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MOMENT OF FORCE; STATIC EQUILIBRIUM

OBJECT: To study the concept of moment of force and the condition that must be satisfied for a body to be in equilibrium insofar as rotary motion is concerned.

METHOD: The concept of moment of force is studied by applying known forces to a meter stick balanced at its center of gravity. The forces and their moment arms are adjusted to place the system into equilibrium. The moments of force that tend to produce clockwise rotation are compared with those tending to produce counterclockwise rotation. Other similar exercises are performed with different sets of forces and also where the axis of rotation is not at the center of gravity of the meter stick. The observed data are tested to check the "law of moments," that is, the condition for equilibrium insofar as rotary motion is concerned.

THEORY: The effect of a force in producing rotation depends upon two factors: the magnitude and direction of the force and the location of the axis of rotation. The factor that determines the effect of a given force upon rotational motion is the perpendicular distance from the axis of rotation to the line of action of the force, a distance called the *moment arm* or *lever arm* of the force.

In considerations involving rotary motion, the acting forces and their moment arms are of equal importance. The product of these two quantities is called *moment of force*, also referred to as torque. The defining equation for moment of force is

$$L = Fs$$

where *L* is the moment of force, or torque, *F* is the acting force, and *s* is the moment arm or lever arm. Since torque is a product of a force and a distance, its unit is any force unit times any distance unit. The pound-foot is the usual unit in the British system. The cgs unit of torque is the centimeter-dyne. In laboratory work the centimeter-gram weight is often used.

Although torque is a vector quantity, in this exercise only forces acting in the same plane will be used. For such forces only the algebraic signs of the torques need be considered. The algebraic sign of a torque is determined by consideration of the direction of the rotation the torque tends to produce. For example, the torque in Fig. 1 (a) tends to produce counterclockwise rotation about O, while the torque in Fig. 1 (b) tends to produce a clockwise rotation. One may refer to these torques as positive and negative, respectively. Note that a given force may produce a clockwise torque about one axis, but a counterclockwise torque about another axis. The direction of a torque is not known from the direction of the force alone.

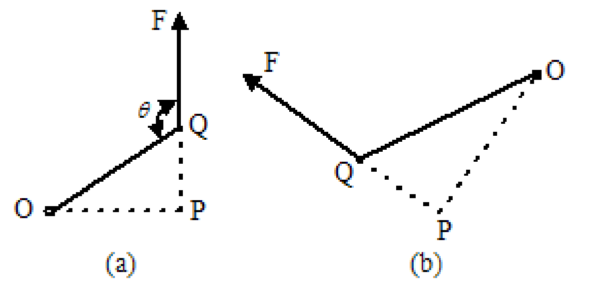

Fig. 1. Algebraic signs of torques. The torque in each case is F x OP. In (a) the torque is counterclockwise (positive), while in (b) the torque is clockwise (negative).

It is important to note that the moment arm of a force is not always the distance from the axis of rotation to the place where the force is attached. Nor indeed is the moment arm always measured along the body acted upon. For example, in Fig. 1 the body OQ may be a meter stick but, the moment arm OP is not measured along the stick in either case.

An object is said to be in equilibrium when there is no change in its linear or rotary velocity. The *first condition for equilibrium* refers to the fact that the vector sum of all the forces acting upon a body must be zero if there is to be no change in the linear velocity of the body.

For an object to be in *equilibrium* with respect to rotary motion when it is under the action of forces in a single plane, the algebraic sum of the torques (about *any* axis) that tend to produce clockwise motion must be equal to the algebraic sum of the torques that tend to produce counter-clockwise motion. In other words the algebraic sum is known as the *second condition for equilibrium*. It may be represented by the equation

$$L = 0 \qquad (2)$$

The most common force acting upon a body is its weight. For any ordinary object, no matter how irregular its shape, there exists a point such that the entire weight may be considered as being concentrated at that point. This point is called the *center of gravity* of the body. The center of gravity may be either within or outside the body, depending upon the shape of the body. If a single force equal to the weight of the body and acting vertically upward could be applied at the center of gravity, it would support the body in equilibrium no matter how the body might be tipped about the center of gravity.

A knowledge of the position of the center of gravity is very useful in problems of equilibrium, for that place is the point of application of the force vector representing the weight.

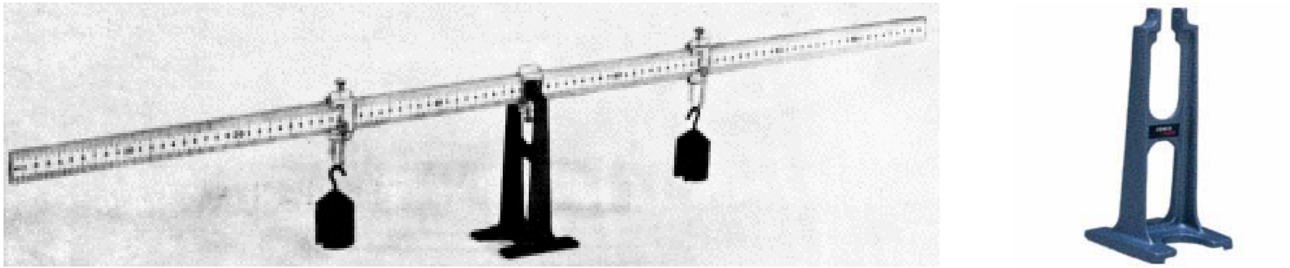

Fig. 2. Demonstration balance.

When the axis of rotation does not pass through the center of gravity the moment of force produced by the weight of the body must be taken into consideration.
The solution of problems involving static equilibrium is simplified by drawing a suitable diagram and the careful choice of an axis of rotation. The best choice is usually a point through which one or more unknown forces act. This point mayor may not be the actual pivot, or the center of gravity of the body. Particular attention must be given to the correct evaluation of the moment arms of the various forces and to the algebraic signs of the torques.

APPARATUS: The principal piece of apparatus (Fig. 2) is called a "demonstration balance." It consists of a meter stick supported on a knife-edge clamp in a heavy iron support. Weight hangars provided with set screws for clamping to the meter stick, provide convenient methods for attaching forces to the balance. A set of hooked or slotted weights, two weight holders, a 250gm spring balance, and a pair of trip scales are needed.

PROCEDURE: 1. Locate the center of gravity of the meter stick by placing it in the clamp and adjusting it until the stick is in equilibrium. Record this position in the data record as the center of gravity of the meter-stick system.

2. Attach a load of about 250gm-weight on the meter stick at a distance of about 15cm from the fulcrum. (Include the weight of the clamp and weight holder in all of the forces in this exercise.) Then attach a weight of about 150gm-weight to the other side of the fulcrum and adjust the position of its clamp until equilibrium is restored. Record the forces and their moment arms.

3. Calculate and record the clockwise torque and the counterclockwise torque. How do they compare? Record the percentage difference between them.

4. Slide the knife-edge clamp as far as possible toward the zero end of the meter stick. Record this position as the axis of rotation. With a loop of thread and a spring balance apply a force near the 100cm end of the meter stick so that equilibrium is produced. Record the force exerted by the spring balance and it's moment arm. The opposing force in this case is the weight of the meter stick as previously located at the center of gravity. Record the moment arm of this weight. From the law of moments calculate the weight of the meter stick. Then weigh the meter stick on the scales and record the percentage difference between the two values.

5. With the fulcrum arranged as in step 4, apply a force of about 300gm-weight at a distance of about 20cm from the fulcrum to exert a force that will produce equilibrium. Record the observed values. Taking into account the measured weight of the meter stick, check the law of moments by noting whether $\sum L = 0$.

6. Fasten the zero end of the meter stick with a loop of thread to a fixed object on the table. Arrange the spring balance at about the 40cm position and a load of about 250gm-weight at about the 90cm position on the meter stick. Raise the spring balance until the meter stick is horizontal and record its reading and the other observed data. Taking into consideration the measured weight of the meter stick calculates the various torques and check the law of moments.

7. Use the demonstration balance to measure the weight of a book or some other "unknown" weight. What is the simplest form of your balance for this purpose?

QUESTIONS: 1. Where is the center of gravity of a doughnut? Explain your reasoning.

2. What relationship exists between a force and its moment arm when the force is varied so as to maintain a constant torque?

3. Describe two or three ways by which an experimenter could locate the center of gravity of an irregular body.

4. Use the data from the first part of step 2 of the procedure of this experiment. What must be the upward force exerted by the fulcrum?

5. Use the data from question 4 and calculate the sum of the clockwise and the sum of the counterclockwise torques about the zero end of the balance. What conclusions can you deduce from your comparison of these sums?

6. Show clearly from your results in this experiment that the position of the center of gravity of the meter stick does not depend in any way upon the position of the fulcrum or upon the magnitude and location of the forces applied to the meter stick.

7. Well-made beam balances, or trip scales, are carefully. manufactured so that the lever arms of the platforms are identical. Suppose the two-pan balance of a grocer's scales are poorly made, so that the balance arm on the right is longer than that on the left. To compensate for this the grocer adds a lead weight to the underside of the pan on the left, When groceries are placed on the left pan and balanced by weights on the right pan; is anyone cheated and, if so, who?

8. Explain why a dressmaker's scissors have long blades and short handles and the reverse is true for a tinsmith's shears.

9. Cite examples to show why some levers are very advantageous although the input force must be much larger than the resisting force.