Selective Experiments In Physics

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CENTRIPETAL FORCE

OBJECT: To make a study of the motion of a body traveling with constant speed in a circular path, and to verify the expression for centripetal force.

METHOD: By means of an electrically driven rotator a body of known mass is rotated about a vertical axis in such away as to produce a definite extension of a spiral spring. From the mass of the body, the radius of the circular path, and the speed of rotation, the centripetal force is computed and compared with the gravitational force necessary to produce the same extension of the spring.

THEORY: Consider the motion of a body of mass *m* about a point O in a circular path of radius *r* (Fig. 1). At any given

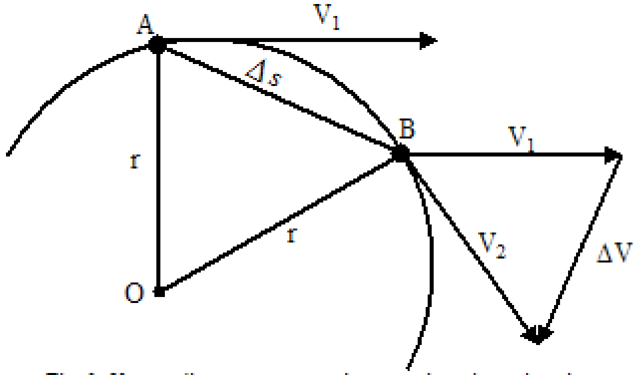

Fig. 1. Vector diagram representing centripetal acceleration.

instant the body at A has a linear velocity v_1 tangent to the circle and it would retain this linear velocity, in accordance with Newton's first law of motion, if not acted upon by external forces. The application of a force along the path (tangent to the circle) would increase or decrease the speed of the body, while a force acting normal to the path (along a radius) would produce a constantly changing direction of motion leaving the speed unaffected.

Suppose the body in Fig. 1 to be traveling in its circular path with constant speed. It has then no acceleration along its path but, since the direction of travel is changing constantly, the velocity is non-uniform and it must have an acceleration normal to path to account for the changing velocity. This radial acceleration is directed toward the center of the circular path and is called *centripetal acceleration*.

According to Newton's second law, the force f required to impart to a mass man acceleration *a* is

$$F = kma \qquad (1)$$

where *k* is a constant of proportionality, the value of which depends upon the units used. In this experiment, the c.g.s. absolute system will be used and *k* will be unity. The centrally directed force *f* on the mass m is called *centripetal force*. In accordance with Newton's third law, the mass *m* exerts an equal and opposite force upon its restraints; this force is referred to as *centrifugal force*. It must be clearly understood that the centripetal force and the centrifugal force do not act upon the same body. An expression will now be derived for the centripetal force. In a small interval of time Δt, let the body move along the arc AB a distance $\Delta s = v \cdot \Delta t$. The linear velocity has the same magnitude at A and B but in going from A to B the direction of motion has changed, the vector difference between v_1 and v_2 representing the change in velocity, $\Delta v = a \cdot \Delta t$. This vector difference is shown in the vector diagram of Fig. 1. Then by similar triangles

$$\frac{\Delta v}{\Delta s} = \frac{a \cdot \Delta t}{v \cdot \Delta t} = \frac{v}{r}$$

whence

$$a = \frac{v^2}{r} \qquad (2)$$

The approximation involved in taking the chord *AB* equal to the arc disappears as the time interval Δt is made vanishingly small. Furthermore, as Δt approaches zero the vector Δv becomes more nearly parallel to *r*. Hence the acceleration, the direction of which is in the direction of the change in velocity, is directed toward the point O. Substitution of the value of a from Eq. (2) in Eq. (1) yields the following expression for the centripetal force

$$F_c = m\frac{v^2}{r} \qquad (3)$$

In Eq. (3) the centripetal force is expressed in terms of the linear velocity of the body. It may be expressed in terms of angular velocity ω by substituting the relationship $v = \omega r$. Thus

$$F_c = m\omega^2 r \qquad (4)$$

ω being measured in radians per second. Usually the frequency of rotation *n* is more readily observable than either the linear or the angular velocity. Substituting $v = 2\pi n r$ in Eq. (3) yields

$$F_c = 4\pi^2 n^2 r m \qquad (5)$$

In accordance with Hooke's law the external force *F* necessary to produce a deformation din an elastic body is given by the equation

$$F = kd \qquad (6)$$

where the "force constant" *k* is defined numerically as the force necessary to cause unit deflection. The value of k depends upon the geometry of the body and the material of which it is composed. For example, the force constant of a coil spring depends upon the dimensions of the coil, the size of the wire, and the elastic property of the material.

APPARATUS: The apparatus required for the performance of this experiment consists essentially of two units: a specially designed centripetal force apparatus (Fig. 2) and an electrically driven, variable speed rotator (Fig. 3). The complete assembly with the centripetal force apparatus mounted on the spindle of the rotator is shown in Fig. 4.
The centripetal force apparatus consists of a metal frame Y within which is mounted a cylindrical mass m attached to a coil spring Z, the entire assembly being rotated about a vertical axis through its center of gravity. The tension in the spring is adjusted by a threaded collar K to which the spring is fastened. In some models the position of the collar is indicated by a millimeter scale S attached to the frame. By means of three guide rods G the cylindrical body is constrained to move only along the axis of the spring, which axis intersects the axis of rotation at right angles. While at rest the cylinder is held by the spring against a stop.

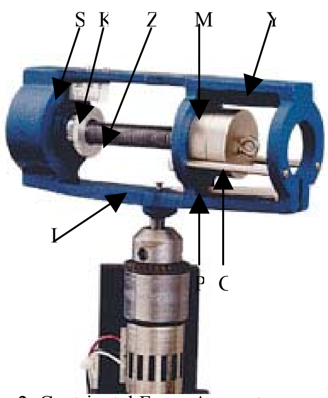

Fig. 2. Centripetal Force Apparatus

When the apparatus is rotated about a vertical axis the mass moves outward producing an extension of the spring. The situation is represented diagrammatically in Fig. 5 in which the identifying symbols correspond to those in Figs. 2 and 4. A specially designed pointer P is loosely pivoted at O and is so shaped that when the cylinder presses against it at Q its tip moves upward through a range of about 5mm. At the middle of this range is a fixed index I. In operation the speed is adjusted until the pointer is opposite the index. Since the index is practically on the axis of rotation, the position of the pointer can be seen clearly while the apparatus is rotating.
The frequency of rotation is determined by counting the number of revolutions occurring in a given interval of time. This observation is facilitated by means of a revolution counter C attached to the frame of the rotator by means of a steel spring which normally holds the counter disengaged from the rotating spindle. By pressing with the finger on the end of the spring the counter gear is engaged with an identical gear on the spindle. The speed of rotation of the spindle is controlled by adjusting the point of contact of the friction disk D with the driving disk W. Turning the milled head H of the screw J carries the friction disk in or out along the radius of the driving disk.
As auxiliary apparatus there are needed a stopwatch or a clock with a sweep seconds hand, a weight holder, miscellaneous weights, a vernier caliper, and rods and clamps for suspending the centripetal force apparatus as shown in Fig. 6.

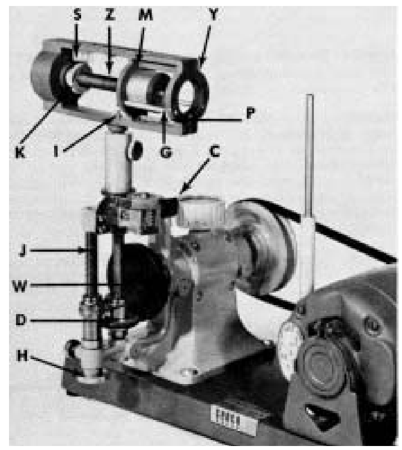

Fig. 4. Centripetal Force Apparatus mounted on Variable Speed Rotator.

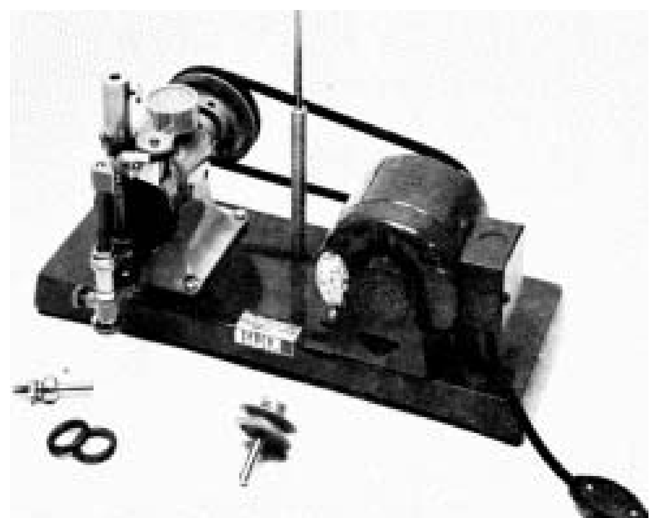
Fig. 3. Variable Speed Rotator

PROCEDURE:
Experimental: By means of the threaded collar adjust the spring to minimum, or nearly minimum, tension. Mount the centripetal force apparatus securely upon the rotator spindle taking care that the axis of rotation is vertical. Set the friction disk so that it is near the center of the driving disk and start the motor. With the eyes on a level with the index, adjust the speed control device until the pointer is just opposite the index. With one hand constantly on the control, practice regulating the speed until sufficient skill is acquired to keep the pointer vibrating about the index as its mean position

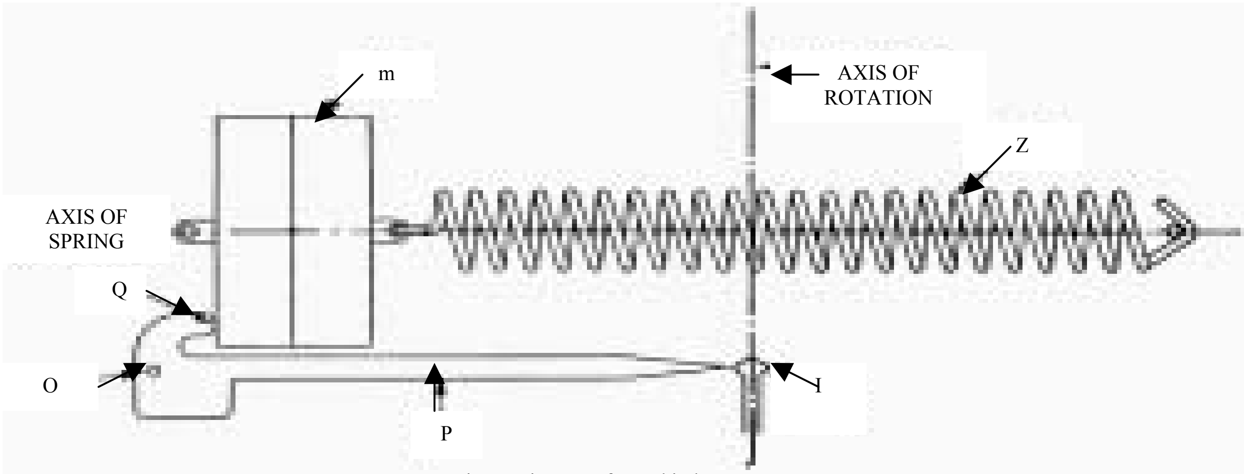

Fig. 5. Diagram of speed index.

the pointer vibrating about the index as its mean position with as little oscillation as possible. When some skill in manipulation has been acquired a run may be made. Record the reading of the revolution counter and, keeping one hand on the speed control, engage the counter with the other. When two experimenters are working together it is a good plan for one to regulate the speed while the other manipulates the counter and observes the time. At the end of one minute disengage the counter and record the reading. The counter may be prevented from spinning after it is disengaged by applying the finger lightly to the gear as a brake at the instant of release. Make this observation for 5 one-minute intervals, taking care that the counter reading is not altered between observations. This data yields 6 readings of the counter spaced at one-minute intervals. Record these 6 readings consecutively in a column.

The second part of the experimental procedure consists in determining the gravitational force necessary to produce the same extension of the spring as that caused by rotating the apparatus. To make this determination remove the centripetal force apparatus from the rotator and suspend it with the mass down as in Fig. 6. Attach a weight holder and add weights until the pointer is again brought to the index. The mass is then in the same position in the frame and hence the force in the spring is the same as when the apparatus was rotating. Record the total weight sustained by the spring including that of the weight holder and of the cylindrical body. The mass of the latter is stamped on it. With the vernier caliper measure the distance r between the axis of revolution (indicated by the scribed line L_1 on the frame) and the center of gravity of the cylinder (indicated by the line L_2). Repeat this measurement several times and record the values.

Change the tension of the spring and repeat the entire experiment.

Calculations: In computing the average value of the frequency of rotation, divide the data into two parts consisting, respectively, of the first three and the last three readings of the counter. The difference between the 4th and the 1st is the number of revolutions occurring in three minutes. Likewise the differences between the 5th and 2nd and between the 6th and 3rd represent three-minute intervals. The data thus represents three over-lapping three-minute time intervals. Note that this method of averaging makes use of all the data, whereas, if the simple arithmetic mean of the 5 one-minute intervals had been taken, all readings except the first and the last would have dropped out. Express the average value of the frequency in revolutions per second. Compute the average value of *r* from the values obtained in the second part of the experiment. Substitute the values of mass, frequency, and radius in Eq. (5) and compute the centripetal force in absolute units. Compute the percentage difference between this force and the weight (also in absolute units) required to produce the same extension of the spring.

ALTERNATIVE PROCEDURE:

Supplementary Theory: The relationship between the force in the spring and the frequency of rotation can be shown graphically. Solving Eq. (5) for n^2

$$n^2 = \frac{1}{4\pi^2 mr} \cdot F_c \tag{7}$$

in which the force F_c satisfies Eq. (6). In this experiment the radius *r* is kept constant, the extension of the spring being taken up by the collar K. Since the pitch of the screw is constant, the force in the spring is directly proportional to the number of turns *N* of the collar. Thus

$$F_c = F_o + k'N \tag{8}$$

where F_o is the initial force in the spring corresponding to the (arbitrary) "zero" setting of the collar, and *k*' is a constant for the spring, namely, the force in the spring for one turn of the collar. Note that *k*' in Eq. (8) and *k* in Eq. (6) are simply related, being merely two equivalent ways of expressing the force constant of the spring. Substituting F_c from Eq. (8) in Eq. (7)

$$n^2 = \frac{1}{4\pi^2 mr}\left(F_o + k'N\right) \tag{9}$$

or

$$n^2 = C\left(F_o + k'N\right) \tag{10}$$

where

$$C = \frac{1}{4\pi^2 mr} \qquad (11)$$

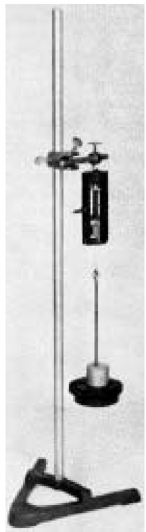

Fig. 6. Arrangement for the application of gravitational forces.

Consideration of Eq. (10) shows that a graph of experimental values of n^2 versus N yields a value of the force constant k' of the spring. This might be called a dynamical method of determining k'. A direct static method of determining k' consists in measuring the weight necessary to balance the elastic force in the spring for each turn of the collar.

Experimental: Having learned how to manipulate the apparatus according to the procedure described above, adjust the spring for (nearly) minimum tension, and identify the initial setting of the collar K either by observing its position on the scale S, or by measuring the distance between it and the end of the frame. Determine the frequency by observing the number of revolutions recorded by the counter in 2 minutes. Since the results are to be determined from a graph of the data involving several independent observations, it will be sufficient to take the average value of the frequency over a two-minute interval. Make four such determinations, each time increasing the spring tension by giving the collar a certain number (say 5) complete turns. Tabulate the data.
Remove the centripetal force apparatus from the rotator and suspend it as shown in Fig. 6. Attach a weight holder of known mass. With the same initial spring setting as before, add weights to the holder until the pointer is opposite the index. Increase the tension by giving the collar a certain number (say 5) whole turns and again observe the total weight W (including the holder) necessary to bring the pointer to the index. In this manner obtain several corresponding values of W and N.

Analysis of Data: Plot curve I, as shown in Fig. 7, taking the square of the frequency n^2 as the ordinate and the total number of turns N of the collar (measured from the initial position) as the abscissa. Plot curve 2 (not illustrated) taking the weight W as ordinate and number of turns N as abscissa. Consideration of Eq. (10) shows that when $N = 0$,

$$F_o = \frac{n_o^{\ 2}}{C} \qquad (12)$$

where n_o is the frequency corresponding to the initial setting of the collar. Thus, the n^2-intercept of curve 1 yields the initial force F_o in the spring. The initial force in the spring may also be obtained from the W-intercept

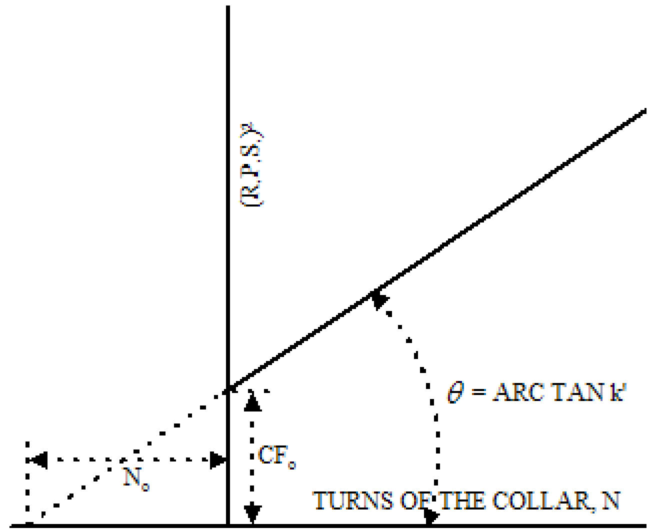

Fig. 7. Graphical method of analysis.

of curve 2. Compare the two values so determined, remembering to express F_o and W_o in the same units. Again referring to Eq. (10), note that when $n = 0$

$$N_o = \frac{-F_o}{k'} \qquad (13)$$

where N_o is the (negative) number of turns from the initial setting necessary to cause zero force in the spring when the mass m is in the operating position. Thus, the N-intercept of curve), together with the value of F_o determined from Eq. (12), yields the force constant k' of the spring. Compare the value so determined with that obtained from the slope of curve 2.

QUESTIONS: 1. Which would be the more serious, a 1 per cent error in observing the time, or a 1 per cent error in measuring the radius? Why?

2. What are the advantages of using the same value of r and varying the spring tension for various speeds rather than keeping a constant spring tension and observing the values of r at different speeds?

3. Could a horizontal axis of rotation be used? Explain.

4. What was the angular velocity in rad/sec of the rotating body in each case?

5. How is the centripetal force on a rotating body affected by (a) doubling the radius, keeping the linear velocity constant; (b) doubling the radius, keeping the angular velocity constant?