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MOMENTUM: BALLISTICS

OBJECT: To study the law of the conservation of momentum and the elements of projectile motion; in particular, (a) to determine the initial velocity of a projectile by means of a ballistic pendulum, and (b) to check this determination by measurements of its range and vertical distance of fall during its flight.

METHOD: A ball is shot by a spring gun into a suspended holder, arranged to swing as a ballistic pendulum (Fig. 1). By equating the momentum of the ball immediately before impact to the momentum of the system an instant after impact, an equation is set up from which the initial velocity of the ball may be expressed in terms of the easily measurable masses of the ball and pendulum bob and the height which the bob rises after impact. This velocity is checked by firing the ball horizontally and observing its range and vertical distance of fall, as the velocity can be calculated from these distances and the acceleration due to gravity.

THEORY: The momentum p of a body is defined as the product of the mass m of the body by its velocity v. In symbols, the defining equation is

$$p = mv \tag{1}$$

The absolute c.g.s. unit of momentum is the gram-centimeter per second; the absolute f.p.s. unit is the pound-foot per second.

It may be easily shown from Newton's second and third laws of motion that momentum is conserved in all impacts. *The principle of the conservation of momentum implies that the change of momentum of one part of a system must be equal and opposite to the change of momentum of some other part of the system*.

The principle of the conservation of momentum may be derived from Newton's laws of motion as follows: Imagine an object of mass m_1 moving with a velocity v_1 when it strikes a stationary object of mass m_2. The deceleration of the first object is, by Newton's second law of motion, due to the force f_1 which the second object exerts upon it. The first object, in accordance with the third law of motion, exerts an equal and opposite force $-f_2$ upon the second body. Representing the respective accelerations by a_l and a_2,

$$f_1 = -f_2 \quad \text{and} \quad m_1 a_1 = -m_2 a_2 \tag{2}$$

These forces necessarily act for the same time interval Δt. From the definition of acceleration

Fig. 1. Spring-Gun Ballistic Pendulum

$$a_1 = \Delta v_1 / \Delta t \quad \text{and} \quad a_2 = \Delta v_2 / \Delta t \tag{3}$$

where Δv_1 is the *change* in velocity of the first body and Δv_2 is the *change* in velocity of the second. Substituting these values of the accelerations in Eq. (2)

$$m_1 \Delta v_1 = -m_2 \Delta v_2 \tag{4}$$

Since the product of mass by change in velocity represents the change in momentum, it follows from Eq. (4) that *the loss in momentum of the first body is just equal to the gain in momentum of the second body.* In other words, the total momentum of the system has remained constant during the impact.

In the present case of inelastic impact it follows from the principle of the conservation of momentum that the momentum of the ball just before impact shall be equal to the combined momenta of the ball and bob an instant after impact. In the ballistic pendulum used in this experiment the velocity of the pendulum before impact is zero, and hence its momentum before impact is zero. The momentum of the ball before impact is the product of its mass m and its initial velocity v just before impact. Since the projectile becomes imbedded in the pendulum bob after impact, the ball and bob an instant after impact have a common velocity V and the combined momenta is *(M + m)V*, where *M* is the mass of the bob system. From the law of the conservation of momentum

Total momentum before impact =
Total momentum after impact

$$mv = (M + m)\, V \tag{5}$$

from which the initial velocity is given by

$$v = \frac{M + m}{m} V \tag{6}$$

As a result of the impact, the pendulum containing the projectile swings about its point of support, and in thus swinging its center of gravity rises through a vertical distance *h*. Knowing this distance, it is possible to determine *V*. The kinetic energy of the system an instant after impact must, by conservation of energy, equal the increase in potential energy gained by the pendulum when it reaches its highest point. By equating the kinetic energy to the potential energy

$$1/2\,(M + m)\, V2 = (M + m)\, gh \tag{7}$$

from which

$$V = \sqrt{2gh} \tag{8}$$

By means of Eq. (8), *V* may be calculated from the measured height *h*. This value, together with the masses *M* and m obtained by weighing, is to be substituted in Eq. (6) for the computation of *v*.

In the second part of this experiment the initial velocity *v* will be checked by computation from measurements of the range and vertical distance of fall of the ball when it is shot horizontally and allowed to fall to the floor without striking the pendulum bob (Fig. 2). In such a case it must be remembered that the horizontal component of the velocity always remains constant and is not affected by the constant downward pull of gravity. Furthermore the vertical component of the motion is unaffected by its horizontal flight and hence the body falls vertically, as does a freely falling object having no horizontal motion. The actual motion of such a projectile is the combination of its horizontal constant velocity and its downward uniformly accelerated velocity. During the time interval *t* required for the projectile to reach the floor, it will have moved horizontally through a distance

$$S = vt \tag{9}$$

and during the same interval, because of the acceleration due to gravity *g*, it will have fallen a distance

$$s = 1/2gt^2 \tag{10}$$

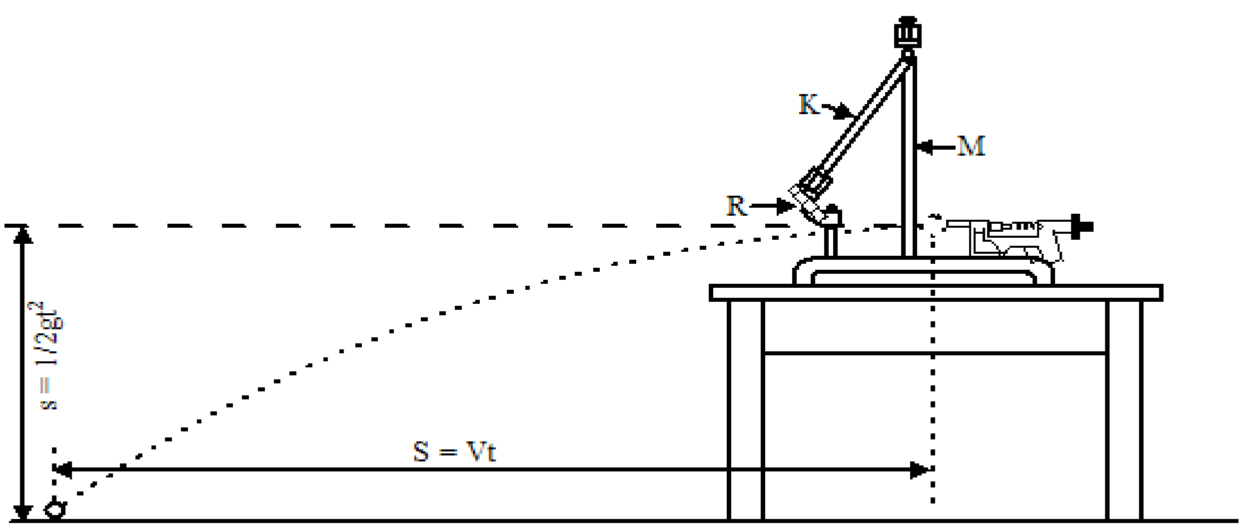

Fig. 2. The Spring Gun arranged for the determination of v from the measurable distance of fall s and the range S.

By eliminating t between these two equations and substituting the measured values of *S* and *s,* the initial velocity may be found from

$$v = \sqrt{\frac{S^2 g}{2s}} \qquad (11)$$

indicator I is attached to the pendulum bob C in such away that its tip indicates the height of the center of gravity.
When the projectile is shot into the pendulum, the latter swings upward and is caught at its highest point by the pawl P which engages a tooth in the curved rack R. The toothed surface of the latter lies on the arc of a circle having its

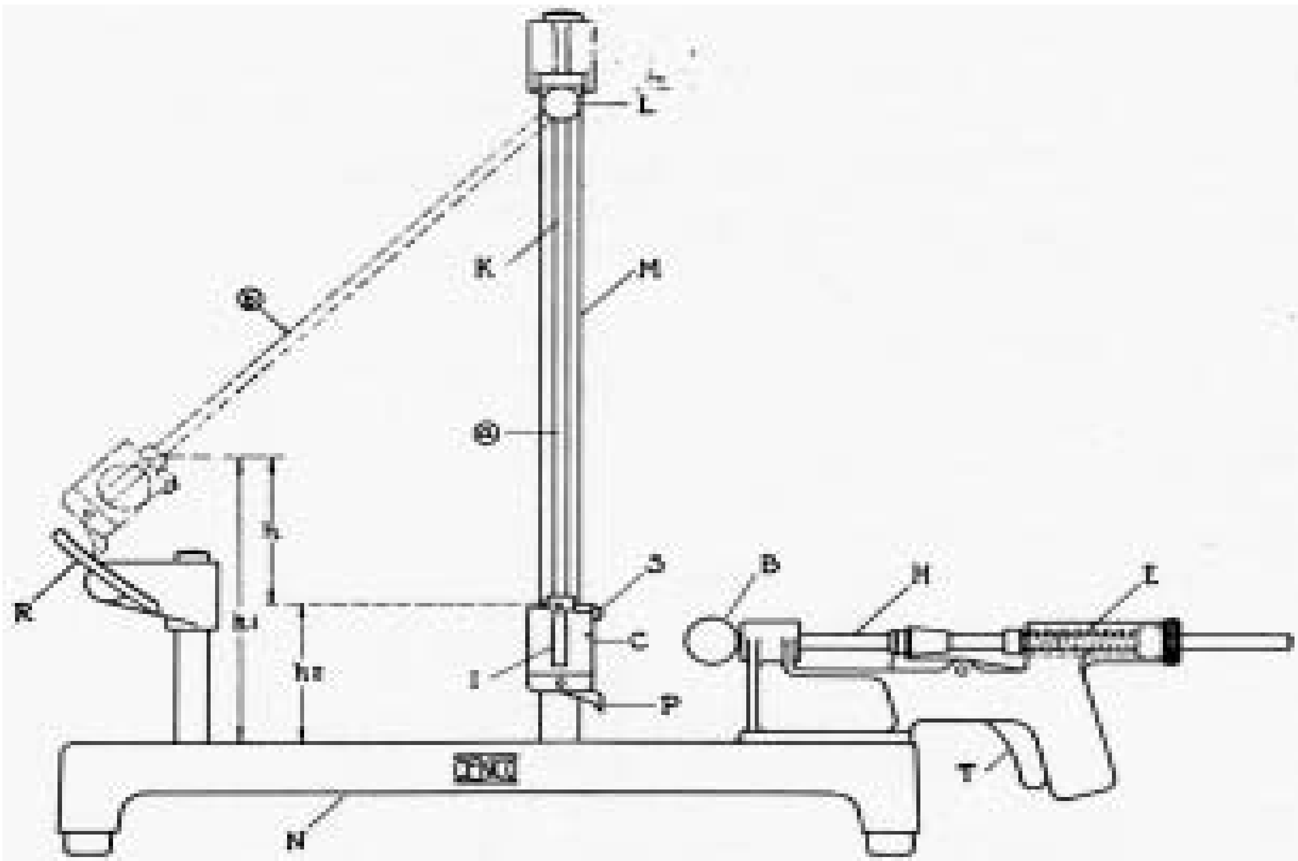

Fig. 3. Details of Spring-Gun Ballistic Pendulum

center in the axis of suspension of the pendulum. A scale along the outer edge of the rack provides a means for noting and recording the position of the pendulum after each shot. The ball is drilled so that it may be held on the forward end of the rod H which is propelled forward by the compressed spring E when the trigger T is pulled.

APPARATUS: A Blackwood ballistic pendulum, trip scales with masses, a metric ruler, a steel tape-measure, a spirit level, a sheet of carbon paper, a table clamp and a wooden box for catching the ball are required.
The Blackwood pendulum, illustrated diagrammatically in Fig. 3, is a combination of a ballistic pendulum and a spring gun for propelling the projectile. The ballistic pendulum consists of a massive cylindrical bob C, hollowed out to receive the projectile and suspended by a strong, light rod K. The latter is pivoted at its upper end in steel cone-pivot bearings at the top of the support rod M. The pendulum may be removed from its supporting yoke by unscrewing the shouldered screw L. The latter, when screwed tightly to its shoulder, automatically adjusts the bearings so that the pendulum is held securely with very little friction.
The projectile is a brass ban B which, when propelled into the pendulum bob, is caught and held by the spring S in such a position that its center of gravity lies in the axis of the suspension rod K. The pendulum therefore hangs freely in the same position whether or not it contains the ball. A brass

PROCEDURE:
I. Determination of Initial Velocity by the Ballistic Pendulum: Set the apparatus near one edge of the table as shown in Fig. 2. If necessary, wedge it up with cardboard until the base is accurately horizontal, as shown by the level. Clamp the frame to the table.
To make the gun ready for shooting, rest the pendulum on the rack R, put the ball in position on the end of the rod H and, holding the base with one hand, pull back on the ball with the other until the collar on the rod H engages the trigger T. This compresses the spring E a definite amount, and the ball is given the same initial velocity every time the gun is shot. Needless to say, it is necessary to be very sure

that the trigger is engaged before releasing the hand from the ball.

Release the pendulum from the rack and allow it to hang freely. When the pendulum is at rest, pull the trigger, thereby propelling the ball into the pendulum bob with a definite velocity. This causes the pendulum to swing from position (a) to (b) (Fig. 3) with the pawl engaged in some particular tooth of the rack.

To remove the ball from the pendulum, hold the latter in a fixed position on the rack and push the ball out with the finger or a rubber-tipped pencil, *meanwhile holding up the spring S.*

Shoot the ball into the cylinder about nine times, recording each time the point on the rack at which the pendulum comes to rest. This in general will not be exactly the same for all cases but may vary by several teeth of the rack. The mean of these observations gives the mean highest position of the pendulum. Raise the pendulum until its pawl is engaged in the tooth corresponding most closely to the mean value and measure h_1, the elevation above the surface of the base of the index point for the center of gravity. Next release the pendulum and allow it to hang in its lowermost position and measure h_2. The difference between these two values gives h, the vertical distance through which the center of gravity of the system is raised after shooting the ball.

Loosen the thumbscrew L and carefully remove the pendulum from its support. Weigh and record the masses of the pendulum and of the ball. Replace the pendulum and carefully adjust the thumbscrew.

From these data calculate the initial velocity v using Eqs. (6) and (8).

II. Determination of Initial Velocity from Measurements of Range and Fall: To obtain the data for this part of the experiment the pendulum is held in the position shown in Fig. 2, so that it will not interfere with the free flight of the ball. One observer should watch carefully to determine the point at which the ball strikes the floor. The measurements in this part of the experiment are made with reference to this point and the point of departure of the ball. Clamp the frame to the table, as it is important that the apparatus not be moved until the measurements have been completed. Apiece of paper laid in a box on the floor at the proper place and covered with carbon paper will help in the exact determination of the spot at which the ball strikes the floor.

Shoot the ball a number of times, noting each time the point at which it strikes the floor. Measure the distances s and S, using the mean position on the floor as the reference point and making due allowance for the box thickness.

Before leaving the apparatus, see that the ball is in the pendulum bob and that the spring is released.

From the average of the values of s and S calculate v by the use of Eq. (8). Note the percentage difference between the values of v determined by the two methods. Try to analyze the probable errors of the two methods and estimate which one should give the more accurate result.

Optional Exercises: 1. Make independent determinations of the initial velocities by the above two methods, but using a different spring tension.

2. Find the time of flight of the projectile by the use of Eq. (10).

3. From the measured and calculated data compute the kinetic energy of the system an instant before impact and an instant after impact. What becomes of the difference? Calculate the percentage difference. Show that the fractional loss is given by the ratio of the mass of the pendulum to the sum of the masses of the ball and pendulum.

4. Repeat Part II of the main experiment, after having blocked up the apparatus until the ball is shot at an angle of 30° with the horizontal. Compare the observed range with that obtained by calculation, using appropriate formulas for projectiles and assuming the previously measured value of v, the initial velocity.

QUESTIONS: 1. Suggest some probable reasons for the difference between the values of vas obtained by the two methods.

2. What might be the procedure to determine the "efficiency" of the spring gun, i.e., the ratio of its energy output to the work done upon it?

3. In the measurement of v by the free flight method, what would be the effect on the accuracy of the measurements (a) if the floor were not truly horizontal; (b) if the base of the apparatus were not accurately horizontal?

4. What becomes of the momentum of a meteorite which comes into the earth's atmosphere? What happens to its kinetic energy? What happens to its center of gravity if the meteorite explodes above the earth?

5. On what physical principle or law is the statement based that the horizontal component of the velocity of an object remains constant?

6. What is the effect upon the observed value of v caused by neglecting the rotational inertia of the pendulum?