CENCO Physics

Selective Experiments In Physics

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STATIC EQUILIBRIUM: THE CRANE

OBJECT: To study force and torque methods of solving problems involving bodies in static equilibrium, utilizing for this purpose a model crane.

METHOD: A model crane (Fig. 1) is utilized, its design being such that the forces in its various parts may be measured. From the dimensions of the crane these forces for any given load are calculated by vector polygon or torque methods or by a combination of both methods. The experiment consists of the comparison of these observed and calculated forces for various loads and configurations of the crane.

THEORY: A body at rest when under the action of forces is said to be in static equilibrium. In order to obtain this state two important conditions must be fulfilled:
(1) For a body to be in equilibrium in so far as linear motion is concerned, the *vector* sum of all the external forces acting upon it must be zero. This statement may be put in several equivalent forms such as: (a) the vector polygon representing all the external forces must be a closed figure; (b) the *resultant* of all the external forces must be zero; (c) the algebraic sum of all the component forces in any direction must be zero- specifically, the algebraic sum of the horizontal and vertical components must each equal zero.

Fig. 1. The Model Crane

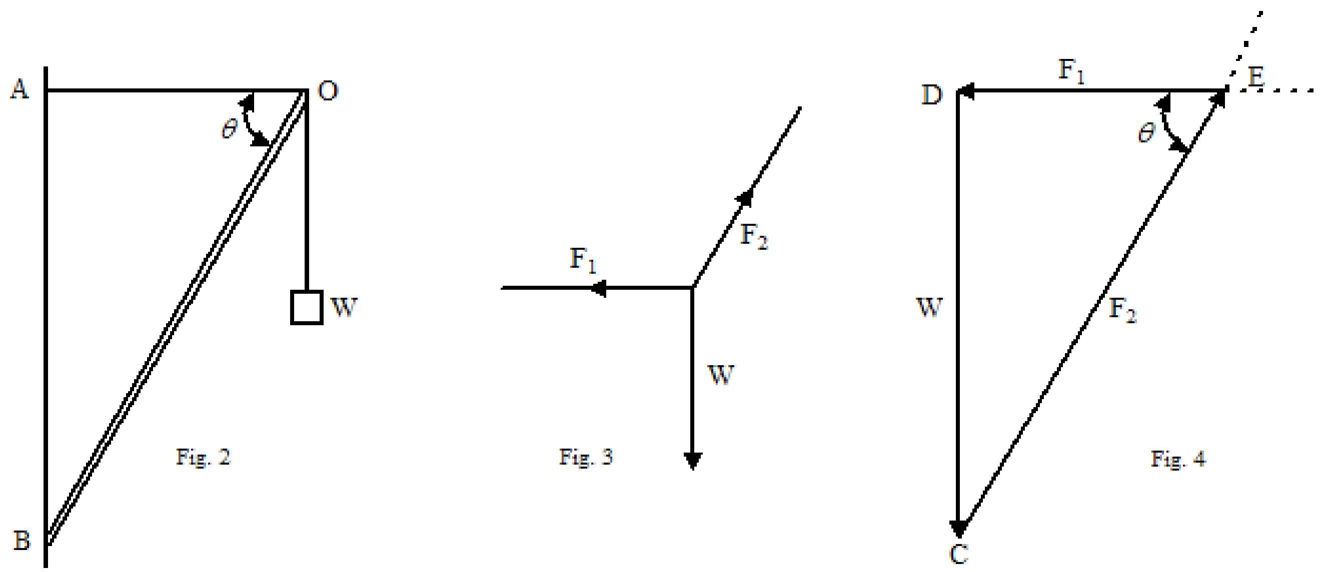
(2) For the body to be in equilibrium in so far as its *rotary* motion is concerned, the algebraic sum of the *torques* of all the coplanar forces about any axis perpendicular to the plane must be zero.
In this experiment both of these conditions are utilized in the calculation of the thrust in the boom of the crane and the tensional force in the rope, assuming as known quantities the load to be lifted and the dimensions of the crane. For example, consider the forces acting upon the point O in the crane sketched in Fig. 2. Let the force in the rope be represented by F_1 and that in the boom by F_2. The directions of their actions on O are shown in Fig. 3. Notice that both the magnitude and direction of *W*, but only the directions of F_1 and F_2 are known. But since the point O is in equilibrium under the action of these three external forces, their vector sum must be zero and the vector polygon representing the forces must be a closed figure. Hence the polygon shown in Fig. 4 may be constructed by drawing, first, a vector *DC* to a suitable scale to represent the known weight *W*; second, a line *ED* in the direction (here horizontal) of the force in the rope; and third, a line *CE* in the direction of the force in the boom. The figure must be a closed polygon for the body to be in equilibrium and hence the intersection at E of the lines *ED* and *CE* determines the length of the vectors representing F_1 and F_2. It is apparent that the vector triangle *CED* is similar to the figure of apparatus *BOA* since their sides are respectively parallel; note, however, that the triangles are not *identical*. (Much confusion may be avoided by a clear differentiation between the two figures; a vector figure should not be superimposed upon a figure of apparatus until the student is thoroughly familiar with this field.) If a graphical solution is desired the lines *ED* and *CE* may be scaled off to the same scale as was used in constructing *DC*, and thus the values of F_1 and F_2 are immediately obtained. If an analytical solution is to be used it may be obtained in various ways, as, for example, since the vector triangle *CED* is similar to the figure of apparatus *BOA*, the sides are respectively proportional and

$$\frac{F_2}{W} = \frac{OB}{AB} \text{ or } F_2 = W\frac{OB}{AB} = W\csc\theta \qquad (1)$$

and similarly

$$\frac{F_1}{W} = \frac{OA}{AB} \text{ or } F_1 = W\frac{AO}{AB} = W\cot\theta \qquad (2)$$

The values of F_1 and F_2 may also be obtained from a consideration of the torques due to the external forces acting upon the crane. The common convention is to consider torques tending to produce counter-clockwise rotation as positive and those tending to produce clockwise rotation as negative. Hence in Fig. 5 the force *W* will produce a negative torque about point B, while F_1 will cause a positive torque. When the second condition for equilibrium is applied and the sum of the torques about the point B is equated to zero



Showing the carious forces acting in the Model Crane.

$$-W \times BH + F_1 \times AB + F_2 \times 0 = 0 \qquad (3)$$

Similarly, writing the expression for the torques about A

$$-W \times AO + F_2 \times AG + F_1 \times 0 = 0 \qquad (4)$$

Knowing *W* and the various distances, Eqs. (3) a may be solved for F_1 and F_2.
Similar procedures may be used to solve for "unknown" forces in terms of the known load and dimensions of the apparatus for various configurations of the crane.

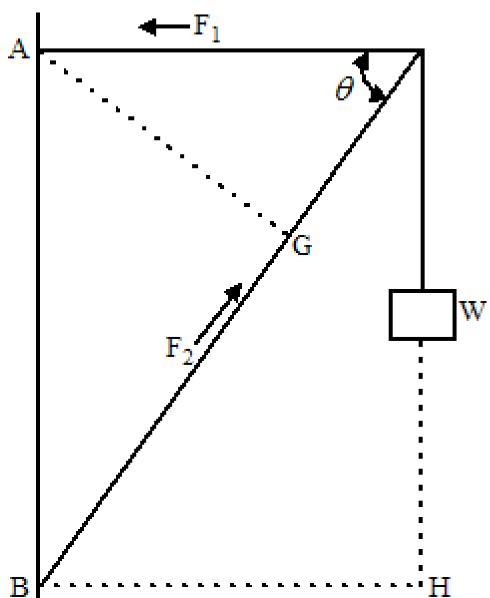


Fig. 5. Illustrating the torque method used in solving for forces in the Model Crane.

APPARATUS: The main apparatus consists of a model crane and means for supporting it in a vertical position. The model crane, Fig. 1, is made up as follows:

(1) A vertical mast. This consists of a 19mm support rod, 185cm long, attached to the wall by suitable end supports.

(2) An experimental crane boom. This is in the form of a compression spring balance having a range of 15 kilograms. The boom terminates at its lower end in a bent rod. The rod is pivoted in a hole in a right-angle clamp, fastened to the mast, so that the boom can rotate freely in the plane of the mast.

The spring balance is vertically suspended from a clamp attached to the mast. The rope from the balance passes over a heavy duty and nearly frictionless pulley, also clamped to the mast. The second stirrup supports a weight holder on which the load is placed. When the load is attached, compensation for the shortening of the compression-scale boom is made by moving the collar upward. The extension of the spring balance attached to the tie-rope is compensated by raising its supporting clamp. Thus the dimensions of the crane may be kept constant from no load to full load.

In addition to the apparatus previously mentioned, a meter stick fitted with caliper jaws, about 10 one-kilogram masses and a one-kilogram weight holder are required. *In addition the student should bring to the laboratory a protractor, a scale and half a dozen sheets of coordinate paper.*

PROCEDURE:

Experimental: 1. Arrange the experimental crane as in Fig. 2. Make the point B, at which the lower end of the crane boom is supported, as nearly as possible vertically below the intersection of the horizontal and vertical cords at A. Be sure that the entire assembly is in the vertical plane (view it from a short distance). Have the collar 0 about half way between the ring and the scale when no load is attached. By the use of a right-angle triangle, a square, or a protractor, make sure that the rope is horizontal.

2. With zero load attached measure with a meter stick provided with vernier jaws the dimensions of the crane, namely, the distances *AO*, *OB*, and *AB*. Read both balances to obtain "zero" or "tare" corrections necessary because of the weight of the boom. Always tap the boom to reduce its friction before taking its final readings. It is well in the record of this experiment to draw a rough sketch of the crane on the data sheet, recording on the sketch all the necessary data.

3. After the no-load readings have been recorded, attach a rather large load (say 10kg, including the scale pan) for *W*. It will now be necessary to move upward both the collar O and the vertical spring balance S in order to restore the configuration of the crane to its original values. After this has been accomplished tap the boom and record its reading and also that of the balance. By subtracting the appropriate "no-load" values recorded in part 2, the corrected load values are obtained. These are to be used as the experimental values of the forces F_1 and F_2.

4. Repeat the above procedure with the crane in other configurations, particularly those in which the apparatus does not form a right-angle triangle.

Interpretation of Data: The major objective of this experiment is to compute the values of F_1 and F_2, assuming as known only the value of *W* and the geometrical dimensions of the crane. This should be done by the following three methods:

(A) ***Torque Method***. Make a good sketch of the crane on graph paper, to a large scale. By writing the equations of torques about points A and B, respectively, the values of F_1 and F_2 can be calculated from the known values of *W* and the appropriate lever arms. The lever arm of F_2 about the axis A is easily expressed in terms of the distance *AO* and the angle *AOB*.

(B) ***Graphical Vector-Polygon Method***. Draw a *vector* triangle (something like Fig. 4) having its sides parallel, respectively, to the directions of the forces about O. USE A LARGE SCALE. Since the magnitude of *W* is known the values of F_1 and F_2 may be immediately obtained by scaling off the appropriate vectors.

(C) ***Analytical Vector Method***. Calculate the values of F_1 and F_2 from the trigonometry of the figure used in part (B), or use simple proportions as described in the theory. Note the percentage differences between the *experimental* values of F_1 and F_2 and the average of the calculated values obtained in (A), (B), and (C).

In the report of the experiment resolve the forces *W*, F_1 and F_2 into X and Y components and check the first condition for equilibrium by noting whether their respective algebraic sums are equal to zero. (This need be done for only one of the arrangements used. Do not select the case where F_1 is horizontal.)

QUESTIONS: 1. Why was it desirable in this experiment to have the load so chosen that a maximum reading was obtained on the scales?

2. Three unequal forces act upon a body at a point so that the body is in equilibrium. If the magnitude of two of the forces be doubled, how must the third force be changed to preserve equilibrium? Justify conclusion by diagrams.

3. Draw a diagram showing the vector polygon of the forces acting upon a ladder which leans against a smooth vertical wall and rests upon the rough ground. (Note: The thrust of the ground is not along the ladder, and the wall cannot exert a force parallel to its surface.)

4. What configuration of the crane would make *W*, F_1 and F_2 all equal? Explain reasoning and illustrate by a vector diagram.

5. Which of the following statements are true? A body to be in equilibrium (a) must be at rest, (b) must move with uniform acceleration, (c) must have uniform linear and rotary velocity, (d) cannot be acted upon by external forces, (e) must have the sum of the upward forces acting upon it just equal to its weight.

6. A crane boom of negligible weight is 20ft. long. It makes an angle of 30° with a vertical mast. The tie rope, fastened from the upper end of the boom to the wall, makes an angle of 90° with the boom. The maximum tensile force that the rope can safely stand is 5.0 tons. What is the maximum load the crane can support and the thrust in the boom for this load? Solve by both an analytical-vector method and a torque method.

7. A derrick boom 30ft. long weighs 400lbs. and is hinged at the bottom to a vertical mast. The boom is held in position by a rope attached at the top, the rope making an angle of 90° with the boom, and the boom making an angle of 30°, with the mast at the hinge. If a load of 1.00 ton is carried at the top of the boom, what is the force exerted on the hinge?

8. A crane is constructed with a 200lb. uniform boom B 30ft. long attached to a vertical mast A. The cable C is fastened to the mast at a point 20ft. above the place where B is hinged to A. The boom inclines 30° to the vertical, and C is attached to B at a point 10.0ft. from the upper end of the boom. Find the thrust of B against A and the pull on C when a load of 1.00kg. is attached to the upper end of B. Solve by the graphical method.