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THE ACCELERATION OF A FREELY FALLING BODY

OBJECT: To study the motion of a freely falling body; in particular, to measure *g*, the acceleration due to gravity.

METHOD: An object is allowed to fall freely, and its positions at the ends of successive equal intervals are recorded on a coated paper strip by means of electric sparks. From these data graphs of distance-time and velocity-time are plotted. The acceleration is determined from the slope of the velocity-time graph.

THEORY: The *average speed* $\bar{v}$ of a body is the quotient of the distance *s* which it traverses and the time *t* required to travel that distance. In symbols

$$\bar{v} = s/t \qquad (1)$$

The *instantaneous* speed *v* of an object is defined as the limit of this ratio as the time is made vanishingly small. Symbolically

$$v = \Delta s/\Delta t \qquad (2)$$

where Δs represents a small increment of distance traversed in the corresponding increment of time Δt.

In Fig. 2 the curve shows the distance-time relationship for a freely falling body. In any such curve Eq. (2) states that the instantaneous speed is given by the slope of a tangent drawn to the curve at the point for the instant in question. If the speed were constant the slope would be constant and the curve would be a straight line. For a freely falling body this is evidently not true, as the speed, and hence the slope of the curve, is continually increasing.

When the velocity of a body varies, the motion is said to be *accelerated*. Acceleration is defined as the time rate of change of velocity; in symbols

$$\bar{a} = \frac{v_t - v_o}{t} \qquad (3)$$

where $\bar{a}$ represents the average acceleration of a body which changes its velocity from v_o to v_t in the time *t*. Since acceleration has the dimensions of a velocity divided by a time, the absolute unit in the metric system will be the centimeter per second per second and in the British system the foot per second per second; usually written, cm/sec^2 and ft/sec^2.

If a body moves in a straight line, making equal changes of velocity in equal intervals of time, its acceleration must be constant, and it is said to be moving with *uniformly accelerated* motion. This is the type of motion produced when a constant force acts upon a body which is free to move. The most common example of this is the motion of a freely falling body. This acceleration *g* is called the "acceleration due to gravity" and has a value of approximately 980cm/sec^2 or 32.2ft/sec^2.

The relationships between the three quantities velocity, distance, and time, in uniformly accelerated motion are readily deduced from the above definitions. Eq. (3) yields directly

$$v_t = v_o = at \qquad (4)$$

which expresses the dependence of v_t upon *t* in terms of the constants v_o and *a*. It is the equation of a straight line, the slope of which is equal to the acceleration.

Since for *uniformly accelerated* motion the average velocity during an interval *t* is the arithmetical mean of the terminal velocities, in view of Eq. (1),

$$s = \bar{v}t = \frac{v_t = v_o}{2}t$$

Substitution of v_t from Eq. (4) yields

$$s = v_o t + 1/2\,at^2 \qquad (5)$$

When v_o = 0, Eq. (5) shows that the distance-time curve is a parabola. The slope of the curve at any point (slope of the tangent) is the velocity at the corresponding instant.

A velocity-time curve for a freely falling body is plotted in Fig. 3. The time interval *T* is the interval between two sparks. A sample record is shown in Fig. 7. Since the graph is a straight line the velocity changes at a uniform rate.

The slope of this curve $\Delta v/\Delta t$ is the acceleration. Since the slope is constant, the acceleration is constant. Hence the *average* velocity during the time interval is identical with the *instantaneous* velocity at the middle of that time interval.

Fig. 1. Free Fall Apparatus.

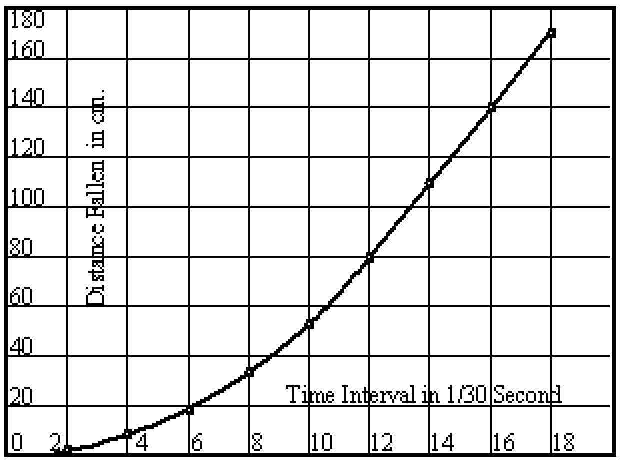

Fig. 2. Relationship between distance fallen and time of fall for a freely falling body.

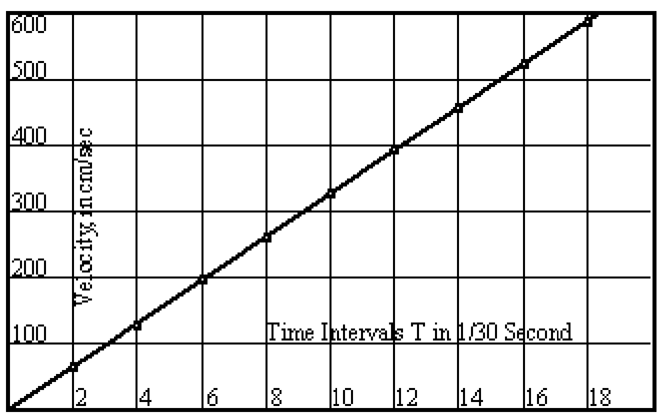

Fig. 3. Relationship between velocity and time of fall for a freely falling body

In the present experiment the value of *g* will be determined from the slope of such a velocity-time curve, as plotted from the experimental data.
The principal points in the preceding discussion may be summarized as follows:

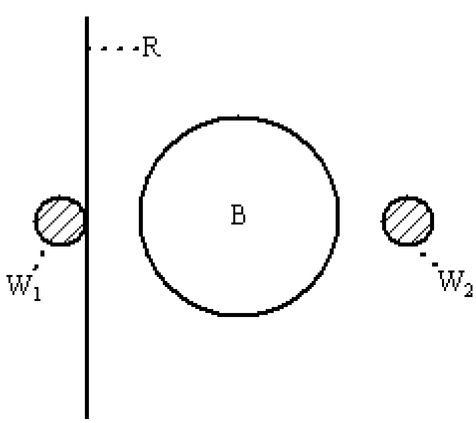

Fig. 4. Horizontal section through Free Fall Apparatus showing relative positions of (B) Falling Body, (R) Record Strip, (W_1) Rear Wire and (W_2) Front Wire.

(a) The average speed of a body is obtained by dividing the distance which it traverses by the time required to travel that distance.

(b) The instantaneous velocity of an object is the limit approached by the ratio $\Delta s/\Delta t$ as Δt approaches zero. This velocity is also equal to the slope of the tangent to the distance-time curve at the point in question.

(c) The acceleration of an object is the time rate of change of its velocity, or $a = \Delta v/\Delta t$. It is also the slope of the tangent to the velocity-time curve at the instant considered.

(d) For a constant acceleration, the velocity-time curve is a straight line and the *average* velocity of the body is also the *actual* (or instantaneous) velocity at the mid-point of the time interval used.

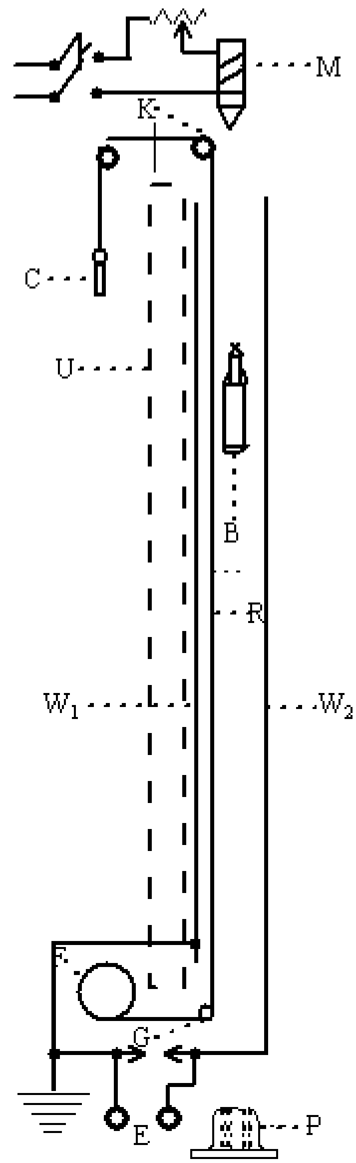

Fig. 5. Schematic diagram of the Free F
Apparatus showing the electrical circuit

APPARATUS: The apparatus consists of two principal units: the fall apparatus and the timing device. As auxiliary apparatus a 6volt storage battery, a 20ohm rheostat, a switch, a vernier caliper and a boxwood or steel metric scale are required.
The fall apparatus provides convenient means for holding the falling body suspended, for releasing it at will, for holding the record strip properly to receive the marks recorded during the fall, and for catching the falling body. The timing device is a unit which produces a series of intense sparks at equal time intervals. Its design permits of easy determination of the length of the interval between sparks.
The fall apparatus is designed so that the fall may be entirely unrestricted, save for air resistance. The marks which define the positions of the body are produced by equally-timed electric sparks which jump from a high-potential vertical wire to the falling body and thence through the record strip to a second vertical wire at ground potential. Fig. 4 shows a horizontal section of this part of the apparatus.

Fig. 6. Spark Timer, producing sparks at intervals of 1/60 second.

Fig. 5 shows a vertical section of the complete fall apparatus, arranged for operation. The falling body B is a metal cylinder and is shown falling, having been released by the electromagnet M. The latter is energized by current from a storage battery connected through a rheostat and switch. When it is desired to have the body fall, this current is interrupted. The apparatus is firmly secured to a vertical wall or supported from a substantial tripod base and carefully aligned so that the falling body, throughout its path, will remain uniformly distant between the rear wire W_1 and the front wire W_2, and finally fall accurately into the dashpot P. The latter has about an inch of sand in its bottom, and its sides are heavily lined with felt. A prepared paper, coated on one side with paraffin, constitutes the record strip. A roll of this paper is carried in a holder at F. When a record is to be made, the end of the strip is pulled through the opening at G, thence upward over the wire W_1 by a weighted clip C. The high-potential terminals of the sparking device are connected to E.

The Timing Device or Spark Timer, Fig. 5, consists of an electrically maintained vibrating steel bar provided with electric contacts for making and breaking a circuit at equal intervals, the length of one interval being the full period of the bar.

The sparking mechanism, Fig. 6, consists essentially of a buzzer type of mechanical interrupter vibrating synchronously with the 60-cycle alternating current power supply and thus interrupting the current at intervals of exactly 1/60 second. This interrupted current, in the primary of a step-up transformer, induces a high voltage in the secondary of the transformer, the design of the electrical circuit being such that a powerful spark occurs only at the instant of peak voltage. The usual modern a.c. power supply is so well regulated in frequency that it may be assumed to be exactly 60 cycles per second. The arrangements of the sparking mechanism are such that it is necessary merely to plug it into the a.c. power supply and to connect the high-voltage side to the terminal of the two wires at E in Fig. 5. A portion of the trace of the spark spots is shown in Fig. 7.

PROCEDURE:

Experimental: It will be assumed that the fall apparatus has been properly aligned so that the falling body will remain equally distant from the two wires and will accurately strike the center of the pocket. This important and delicate adjustment should be made only under the personal supervision of the instructor.

Energize the electromagnet by closing the switch connecting the storage battery through the 20ohm rheostat to the binding posts on the electromagnet. With all the resistance cut out, suspend the body from the electromagnet. Then, holding one hand just under the body, increase the resistance until it is released. Now decrease the resistance slightly until the body will just hang from the electromagnet. When the body hangs motionless, open the switch and the body should fall directly into the pocket. Use suitable precautions to prevent the body from becoming damaged by striking any object.

With the body *not hanging* draw the sensitized paper through the opening at the lower end of the casting, then upward and back through the upper opening. The light coated side of the paper must be on the outside. Attach the weighted clip to the end of the paper to hold it taut.

Make the necessary electrical connections to the timing device. Connect the high-potential leads to the fall apparatus, the frame of which should be grounded.

Suspend the body from the electromagnet and lightly touch the body until it hangs motionless. Start the spark timer. Observe that the sparks now jump from the outer wire through the body to the electromagnet and grounded support. When these conditions are realized, release the body by opening the switch in the electromagnet circuit.

When the body has fallen, stop the spark timer and examine the record of the dots on the paper. If any of the spots are missing, shift the paper to one side and repeat. Remove the record from the apparatus by drawing it upward, which at the same time puts a fresh strip in place. Repeat the process until each individual observer has one good trace.

Since in time intervals of only 1/60 second the spots are too close together for accurate measurements of the distances between individual spots, it is best to mark off and use only each *second* spot. A convenient plan is to scribe on the trace with a sharp pencil the ordinal number of each second spot, numbering these marked spots 0, 1, 2, 3, etc. Confusion between the different groups of traces may be avoided by numbering one group on the right and a second group on the left of the row of spots. For the second group the numbering should begin with the spot immediately below the zero spot of the first group. Two traces are ordinarily for a good experiment, the third group of spots being used only if desired. This apparatus is easily capable of yielding results if high precision and the data will justify great care in observation and recording.

Tabulation of Data: With a metric scale, measure the total distances S_1, S_2, S_3, etc., between the zero spot and succeeding number spots. To insure accuracy in these measurements, place the strip on a flat surface

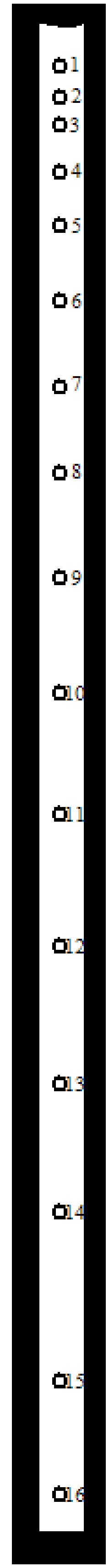

Fig. 7. A portion of the spark record of the freely falling body of the Free Fall Apparatus. To avoid confusion only one set of spots has been inked in. There is another spot between each one shown in the figure.

where there is good light. Place the meter stick *edgewise* on the trace in such a manner that the graduations are directly touching the dots. *Leave the meter stick stationary* and read the positions of the dots, estimating readings to fractions of a millimeter.

Subtract each position reading from the one immediately following it. This difference gives the distances s_1, s_2, s_3, etc., fallen during successive equal time intervals. By dividing these distances by the time interval, the average velocity for that interval may be calculated. The time interval is 1/60 second. These values of *average* velocities are also the *instantaneous* velocities for the mid-point of the interval considered.

An alternative method for measuring the distances s1, s2, s3, etc., is to measure them directly with a vernier caliper. The shape tips on the caliper are ideal for this purpose and readings may be made to tenths of millimeters.

The data may be tabulated as in Table I. A similar tabulation may be made for the data from the spots in the second trace.

Interpretation of Data:

Required Analysis: Plot a curve showing the relation of average velocity to time, using velocity as ordinates and time as abscissas, and plotting the points in the first quadrant. Locate each average velocity at the *mid-point* of the corresponding time interval, since for uniform acceleration the average velocity is identical with the actual velocity at the middle of the interval. Place the zero of abscissas (time intervals) somewhat to the right of the left-hand edge of the graph paper, since at "zero" time (the first spot) the body already had a small initial velocity. From the slope of this velocity-time curve determine the acceleration g of the falling body. Calculate the percentage difference between this value of *g* and the standard value. Draw a second graph using the other set of data, and calculate *g* from these data.

On another graph sheet plot a curve to show the total distance *S* fallen against the time. The times are simply the product of the ordinal numbers by the sparking interval *T*.

Thoroughly interpret the graphs in the report of the experiment. This interpretation should include conclusions to be drawn from the shapes of the curves, their slopes, and their intercepts. Careful explanations of the reasons for all conclusions should be given.

Optional Analyses: 1. Compute the value of *g* by applying the method of equal intervals to the last column of your table.

2. By taking corresponding values of distances and velocities for particular times from curves as in Figs. 2 and 3, plot a velocity-distance curve. Explain its shape.

3. Select some point on the distance-time curve and draw a tangent to the curve. From the slope of the tangent determine the velocity at that instant and compare it with the computed value.

4. Using the data for any two points on the record, compute the initial velocity by the use of Eq. (5). Compare this value with the initial velocity determined from the intercept of the velocity-time curve.

QUESTIONS: 1. If by some suitable mechanism the falling body had been given an initial downward push instead of being just released, would the resulting observed value of *g* have been different? Explain.

2. Classify the following as to whether they would introduce *systematic* or *random* errors in this experiment: (a) air friction, (b) estimations of fractional parts of millimeters on the scale, (c) zero error of meter stick, (d) variations in frequency of the a.c. power supply.

3. Neglecting friction, which of the following statements properly characterizes the motion of a heavy object thrown violently downward from a tall building: (a) uniform speed, (b) uniform deceleration, (c) constant acceleration, (d) uniformly increasing acceleration, or (e) a non-uniformly changing acceleration?

4. When the switch is opened the electromagnet does not instantly lose quite all of its magnetism. What effect does this have on your results?

5. What would be the appearance of the velocity-time curve if the falling body were so light that the effect of air friction could not be neglected?

TABLE I

Time Intervals, in 1/30 sec	S - Total Distance Traversed, in cm	s - Distance Traversed in One Time Interval, in cm	$\bar{v}$ - A. Velocity During One Time Interval, in cm/sec
0	......	......	......
....	......	0.96	38.4
1	0.96	......	......
....	......	1.58	6.2
2	2.54	......	......
....	......	2.23	89.2
3	4.77	......	......