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# THE SIMPLE CRANE; STATIC EQUILIBRIUM

**OBJECT**: To determine by the use of a simple crane boom the conditions that are necessary to produce static equilibrium.

**METHOD**: A simple crane, employing a boom of negligible weight, supports a known load at its upper end. The force in the tie cord and the compressional force in the boom are measured. These three forces, acting at a point at the upper end of the boom, are represented by vectors. The resulting vector polygon is examined and a general conclusion is deduced that leads to the first condition for static equilibrium. The second condition for equilibrium is developed by considering the torques that tend to produce rotation of the boom. This second condition is further developed by making similar observations and calculations when a crane boom of appreciable weight is used.

**THEORY**: A body is said to be in *equilibrium* when there is no change in its motion. Obviously a body at rest must be in equilibrium. Many important problems in static equilibrium are involved in the study of structures, such as buildings, bridges, derricks, and a wide variety of familiar devices.

*First Condition for Equilibrium*. In so far as linear motion is concerned a body is in equilibrium when it has no linear acceleration. To produce this first condition for equilibrium the *vector sum* of all the forces acting upon the body must be equal to zero. For this case the vector polygon that represents all the forces acting upon the body must form a *closed* figure. If the lines of action of these forces intersect at a common point the forces are said to be *concurrent*. If the forces acting upon a body are concurrent and their vector sum is zero the body must be in equilibrium.

*Torque and the Second Condition for Equilibrium*. The effect of a force used to produce or prevent a change in the rotation of a body depends not only upon the magnitude of the force but also upon the point at which it is applied and the direction in which it acts. The *torque,* or *moment* of force, about any axis is the product of the force and the perpendicular distance from the axis to the line of action of the force. This distance is called the *moment arm*. The defining equation for torque is

$$L = Fs \tag{1}$$

where $L$ is the torque developed by the force $F$ and $s$ is the moment arm.

Since torque is the product of a force and a distance, its unit is a force unit multiplied by a length unit. The pound-foot is the usual British unit. The mks unit is the meter-newton. The cgs unit is the centimeter-dyne.

Torque is a vector quantity, with the direction assigned to the vector parallel to the axis of the torque. In this experiment we shall consider only cases in which all of the forces act in the same plane. Hence the resulting torques are parallel and only their algebraic signs need be considered. For convenience torques tending to produce counter-clockwise rotation are usually called positive and clockwise torques are considered negative.

For an object to be in rotational equilibrium under the action of forces in a single plane, the algebraic sum of the torques (about any axis) acting upon the body must be zero. This statement is known as the second condition for equilibrium. It may be represented by the equation

$$\sum L = 0 \tag{2}$$

In the first and second conditions for equilibrium we have a complete system for solving problems involving forces that are all in a single plane.

*Center of Gravity*. The most common force acting upon a body is its weight. For every body, no matter how irregular its shape, there exists a point such that the entire weight may be considered as concentrated at that point. This point is called the *center of gravity* of the body. The center of gravity may be either within or outside the body. If a single force equal to the weight of the body and acting vertically upward could be applied at the center of gravity, it would support the body in equilibrium.

A knowledge of the position of the center of gravity is very useful in problems of equilibrium, for that position is the point of application of the vector representing the weight. (It is never necessary and seldom convenient to break the weight up into parts.)

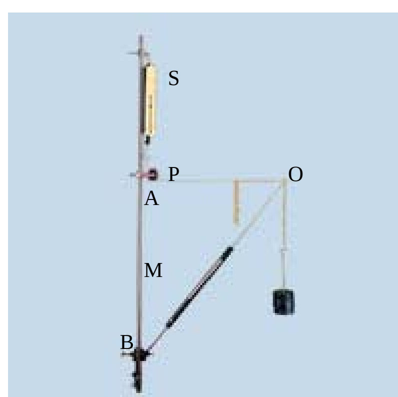


Fig. 1. Simple crane.

**APPARATUS**: A simple crane (Fig. 1) is made from alight but rigid boom (hollow metal tube or wooden rod) with a ring screw O at one end for attaching a tie cord and a loose hinge B at the other for attaching the lower end to the vertical mast M. In one form of boom this hinge may be a knife edge which serves as a bearing in a special V-groove clamp. Or a wooden-rod boom may be provided with a hole at the end B with the hole loosely held by a pin that is smaller than the hole. The tie cord passes around a low-friction pulley P to a 15kg spring balance S, used for measuring the tension in the tie cord. A second 15kg spring balance may be attached at O to measure the compression in the boom. Since the boom is rigid the length OB is constant. Auxiliary apparatus needed includes a weight holder, several kilogram masses, meter stick with caliper jaws, and a second boom that has an appreciable weight.

**PROCEDURE**: 1. Set up the crane as in Fig. 1. Arrange the dimensions so that when the crane is loaded no two sides of the triangle AOB are equal and no angle is aright angle. Make the point B, at which the lower end of the boom is supported, as nearly as possible vertically below the vertical cord at A.

2. Hang a load of about 9kg, including the weight hanger, from the ring O. Measure and record the lengths, AO, BO and BA, using the meter stick with caliper jaws. In the measurement of AO, and BA be careful to locate the point A at the intersection of the cords (Fig. 2). Record the load and the tension in the tie cord as indicated by the spring balance S. Measure the force exerted by the hinge at B by the use of a second spring balance at O pulling in a direction parallel to the boom until the boom hangs loosely in the support at the hinge B. Record the reading of this spring balance.

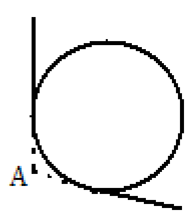


Fig. 2. Intersection of two segments of tie cord.

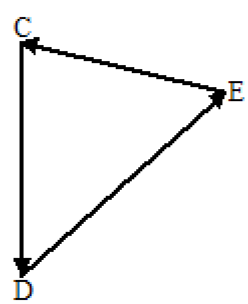


Fig. 3. Vector diagram of forces acting upon the crane boom.

3. Replace the light boom with a heavy boom whose weight has been measured. Attach a suitable load, as in Step 2, and record the force at O exerted by the cord, as read from the balance S. Record the lengths OB, OA and AB.

**INTERPRETATION OF DATA**: 1. Use the measured lengths AO, BO and BA to make a scale diagram of the triangle AOB. Choose a scale such that the diagram occupies most of a sheet of paper. On the diagram draw dotted lines to represent the moment arms of each of the forces about B as an axis and about A as an axis. Measure and record the lengths of the moment arms as determined from the lengths of the lines and the scale chosen.

2. Consider the load as the only known force. Write the equation of torques about the axis B and use it to compute the tension in the tie cord. Similarly write the equation of torques about A and use it to calculate the thrust at B. Compare each of these values with the corresponding force determined experimentally. What conclusion can be drawn from these data with respect to the second condition for equilibrium?

3. Draw a vector diagram to represent the forces acting about the point O. Use a large scale, so that the figure will fill a sheet of graph paper. This vector diagram, which is similar to the physical triangle, may be drawn as follows: draw the vertical vector CD (Fig. 3) to scale to represent the load. From the scale diagram of the triangle AOB measure the angle ABO and OAB. Draw the line DE from D so that the angle CDE equals the angle ABO. Draw the line CE from C so that the angle DCE equals the angle BAO. Insert the arrows at C and E and measure the length of the vectors DE and EC. These lengths, in terms of the scale chosen for CD, will make it possible to determine the forces of the pull of the cord on O and the thrust of the hinge along BO. Compare these values with the experimental values. How do these data give evidence concerning the first condition for static equilibrium?

4. On the diagram of Part 1 measure the angle each force makes with the horizontal. From the first condition for equilibrium write the equations for the horizontal and the vertical components of the forces, using only the load as a known force. Solve these equations for the tension of the tie cord and the thrust of the hinge along BO. What conclusion can be drawn from these data with regard to the X components and the Y components of all of the forces that act on a body in static equilibrium?

5. Use the second condition for static equilibrium to calculate the horizontal and vertical components of the thrust of the hinge at B for the case of the heavy boom used in Step 3 of the Procedure.

**QUESTIONS**: 1. What configuration of a simple weightless crane would make the load lifted, the tension in the tie rope and the compression in the boom equal in magnitude? Explain the reasoning by a vector diagram.

2. Which of the following statements are true? A body to be in equilibrium (a) must be at rest, (b) must move with uniform acceleration, (c) must have uniform linear and rotary velocity, (d) cannot be acted upon by external forces, (e) must have the sum of upward forces acting upon it just equal to its weight.

3. The center of gravity of a 50.0gm meter stick is located at its 51.0cm mark and the stick is supported at the 70.0cm mark. Where must an 80.0gm object be hung in order to have equilibrium?

4. A crane boom of negligible weight is 20ft long. It makes an angle of 30° with a vertical mast. The tie rope, fastened from the upper end of the boom to the wall makes an angle of 90° with the boom. The maximum tensile force that the rope can safely stand is 5.0 tons. What is the maximum load the crane can support and what is the thrust in the boom for this load? Solve by both a vector method and a torque method.

5. In order to assist a horse to pull a wagon out of a rut, at what place on a wheel and in what direction should one

apply a force most effectively? Explain by the aid of a diagram.

6. A 250lb cubical block, 5.0ft on each edge, rests on a horizontal floor against a small obstacle on one edge. In order to overturn the block most easily, (a) how should one push, and (b) what force must one apply? (c) How will this force vary as the block starts to tip?

7. The beam of a wall crane 7.5ft long is held at right angles to the wall by a tie that is attached to the wall 6.0ft above the foot of the beam. If the load lifted is 3.0 tons, find the tension in the tie and the compressional force in the beam. Neglect the weight of the beam.

8. A load of 50lb. is suspended from the ceiling by a cord 8.0ft long. A second cord is tied to the first 2.0ft above the load, and a pull is exerted by this attached cord always making an angle of 30° above the horizontal. When the tension in the second cord becomes 20lb., it is tied fast. (a) Find the tension in the first cord, above and below the knot. (b) What angle does the first cord make with the horizontal?

9. The uniform boom of a crane is 30.0ft long and weighs 2.00 tons. The tie rope is horizontal and is attached 10.0ft from the upper end of the boom, which makes an angle of 60° with the horizontal. If a 10.0 ton load is attached to the end of the boom, calculate the tension in the tie and the force exerted by the boom on its lower support.

10. The uniform boom of a crane is 40.0ft long and weighs 400lb. It is hinged at the bottom and held at an angle of 45° by a tie rope attached 10.0ft from the upper end. The tie rope makes an angle of 60° with the vertical. A load of 3600lb. is supported at the end of the boom. Find (a) the tension in the tie rope, (b) the vertical and horizontal thrusts at the hinge, and (c) the resultant thrust at the hinge.