• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

THE PRECESSION OF A GYROSCOPE

OBJECT: To check quantitatively the relationship between the applied torque and the precessional angular velocity of a gyroscope.

METHOD: The gyroscope wheel is set into rotation, and its angular velocity is measured. A known torque is then applied perpendicularly to the axis of rotation, and the angular velocity of precession is measured. This angular velocity of precession is then calculated from the relation between the known torque and the known angular momentum of the rotating wheel.

THEORY: It was the French physicist Leon Foucault who, about 1851, first investigated the properties of a gyroscope and also invented the name gyroscope. The name was derived from two Greek words: "gyros" meaning revolution and "skopein" meaning to view. Thus literally the word gyroscope means "viewing the revolution."

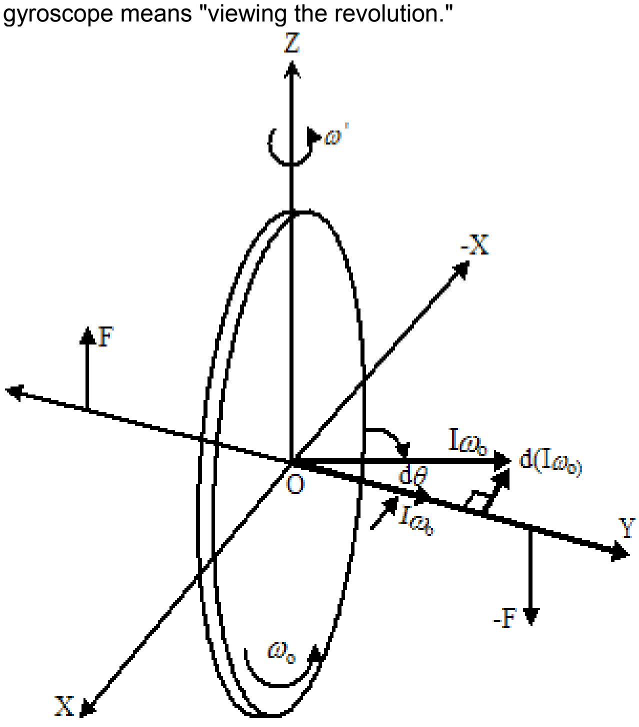

Fig. 1. Wheel rotating and precessing.

The essential part of a gyroscope is a rotating wheel mounted so that its axis of rotation is free to turn in any direction. In order to investigate the properties of a rotating wheel, consider a right-handed set of Cartesian axes *xyz* placed with its origin Oat the center of the wheel, as shown in Fig. 1; initially the plane of the wheel lies in the *yz* plane. The wheel is considered to be rotating with a constant angular velocity ω_o about the *y*-axis. Also, the wheel is turning, or precessing, with an angular velocity ω' about the *z*-axis due to a torque *L* produced by the forces *-F* and *F* acting perpendicularly to the *y*-axis.

These angular velocities, ω_o and ω', and the torque *L* can be represented by vectors whose lengths are proportional to the magnitude of the quantities and whose directions are given by the right hand rule of rotations. This rule says that if the fingers of the right hand are curled in the direction of the rotation, then the outstretched thumb gives the direction of the vector that represents the angular velocity. According to this rule, the angular velocity ω_o is directed along the *y* axis, and the precessional angular velocity ω' is directed along the *z*-axis. The forces *-F* and *F*, acting perpendicularly to the y-axis, produce a torque *L* in a clockwise direction. The torque is represented by a vector directed along the *-x* axis.

Newton's second law as applied to rotational motion states that the rate of change of the angular momentum about an axis of rotation of an object is equal in magnitude and direction to the torque applied to the object. The angular momentum of the rotating wheel is $I\omega_o$, where *I* is the moment of inertia of the wheel about its axis *y* of rotation.

Thus Newton's second law for rotational motion leads to the equation

$$L = \frac{d(I\omega_o)}{dt} \tag{1}$$

Since the wheel is precessing with an angular velocity ω' about the *z*-axis, the wheel depicted in Fig. 1 rotates through an angle $d\theta$ in the *xy* plane in a time *dt*. Thus the change in angular momentum $d(I\omega_o)$ is equal to $(I\omega_o)\,d\theta$, as can be seen from the infinitesimal triangle having sides of length $I\omega_o$ and angle between them of $d\theta$. From Eq. (1) it follows that

$$d(I\omega_o) = (I\omega_o)d\theta = L dt \tag{2}$$

Thus

$$L = I\omega_o \frac{d\theta}{dt} = I\omega_o\omega' \tag{3}$$

where $d\theta/dt$ is the angular velocity ω' about the *z*-axis. These equations describe relationships between vector quantities. The change in angular momentum $d(I\omega_o)$ occurs in the same direction as the torque *L*. This fact can readily be seen in Fig. 1. It is assumed in this derivation that the

angular momentum about the vertical *z*-axis is negligible compared to the angular momentum $I\omega_o$ of the rotating wheel about the *y*-axis.
The precessional motion previously described does not occur when the torque *L* is first applied to the rotating wheel. If the axis of rotation is held in a horizontal position and then released, it falls only a small amount, and a new kind of

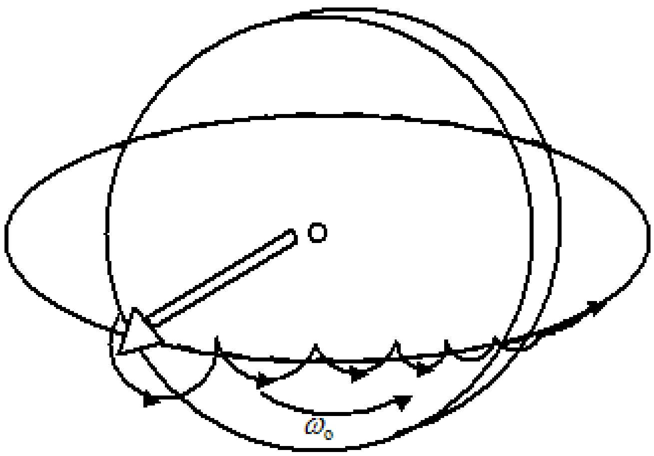

Fig. 2. When rotating wheel is first started, with a weight at the end of the pointer, the tip of the pointer follows a curved path and this motion is called nutational motion.

motion, called *nutation*, sets in. This motion is shown in Fig. 2 and can be readily demonstrated with the Mitac gyroscope. The nutational motion is rapidly damped and then the normal precessional motion ensues.

APPARATUS: The Mitac gyroscope, Fig. 3; a stroboscope, Fig. 4; a stop watch; inside and outside calipers; a platform balance.

Fig. 3. The Mitac Gyroscope.

PROCEDURE: Connect the Mitac gyroscope to an ac power line to energize the gyroscope, thereby setting the wheel into rotation. Measure the angular velocity of rotation ω_o, using either a stroboscope or a stopwatch for timing, say, twenty complete turns. If the stroboscope is used it should first be calibrated according to the instructions given with the instrument.
As an aid in synchronizing the frequency of the stroboscopic flashes to the speed of the wheel, a series of short dark lines can be marked near the edge of the wheel at 30° intervals. (It is possible to make this adjustment without the added dark lines.) Repeat the measurement for ω_o at least five

Fig. 4. Stroboscope for measuring the speed of rotation of the rotating wheel.

times so that an average value and its order of accuracy can be obtained. For the order of accuracy, the maximum deviation from the average value can be used. Some judgment must be used in obtaining the maximum deviation; if one reading is very far from the average value the reading should be discarded and another reading taken. This procedure should be used for all of the measurements made in this experiment.
Weigh the two slotted weights on a platform balance noting the maximum deviations of the measurements. Hold the axle of the rotating wheel horizontal and at rest and place the lighter slotted weight in the depression on the axle at a distance L_a from the center of the wheel, Fig. 5. Release the axle and observe the nutational motion. When this is damped out, start the stopwatch and measure the time of one or more complete turns. Repeat this measurement at least five times and obtain an average value and a maximum deviation for the angular velocity of precession ω'. Using the inner and outer calipers, measure the slot distances L_a, L_b, L_c and again obtain average values and maximum

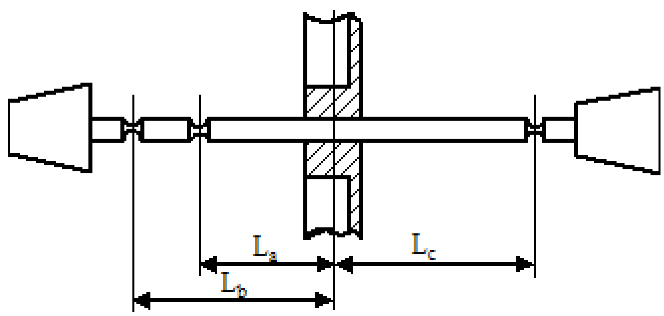

Fig. 5. Axle of rotating wheel showing depressions where slotted weights are inserted.

deviations. Measure the precessional motion for the slot distances L_b and L_c and the lighter slotted weights. As an optional experiment, the procedure can be repeated with the heavier slotted weight.

Analysis of Data. Find the values of the torques used in the experiments. In each case give the maximum deviations from the mean values of the torques. Given the amount of inertia of the rotating wheel about its axis of rotation $I = (2.00 \pm 0.02) \times 10^5 gm\text{-}cm^2$, and the measured value of ω_o, find the values of $L/I\omega_o$. From Eq. (3) these values should be equal to ω', the measured values of the angular velocity of precession. Give the percent differences of the two values obtained for ω'. The difference in the two values for ω' should be within the maximum deviations of the various measured quantities. If it is not, try to account for the difference.

QUESTIONS: 1. If a hoop is rolling in a vertical plane along the ground and it is desired to change the direction of motion of the hoop, where must a force be applied? At the top or the side of the hoop? Explain your answer.

2. As the precessional motion of a gyroscope continues, explain why the axis of rotation falls. Identify the torque producing this change.

3. Suppose the rotating wheel of a gyroscope were suspended so as to be completely free to turn in any direction. Explain why the axis of rotation would remain fixed in space and why the axis of rotation would appear to rotate in a westward direction, that is, the same as that of the sun or stars.

4. Suppose the wheel of the Mitac gyroscope is rotating freely and a horizontal force is applied at a level with the axis of rotation on the edge of the rotating wheel. Explain the ensuing motion of the rotating wheel.

5. If a left-hand rule had been used for determining the direction of the vectors in rotational motion, would this have changed the final result as given in Eq. (3)?

6. Show that the units for the quantities on the left and right sides of Eq. (3) agree.