CENCO Physics — Selective Experiments In Physics

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MOMENT OF INERTIA

OBJECT: To determine experimentally the moment of inertia of a body about an axis, and to compare this with the theoretical value computed from the mass and dimensions of the body.

METHOD: A body is set in rotation about a vertical axis, the applied torque arising from a constant force produced by a falling mass. The assumption is made that the energy of the system, consisting of the rotating body and the falling body, is conserved. On equating the energy of the system when the two bodies are at rest to the energy of the system when the falling body is in a displaced position, an expression is obtained in which the moment of inertia of the rotating body is simply related to measurable quantities. Since the moments of inertia of certain bodies may be calculated from their masses and physical dimensions, at least one of the moments of inertia may be computed as a check.

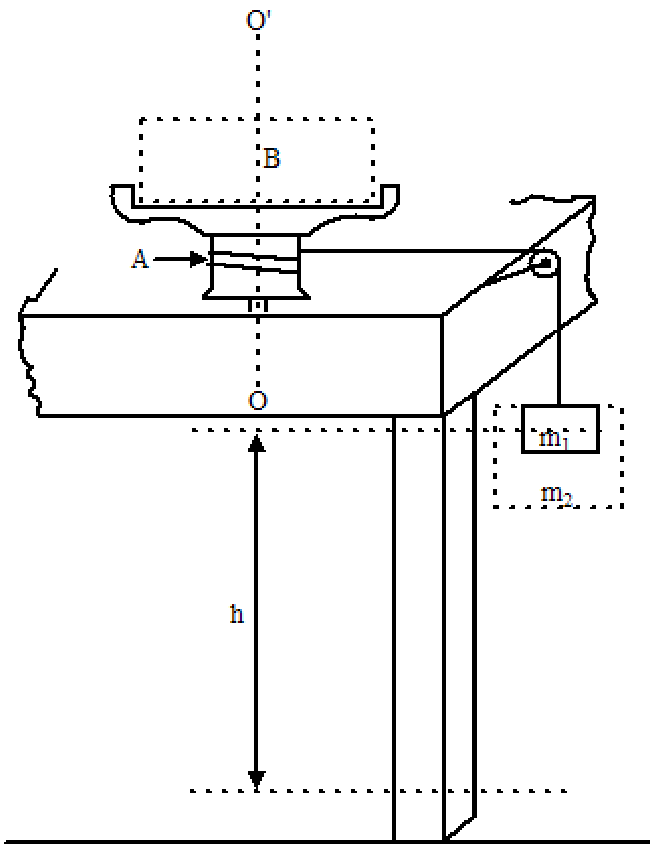


Fig. 1. Schematic diagram of body set in rotation by a falling mass.

THEORY: Consider the system schematically illustrated in Fig. 1. The body A, free to rotate about the vertical axis OO', is set in rotation as the result of the mass m_1 falling through a height h. The cord which constrains the motion of the mass m_l is wound around the drum of the body A, the radius of the drum being equal to r. In the experiment the axis OO' passes through the center of mass of the body A, the moment of inertia of which is I_1. When the mass m_1, starting from rest, is displaced a distance h under the action of the force of gravity, the body A is set in rotation. When in the displaced position, the mass m_l has a velocity V_1 and the body A an angular velocity ω_1; the two quantities are simply related by

$$v_1 = r\omega_1 \tag{1}$$

On the assumption that energy is conserved,

$$m_1 gh = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} I_1 \omega_1^2 \tag{2}$$

Since the driving force is constant, the mass m_1 will be uniformly accelerated; consequently its displacement h, time of fall t_1 and velocity v_1 are related by

$$v_1 = \frac{2h}{t_1} \tag{3}$$

Substituting Eqs. (1) and (3) into Eq. (2),

$$m_1 g = 2m_1 h / t_1^2 + 2I_1 h / t_1^2 r^2 \tag{4}$$

and

$$I_1 = m_1 r^2 \left(\frac{g t_1^2}{2h} - 1 \right) \tag{5}$$

from which the moment of inertia may be determined in terms of measurable quantities and the acceleration g due to gravity.

As illustrated in Fig. 1, the body A may have such shape that it should be either difficult or impossible to compute its moment of inertia about the axis OO' from its mass and dimensions. In general, therefore, the moment of inertia of a body A could be determined only experimentally by this method using Eq. (5) or by a similar experimental procedure. It is desirable, however, to determine experimentally the moments of inertia of certain regularly shaped objects and to calculate their theoretical values as a check.

Suppose the system to be altered to include a regular geometrical mass B (Fig. 1) having a moment of inertia of I_2 about its center of mass located on the axis OO'. The total moment of inertia of the system about the axis OO' is $I_1 + I_2$ and a larger mass m_2 is required to produce the displacement h in a conveniently measurable time t_2. As before, however, Eq. (5) applies and

$$I_1 + I_2 = m_2 r^2 \left(\frac{g t_2^{\ 2}}{2h} - 1 \right) \qquad (6)$$

From observed values of m_l, t_1, h and r, I_1 may be computed for the body A. Similarly from observed values m_2, t_2, h and r, $I_1 + I_2$ may be experimentally determined, and hence the moment of inertia I_2 of the body B.

APPARATUS: The apparatus consists of a light metal cross mounted in ball bearings so as to rotate in a horizontal plane about a vertical axis. The cross serves as a carrier for the

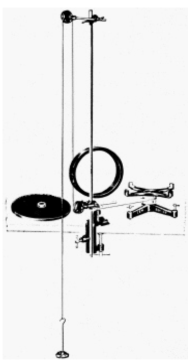


Fig. 2. Moment of Inertia Apparatus

object whose moment of inertia is to be determined. The drum is driven by means of a falling weight connected to the drum by means of a cord and pulleys. The bodies whose moments of inertia are to be determined may be a ring, a disk and a rectangular bar. It is instructive to measure the moment of inertia of a system of bodies also. A simple system, whose moment of inertia may be calculated as well as experimentally determined, consists of two cylindrical masses placed on opposite arms of the metal cross at equal distances from the axis of rotation. The accessory apparatus which is needed includes a meter stick, a stop watch, a trip scale balance, suitable weights, a table clamp, a one-meter support rod, two right angle clamps and pulleys as illustrated in Fig. 2.

PROCEDURE:

Experimental 1. With the vernier caliper measure the diameter *2r* of the drum. Determine the linear dimensions and the masses of the regularly shaped objects whose moments of inertia are to be determined.

Take a cord about 4 meters long and, after passing it over the top pulley and under the bottom one (Fig. 2), wind a little more than a meter of it on the drum. Adjust until the possible distance of fall is at least one meter. Make a loop in the free end of the cord and in this loop insert small riders, 1/2 to 1gm each, one at a time until the cross rotates with constant speed when started. Since these masses are small in comparison with the accelerating mass, their effect on values obtained from Eqs. (5) and (6) is negligible. Of course, the mass of the riders should not be included as part of the mass mused in these equations.

2. Now attach an additional mass m_1 (20 to 50gm) to the cord. This mass is the body whose weight furnishes the driving force. Select and accurately measure the distance of fall h and make three or four determinations of the time of fall t_1 through this distance.

3. Place one of the regularly shaped objects on the metal cross so that its center of mass is on the axis of rotation. Add appropriate riders until the system rotates at a constant speed. As before, these may be neglected in making computations for moment of inertia. Now add a mass m_2 and observe time of fall t_2 over the distance h. The mass m_2 should be much larger (100 to 200gm), since the moment of inertia of the cross, drum and object is much larger than the moment of inertia of the cross and drum alone.

4. Repeat the measurements as in Part 3, placing other regularly shaped objects on the cross as directed by the instructor.

5. (*Optional*) The experimental procedure for finding the moment of inertia of a system of bodies is the same as that for finding the moment of inertia of a simple body. A simple system consists of two similar cylindrical masses placed on opposite arms of the metal cross. The radii of the cylinders, their masses, and the distances of their centers of mass from the axis of rotation must be recorded. Although it is not necessary that the cylinders be placed equally distant from the axis of rotation, it is recommended that this be done.

Interpretation of Data: 1. Use Eq. (5) and the data obtained in Parts 1 and 2 above and obtain the moment of inertia I_1 of the cross and drum.

2. In the same manner, using the data in Parts 1 and 3 above, compute the moment of inertia $I_1 + I_2$ of the cross, drum and the regularly shaped object. By subtraction determine the experimental value of the moment of inertia I_2 of the regularly shaped object.

3. In Table I the theoretical value for the moment of inertia of regularly shaped bodies about various axes is indicated. Compute the theoretical values of the moments of inertia of the regularly shaped objects used and compare these with the experimental values obtained.

4. (*Optional*) If the moment of inertia of a system of cylindrical masses has been measured, the theoretical value of this moment should be calculated also. In making the calculation, it is convenient to use the formula $I = I_o + Mh^2$ given in Table I. The formula must be applied to each cylinder. Compare the value so obtained with the experimental value.

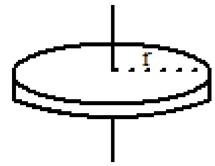
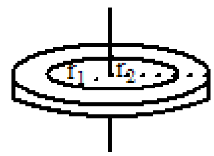
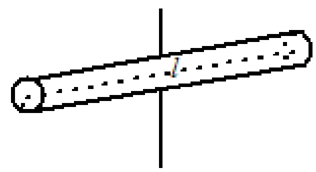
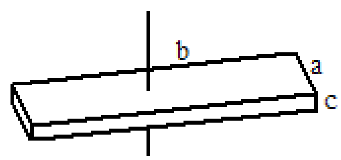
QUESTIONS: 1. Mention two retarding torques for which attempts are made to compensate by the use of the riders.

2. What would be the effect on the motion of the cross if the center of mass of the body were to one side of the axis of rotation?

3. How would the results be affected if the drum on which the string is wound were slightly eccentric?

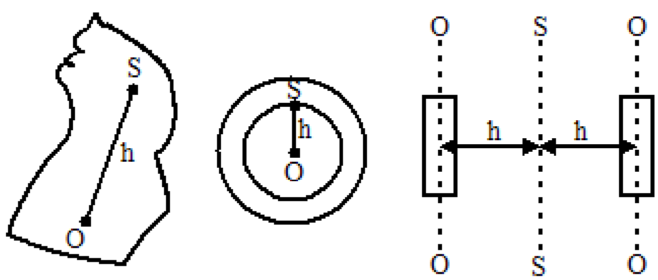
4. Which of the measurements, *m, t* or *h*, is the least precise? Explain, suggesting the possible magnitude of the error in each.

TABLE I

Body	Axis	Moment of Inertia	
Solid cylinder or disk	Longitudinal through center	$Mr^2/2$	
Cylindrical ring of rectangular cross section	Perpendicular to plane of ring, through center	$M(r_1^2/2 + r_2^2/2)$	
Solid cylinder	Transverse through center	$M(r^2/4 + l^2/12)$	
Rectangular bar having edges a, b, and c	Perpendicular to face ab at center	$M(a^2/12 + b^2/12)$	

Note: If I_o represents the moment of inertia of a body about an axis through its center of mass O, the moment of inertia I about any axis S parallel to it at a distance h may be calculated from the following expression

$$I = I_o + Mh^2$$



5. What percentage of error will be introduced in observed value of *I* by the indicated deviations in the values of *m, t* and *h?*

6. If the effect of friction when the rotating body is being accelerated is the same as the static friction measured experimentally, the correct form of Eq. (5) when the effect of friction is not neglected is

$$I = (m + \mu)r^2\left(\frac{t^2 g}{2h} - 1\right) - \frac{\mu g t^2 r^2}{2h}$$

In this relation, μ is the mass of the riders used to compensate for the frictional resistance.

(a) From this equation justify neglecting the mass of the riders for experimental purposes. Would the procedure for balancing out the friction with riders be valid if the frictional effects were large?

b) Find an algebraic expression for the error introduced by neglecting the effects of friction as was done in Eq. (5).

c) Find the percentage of error introduced by the approximation employed in experimentally determining the moment of inertia of the cross.

d) Let $M = m + \mu$, the sum of the accelerating mass and the mass of the rider. Calculate an algebraic expression for the error introduced by using *M* instead of *m* in Eq. (5). Justify the procedure of neglecting the mass of the riders used in this experiment.