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COEFFICIENT OF FRICTION

OBJECT: To determine the values of the coefficient of kinetic friction and the coefficient of static friction for two particular surfaces.

METHOD: A loaded block is placed on a horizontal surface and the tractive force for each load, required to pull the block along the plane, is determined. These observations are plotted and the values of the coefficient of kinetic friction and the weight of the unloaded block are determined from the graph. The plane is then inclined so that the block slides down it without acceleration and the coefficient of kinetic friction redetermined from the angle the plane makes with the horizontal. The coefficient of static friction is determined in a similar manner. The block is placed on the plane, the plane is gradually elevated, the angle at which the block begins to slide is noted and this angle is used to determine the coefficient of static friction.

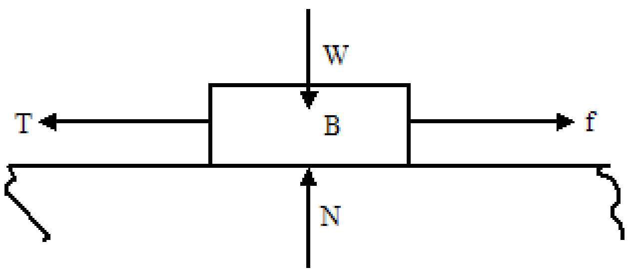

Fig. 1. A body moving with constant velocity is in equilibrium.

THEORY: If a body is at rest or moving with constant velocity, it is in equilibrium and the vector sum of all the forces acting on it is zero. It is also true that, if all the forces acting on the body are resolved into horizontal and vertical components, the sum of the horizontal components is equal to zero and the sum of the vertical components is equal to zero.

From this it follows that, if the body B (Fig. 1) is moving with constant velocity along the horizontal surface in the direction of the tractive force *T,* the force of friction *f* is equal and opposite to *T*. The force of friction always opposes the motion and is, therefore, always opposite in direction. Similarly, the upward thrust of the plane on the block (the force *N* normal to the plane's surface) is equal and opposite to the pull of gravity on the block (the weight *W*). Obviously, if the block is loaded, *W* includes the weight of the block and the weight of the load. Although *N* and *f* are here treated as two independent forces, the reaction of the plane may be thought of as a single force and the forces *N* and *f* as the normal and tangential components of this force. It is found experimentally that the force of friction *f* is proportional to the normal component *N* of the plane's reaction. Stated algebraically

$$f = \mu N \qquad (1)$$

where the constant of proportionality μ is called the coefficient of kinetic friction. In other words, μ is the factor by which the thrust of the plane must be multiplied to give the force of friction. Transposing terms in Eq. (1) yields

$$\mu = \frac{f}{N} \qquad (2)$$

From Eq. (2) it is clear that the coefficient of friction is the ratio of the two components of the reaction of the plane on the block and is a pure number. It is obvious that in the case above, where the body is pulled *without acceleration* along a *horizontal* surface by a tractive force *T*, the coefficient of friction is also equal to the ratio T/W, or the tractive force is μ *W*.

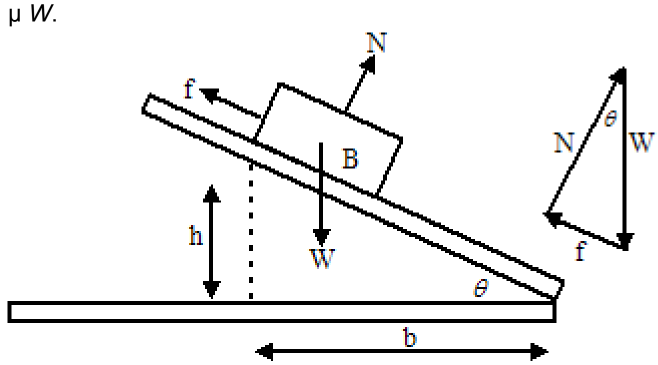

Fig. 2. If the body slides down the plane without acceleration, the vector diagram of forces is a closed triangle.

If a body slides down an inclined plane without acceleration, it is in equilibrium and the vector diagram of forces is a closed polygon. For example, if the body B (Fig. 2) is sliding down the incline with constant velocity, the vector diagram formed by the weight *W* of the block, the force of friction *f* and the normal component *N* of the plane's reaction is a closed triangle. Since the coefficient of friction is the ratio f/N, it follows from the similarity of triangles that

$$\mu = h/b = \tan\theta \qquad (3)$$

Assume that the body is at rest on the horizontal surface (Fig. 1) and that the force *T* (initially zero) is gradually increased. While the body remains at rest, *f* must be equal and opposite to *T*, so that as *T* increases *f* also increases. There is, however, for a particular weight and for a particular pair of surfaces, a maximum value f_o for the force of friction, and if *T* is greater (infinitesimally greater) than f_o the body starts to move. Since the force of friction is greater for starting than for sliding, once the body starts to move it will move with an acceleration. The coefficient of static friction μ_o is given by the equation

$$\mu_o = f_o / N \quad (4)$$

Fig. 3. The Inclined Plane

If the plane is gradually elevated it is found that there is a particular angle at which the block begins to move. This angle, the maximum angle that the plane can make with the horizontal with the block remaining at rest on it, is called the *limiting angle of repose*. The tangent of the limiting angle of repose is equal to the coefficient of static friction.

Leonardo da Vinci (1452-1519) was probably the first man to do experimental work on friction and along series of experiments since his day has led to the formulation of the following laws for the friction of solid surfaces: (1) The force of friction is proportional to the normal thrust of the plane. (2) Within wide limits it is independent of the area of contact. (3) It is nearly independent of the speed. (4) It depends upon the condition of the surfaces and in general is greater when the two bodies are made of the same material than when they are dissimilar.

APPARATUS: An inclined plane, a block of wood and a box of hooked weights are required.

The inclined plane is shown in Fig. 3. In use the base rests on a horizontal surface and the angle the plane makes with the horizontal is read from the circular scale. For friction measurements the ratio of the height *h* to the base *b* (Fig. 2) may be determined from the two straight scales, the vertical one and the one on the plane. To determine *b* the plane is lowered to the horizontal position. If the surface of the plane is painted or rough, a smooth flat plane should be laid or clamped on it.

When used in the horizontal position the sliding block is

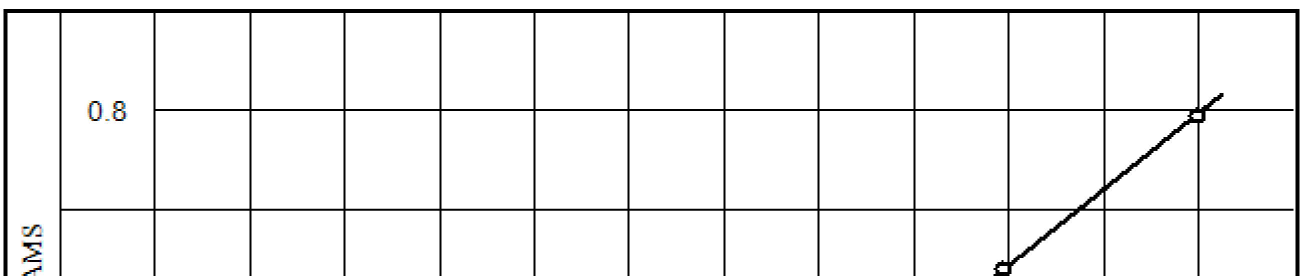

Fig. 4. The force of friction is proportional to the normal component of the plane's reaction.

pulled by hooked weights that are attached to the block by means of a string which passes over the pulley. The pulley is mounted in cone pivot bearings and runs with very little friction.

PROCEDURE: With the plane in the horizontal position, place the block on the plane and pass the attached string over the pulley to a hooked weight. Adjust the pulley so that

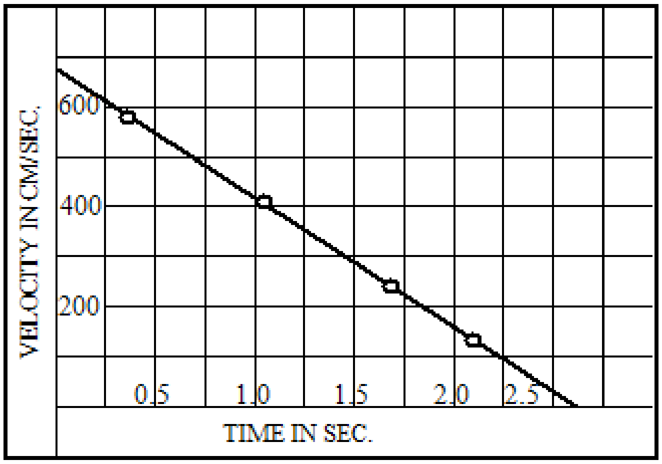

Fig. 5. The velocity of a body, sliding on a horizontal surface decreases uniformly with time.

the string is accurately parallel with the plane. Add weights until the force is just sufficient to keep the block, when once started, moving slowly and uniformly over the surface. Since all parts of the surface will not have the same coefficient of friction, the experiment should be limited to a certain part, say the middle section, of the plane. Repeat the experiment with 1, 2, 3 and 4 kilogram weights resting on the block. Plot the results on a graph similar to the one shown in Fig. 4. Interpret this graph and from the graph determine the weight of the unloaded block and the coefficient of kinetic friction for the pair of surfaces. Check the weight of the block on a balance.

Elevate the plane to the position where the block, once started, will slide down the plane without acceleration. This part of the experiment should be performed on the section of the plane used in the previous case. Read the angle of inclination from the circular scale on the plane and use Eq. (3) to determine μ. Compare with the result previously determined.

Place the block on the plane and gradually elevate the plane until the block begins to slide. Clamp the plane in this position and read the value of the limiting angle of repose from the circular scale. Determine the coefficient of static friction.

Suggestion: For those not familiar with the use of trigonometric tables, the ratio h/b may be determined from the two straight scales on the plane. The distance *h* is read from the vertical scale and the distance *b* (from the vertical scale to the hinge) is read directly from the scale on the plane when the plane is in the horizontal position.

QUESTIONS: 1. A certain block rests on a horizontal surface and a horizontal force just sufficient to start the block is applied to it. What is the acceleration of the block? Assume that the coefficient of kinetic friction is 0.18 and the coefficient of static friction is 0.26.

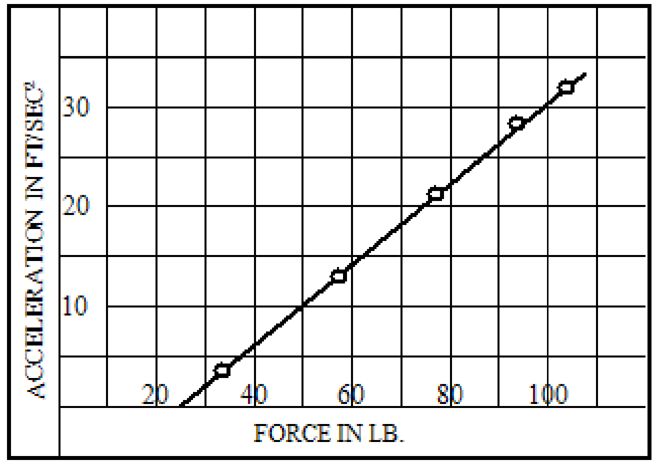

Fig. 6. The acceleration of a body is proportional to the net force acting on it.

2. Why does a locomotive engineer shut off the steam when the drivers begin to spin?

3. Why is it unwise to apply the brakes of a car suddenly on a slippery pavement?

4. Let the two hands support a meter stick, one at the 10cm mark and one at the 70cm mark. Bring the two hands together and note the point on the meter stick at which they meet. Try starting the hands at various points on the stick. Explain the results.

5. What force is required to pull a 150-pound sled along a horizontal surface? The coefficient of friction is 0.10 and the sled is pulled by a rope which makes an angle of 30" with the horizontal.

6. A shuffleboard piece is given a shove on a horizontal deck. The manner in which its velocity changes after leaving the shovel is shown graphically in Fig. 5.5. What is the coefficient of friction?

7. A box rests on a level floor. Various horizontal forces are applied to the box and the corresponding accelerations determined. These results are plotted in Fig. 6. What is the coefficient of friction?

8. A 100-pound box is pulled up a plane inclined at an angle of 10˚ with the horizontal. If the coefficient of friction is 0.20, what force (parallel to the plane) is required? What force is required to pull the box down the plane?