• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

CENTRIPETAL FORCE BY GRAPHICAL INDUCTION

OBJECT: To determine empirically the functional relationship of the centripetal force (a) to the mass of the moving body, (b) to its tangential velocity and (c) to the radius of the circular arc in which the mass moves.

METHOD: The centripetal force acting on a pendulum bob, when the bob passes across its rest position, is measured directly with an equal arm balance. Data are taken to show how this force is dependent on the mass of the bob, on the radius of the arc, and on the tangential velocity of the bob. The centripetal force equation is derived from these data and the graphs are drawn.

THEORY: In 1687 Sir Isaac Newton announced his famous three laws of motion. The second law, $F = ma$, expressed the fact that a force is required to accelerate a given mass. Acceleration is defined as the ratio of change in velocity to time involved. Since velocity is a vector quantity, this change in velocity may be either one of magnitude (speed), or of direction, or a combination of both.
When the path of motion of a mass is constrained to a circular arc (Figure 1), the direction of motion is continuously changing. The force producing this directional change acts along the radius of the arc and points toward the center of rotation. This force, therefore, is designated the radial, or the *centripetal*, force.

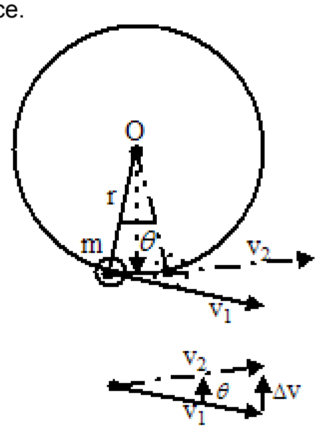

Fig. 1. Change in velocity Δv for circular motion.

In Figure 1, the mass m has a velocity of v_1 tangent to the circular arc at that point. Assume that a moment later, after traversing the small angular displacement θ, the mass had the velocity of v_2. When v_2 has the same magnitude as v_1 the change in velocity Δv is a direction change only. Thus the force constraining the mass to the circular arc is the centripetal force

$$F_r = m\frac{\Delta v}{\Delta t}$$

The moving object of Fig. 1 may be the bob of a simple pendulum swinging in the circular arc *ABC* or radius *OA*, depicted in Fig. 2. The speed of the bob varies from zero at the end positions to a maximum velocity v_B at its lowest or rest position. At the rest position there is momentarily no force to change the tangential velocity of the bob. At this position the string must provide the supporting weight force of the bob and the centripetal force associated with the circular arc path. Thus the string tension force less the weight force of the bob is the acting centripetal force.

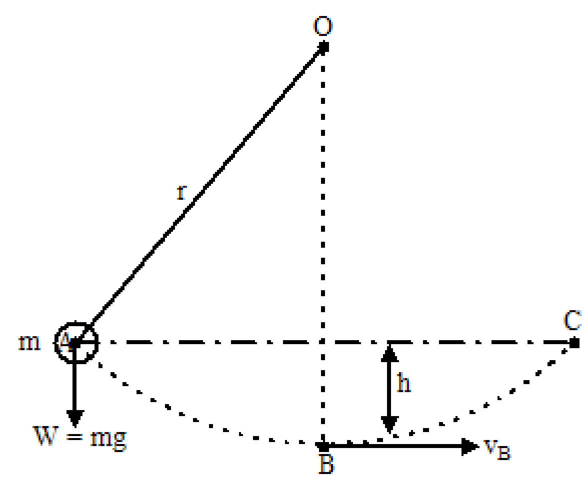

Fig. 2. Motion described by a simple pendulum.

The variables which might influence the magnitude of the centripetal force are (a) the mass of the bob, (b) the radius of the arc, and (c) the instantaneous velocity of the bob at its bottom position.
To the degree that a swinging pendulum has a negligible air resistance to dissipate energy, and this resistance is ordinarily accepted as negligible, the sum of the kinetic energy plus the potential energy of the bob remains constant throughout the bob's swing. At the end positions *A* and *C* the energy is all potential and is equal to $E_p = mgh$. At the lowest position it is all kinetic and is equal to $E_k = \frac{1}{2} m v_{B^2}$.
Therefore, equating the two,

$$\frac{1}{2}mv^2 = mgh$$

$$v_B = \sqrt{2gh}$$

where v_B is the tangential velocity of the bob.
The object of this experiment is to find the functional relationships of *m, v*, and *r* to the centripetal force F_r and, thereby, obtain the laboratory equation. Measurements will be made of the relationship of F_r to each of these factors in turn, holding the other two factors constant. The data will then be graphically analyzed.

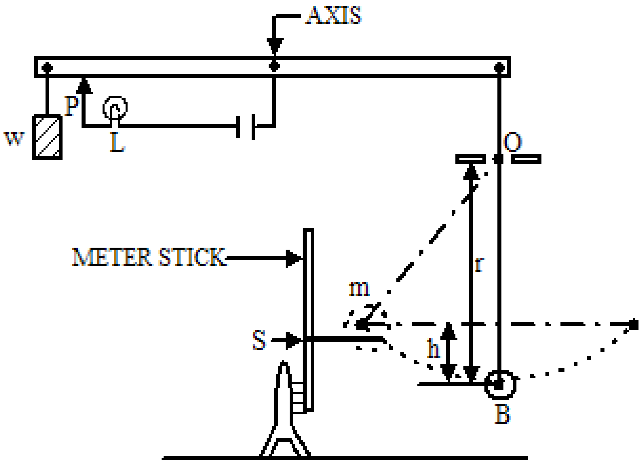

Fig. 3. Schematic view of apparatus for measuring centripetal force.

To facilitate these measurements an equal arm balance is used. See Fig. 3. A pendulum, consisting of a mass tied to a cord, is attached to one end of the balance arm. A balancing load *W* may be applied to the opposite end of the arm to measure the tension force in the pendulum cord. A light *L* in the electrical circuit glows as long as the weight force of the balancing load *W* can close the electrical contact at *P*. Thus, the pendulum bob can be weighed by finding the minimum value of *W* needed to close the contact when the pendulum hangs at rest.
When the pendulum bob is placed on the starting platform *S* and then released by pulling the starting platform to the left, the bob will swing on the arc of radius *r*. The velocity of the bob at *B* is dependent on the initial height h through which the bob drops. The light will then blink each time the bob passes *B* because of the centripetal force, which adds to the weight of the bob. The minimum load which must be added to *W* to stop the blinking gives the value of the centripetal force producing the circular motion.

APPARATUS: Centripetal force balance (Fig. 4), three different mass bobs, measuring stick and attached starting platform, weight hanger and set of weights ranging from one to five grams, battery for the electrical circuit.

PROCEDURE: Arrange the apparatus as shown in Fig. 3. The desired value of r is obtained by adjusting the position of the plate containing the string guide hole. The radius *r* is measured by means of the starting platform and its meter-stick support.

Note that the pin set through the bob locates its center of gravity. This pin rests on the starting platform *S.* See also

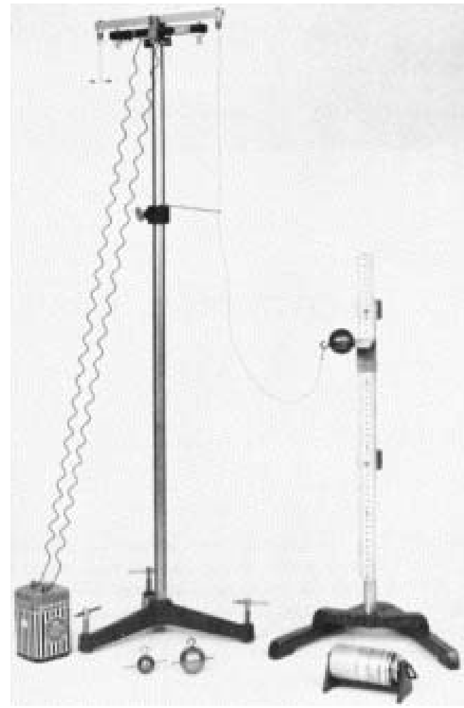

Fig. 4. Pictorial of Centripetal Force Apparatus

Fig. 5. When the starting-platform tripod is pulled gently to the left, the bob is released to swing through its predetermined arc. Thus force readings may be made repeatedly. Care must be taken to minimize vibrations which would cause a false measurement of the balance force.

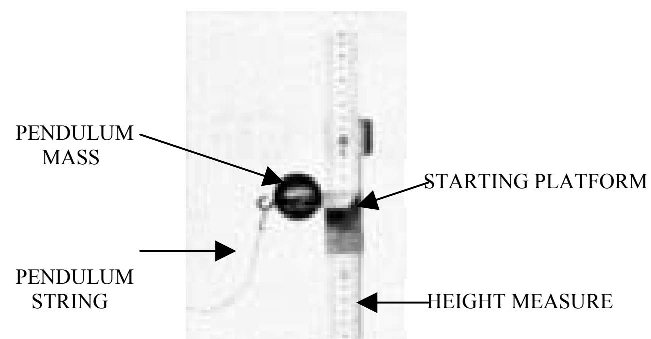

Fig. 5. Starting-platform design.

EXPERIMENT:
Centripetal force as a function of *v*; *r* and *m* constant. Hang a pendulum bob on the balance. Adjust the plate containing the string guide hole so that the pendulum string centers in the hole and radius r has a value of about 70cm.
Measure *r* using the starting-platform meter stick. Determine the weight of the pendulum bob.
Using a total load of approximately twice the weight of the bob, find the minimum value of *h* (use starting platform as shown in Fig. 5), which causes the light to flash only on the first two swings of the pendulum. Record F_r and *h*.

Set the starting platform, in turn, for heights which are 3/4, 2/4 and 1/4 of the value of h previously determined. Find the corresponding values of F_r, using the method stated, for each of these heights of fall.

Compute the values of v and v_2 for each h value and plot F_r vs. v and F_r vs. v_2.

What must be the relationship of the tangential velocity to the centripetal force?

Centripetal force as a function of *r*; *m* and *v* constant. Using the same pendulum as in the previous experiment, adjust the starting platform for a drop h of about 15cm. Record h and measure F_r.

Move the plate containing the string guide hole to positions, in turn, at which r = 3/4, 2/4 and l/4 of the value just used. Find for each r the corresponding value of centripetal force.

Plot a graph of F_r vs. r.

What conclusion can be drawn from this curve?

Centripetal force as a function of *m*; *r* and *v* constant. Adjust the pendulum as used in Fig. 1. Set the starting platform at a fixed height h of about 30cm. Record h. Measure, in turn, the centripetal force for each mass bob provided.

Plot F_r vs. m.

What conclusion can be drawn from these data?

Relationship of Variables. Relate the three variables by a single expression, namely, F_r in as much? Using your data find the value of the proportionality constant and write the laboratory equation.

QUESTIONS: 1. A car traveling on a circular track has a constant speedometer reading of 30 miles per hour. Is the car accelerated? State reason for the answer given.

2. The car in question 1 weighs 3200 pounds and the track has a radius of 1000 feet. What is the centripetal force acting on the car when its speed is 30 miles per hour?

3. By what factor must the centripetal force of question 2 be multiplied to give the new value of the centripetal force when

(a) the mass of the car is doubled?
(b) the radius of the track is doubled?
(c) the speed of the car is doubled?

4. Assume that in Fig. 3m = 200gm, r = 60cm, and h = 20cm. What is the tension in the cord immediately after the bob is free from the starting platform? What is the magnitude of the centripetal force?

5. Using the values m, r, and h of question 4 compute the tension in the cord when the mass has descended a distance $h/2$. (Suggestion-construct the several force vectors involved.)

6. Suppose that a rigid horizontal pin were placed at a point $h/3$ directly above B (Fig. 3) so that the cord of the released pendulum would strike the pin. Describe the resulting motion.

7. A mass of x kg placed on a frictionless horizontal surface is constrained by a cord to move in a circular path at a constant speed. The centripetal force is 180 newtons. How much work is done during the time the mass moves a distance of 50 meters? Support your answer with an explanation.

8. Suppose an astronaut were placed in orbit inside a spinning hollow sphere having a radius of 20 feet. At what speed must the sphere spin to give him his earth "weight" effect? Describe his experiences as he moves about inside the sphere.

9. Assume that the earth is a perfectly uniform revolving sphere with a radius of 4000 miles. A spring balance registers a chunk of gold as "1 pound" at the geographic North Pole. What "weight" reading would the spring balance give for the gold chunk at the equator?