reaction *R* has both vertical and horizontal components, R_y and R_x.

To simulate conditions of the ladder problem on a laboratory scale, the model ladder is used. In the model ladder it is not convenient to apply the forces at the actual points of contact A and B (Fig. 2). Forces corresponding to *P*, R_x and R_y are applied at the points E and D. When the ladder is in equilibrium in the position shown in Fig. 2, the following conditions obtain among the forces:

$$\sum F_y = 0, \quad \text{i.e.,} \quad R_x - P = 0 \tag{3}$$

$$\sum F_y = 0, \quad \text{i.e.,} \quad R_y - w - W = 0 \tag{4}$$

and taking moments about the point D,

$$\sum Fl = 0, i.e., w \times DC \cos\theta + W \times DH \cos\theta - P \times DE \sin\theta = 0 \tag{5}$$

The magnitude of *R* is given by

$$R^2 = R_x^{\ 2} + R_y^{\ 2} \tag{6}$$

and its direction by

$$\tan\phi = R_y / R_x \tag{7}$$

where ϕ is the angle the resultant *R* makes with the horizontal.

APPARATUS: The apparatus consists of a rectangular frame built of tripod bases (or table clamps), rods and clamps in which a model ladder is supported. The model ladder, the center of gravity of which is marked, has safety clips at either end and rungs for supporting the load. Three spring dynamometers, two turnbuckles, two special clamps for holding horizontal wires and a hook clamp for the vertical wire, and a supply of wire are required in assembling the apparatus.
The load *W* consists of a weight hanger and slotted weights. A trip scale and weights are used to weigh the ladder. The extension of the springs is measured with a vernier caliper.

PROCEDURE:
Experimental. Tie small tags to the three dynamometer springs, designating them as No. 1, No. 2 and No. 3. Measure the length S_1, S_2, S_3 of each spring over the tight portion of the spiral when unloaded. Suspend them in turn from the collar L (Fig. 2) and measure their lengths, S_1', S_2', S_3', when supporting a load of three kilograms. From the load and stretch, calculate the constants k_1, k_2, k_3 of the dynamometers, expressing the result in grams weight per centimeter stretch.
Weigh the ladder and attach it to the bottom and right hand supports by means of the safety clips. Fasten a wire to dynamometer No. 1 and attach it near the bottom of the ladder, adjusting the length of the wire so that the angle which the ladder makes with the vertical is between thirty and forty-five degrees. Use dynamometer No. 2 to apply a horizontal force near the other end of the ladder, and dynamometer No. 3 to apply a vertical force at the same point of attachment as No. 1. Attach a total load of five kilograms at some point on the ladder. With the turnbuckles adjust the forces in springs No. 2 and No. 3 until the ladder is "free," i.e., until the forces between the ladder and support rods at A and B (Fig. 2) are zero. The three wires supporting the ladder should be horizontal and vertical as shown in the figure.
Determine sine and cosine of the angle θ from measurement of the distances *BK* and *AK*. Measure with special care the arm, *DC, DH, DE*. Measure with the calipers the length of the dynamometer springs and, using the constants of the springs, calculate the forces exerted.

Interpretation of Data: From the known values of *W, w* and θ, calculate the values of *P*, R_x, R_y using Eqs. (3), (4) and (5), and compare with results from dynamometers. Make a sketch of the apparatus to scale, indicating the directions of the observed forces. Using the values obtained above for the measured forces, determine the sum of the moments around a point chosen at random.
Calculate the magnitude and direction of the resultant *R* of R_x and R_y and indicate it on the sketch.

QUESTIONS: 1. Will the force against the wall change with the position of the load? If so, in what way?
2. If the ladder rested on rough ground, would the force of friction of the ground (a force parallel to R_x) change with the position of the load *W*?
3. Assuming no load on the ladder (three forces only), make a sketch showing the directions of the three forces.
4. A ladder 20 feet long and weighing 90 pounds rests against a smooth wall at an angle of 30 degrees. Find the force which it exerts on the wall and the horizontal and vertical components of the reaction of the ground.

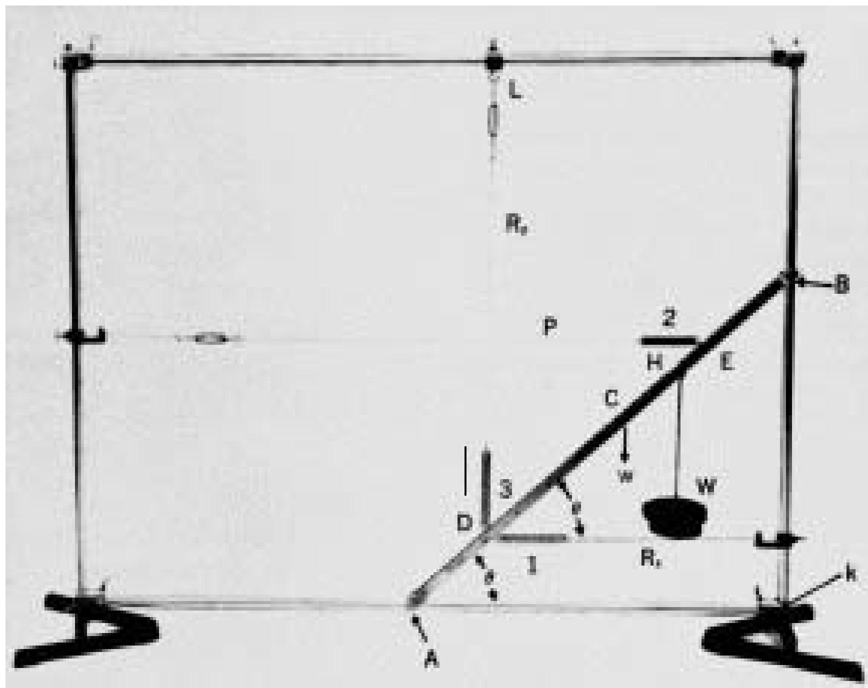

Fig. 2. Apparatus for studying the forces acting on a ladder.

5. For $\phi = \theta$ the ladder shown in Fig. 1, under what conditions will? In a practical situation, will the angle ϕ be equal to, less than or greater than θ?

Fig. 3. Method of measuring the dynamometer springs.