CENCO Physics Selective Experiments In Physics

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PARTIALLY ELASTIC IMPACT

OBJECT: To study the law of conservation of momentum for the case of two colliding objects, and to measure the loss in kinetic energy at impact.

METHOD: Two objects of known masses are hung from the same point of support by cords of the same length so that both are capable of swinging as pendulums. One of the objects is pulled back through a measured angle and allowed to strike the other which is hanging vertically at rest. After collision both objects swing through angles which are measured by means of a small rider moved along by the moving objects. From these angles and the length of the pendulum, the vertical heights to which the objects are raised may be calculated and thus the velocities of the objects after collision may be obtained. The velocity before collision is similarly calculated from the angle through which the colliding object falls. Knowing the velocities before and after collision and the masses of the objects, the momentum and kinetic energy before and after collision may be calculated.

Fig. 1. Collision of two objects: (a) Velocities before impact; (b) Velocity after impact.

THEORY: Suppose an object (Fig. 1) of mass M_1 and velocity u collides with another object of mass M_2 which is at rest. After collision let the velocities of the two objects be v_1 and v_2 respectively. The change in momentum of object 1 is $M_1(v_1 - u)$ and of object 2 is $M_2(v_2 - 0)$. Let the time of duration of impact be Δt. Then the rate of change in momentum for object 1 is $M_1(v_1 - u)/\Delta t$ and is equal to the force exerted on object 1 by object 2. The rate of change of momentum for object 2 is $M_2(v_2 - 0)/\Delta t$ and is equal to the force exerted on object 2 by object 1. By Newton's third law of motion the force exerted by object 2 on object 1 is equal and opposite to the force exerted by object 1 on object 2, or

$$M_1\left(v_1 - u\right)/\Delta t = M_2\left(v_2 - 0\right)/\Delta t \qquad (1)$$

Hence

$$M_1 u = M_1 v_1 + M_2 v_2 \qquad (2)$$

Thus the *momentum of the system before impact is equal to the momentum of the system after impact*. This is the law of *conservation of momentum.*

Coefficient of Restitution: The coefficient of restitution *e* for two colliding objects is defined as the ratio of the relative velocity of separation immediately after impact to the relative velocity of approach immediately before impact, or for the system considered here,

$$e = \left(v_2 - v_1\right)/u \qquad (3)$$

The coefficient of restitution depends on the type of material and hardness of the surfaces, etc., of the two colliding object and consequently in practice may have all possible values between zero and unity.

Relative Loss of Kinetic Energy at Impact: Since the impact in this experiment is only partially elastic, there is some kinetic energy lost at impact. The relative loss l of K.E. at impact is defined as

$$l = \frac{\text{Total K.E. before impact} - \text{Total K.E. after Impact}}{\text{Total K.E. before Impact}}$$

$$l = \frac{\frac{1}{2}M_1 u_2 - \frac{1}{2}\left(M_1 v_1^2 + M_2 v_2^2\right)}{\frac{1}{2}M_1 u^2} \qquad (4)$$

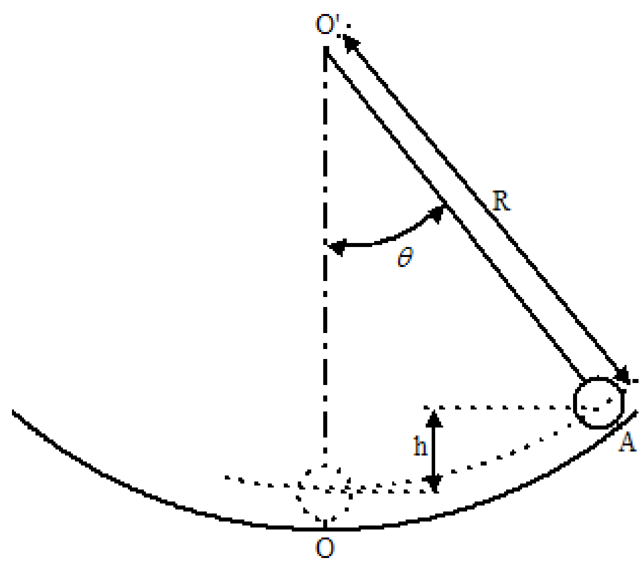

Fig. 2. Determination of velocity of ball from vertical height through which it falls.

The velocities of the two objects are obtained by suspending them as bifilar pendulums of the same length and measuring the angles or vertical heights through which the objects move. Let the length of the pendulums be R (Fig. 2). Suppose the object is raised to position A so that the pendulum makes an angle θ with the vertical. The vertical height *h* through which the center of mass of the pendulum is raised is

$$h = R(1-\cos\theta) \tag{5}$$

The velocity *v* acquired by the object in falling through the vertical height *h* is given by $v^2 = 2gh$, since the P.E. at A equals the K.E. at O or $Mgh = 1/2Mv^2$. Thus

$$v = \sqrt{2gR(1-\cos\theta)} \tag{6}$$

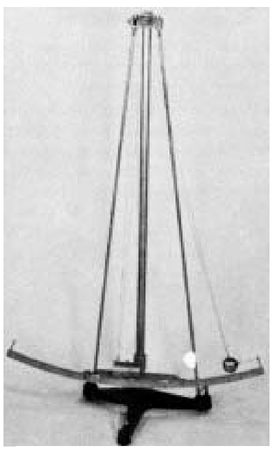

Fig. 3. Impact Apparatus

APPARATUS: The apparatus consists essentially of a metal cylinder and a steel sphere suspended as bifilar pendulums of the same length which can swing symmetrically over a graduated arc of a circle whose center is the axis of suspension (Fig. 3). On their under sides the spheres are provided with indices which can move light riders along the graduated scale. In this manner positions of the spheres may be determined.
A meter stick and a small quantity of wax are essential accessories.

PROCEDURE: Attach four long cords to the cylinder and two to the sphere and connect each of the upper ends of the cords to one of the suspension screws at the top of the apparatus. Adjust the lengths of the cords until the spheres hang slightly above the graduated scale so that the indices can move the riders along the scale. Level the apparatus by the three screws on the base until the spheres hang symmetrically over the graduated scale. Adjust the lengths of the cords so that each sphere hanging alone has its position marker at zero. After the adjustments have been made correctly, the two spheres swing symmetrically over the graduated scale and have no sidewise motion after impact.
Displace the larger sphere about 20° to the position A in Fig. 4, using a piece of thread attached to the sphere and to the

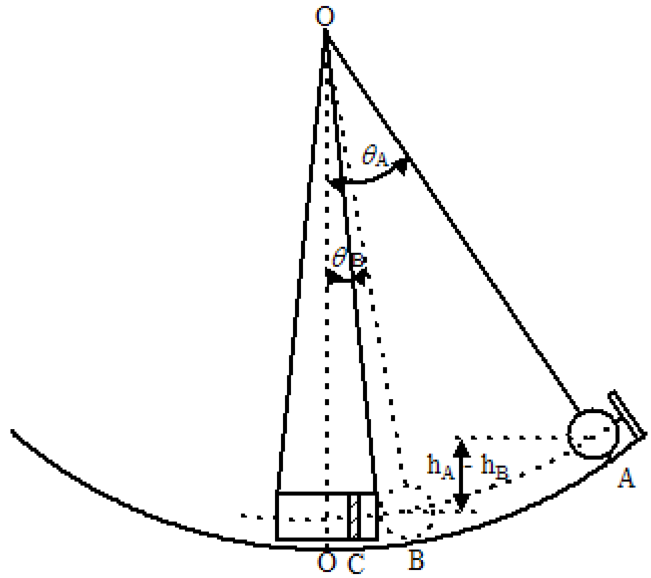

Fig. 4. Positions of cylinder and sphere before and at impact.

vertical post on the scale. Burn the thread near the sphere and allow it to strike the small sphere. Note the positions of the two spheres by means of their riders. Now place the two riders close to these determined positions so that when final readings are taken the spheres expend a negligible amount of their energy in moving the riders. The rider with the shorter projection should be placed near position B (Fig. 5) so that the smaller ball will pass over it after the impact occurs.
Again attach a thread to the larger sphere and displace it to position A, noting its position by its rider. Move the rider back to a position somewhere near B. Burn the thread and determine accurately the readings θ_B and θ_C (Fig. 5). Repeat several times with the same setting at A until consistent results are obtained. Lower the larger sphere until it just touches the smaller one hanging at its zero position, and determine θ_D (Fig. 4). Measure the length of the pendulums *R* from the support to the center of the spheres. If time permits, repeat the experiments with a different setting at A.
The height through which the larger ball falls before striking the smaller one is given by

$$\begin{aligned} h_A - h_D &= R\left(1-\cos\theta_A\right) - R\left(1-\cos\theta_D\right) \\ &= R\left(\cos\theta_D - \cos\theta_A\right) \end{aligned} \tag{6}$$

The velocity *u* immediately before impact is

$$u = \sqrt{2gR\left(\cos\theta_D - \cos\theta_C\right)} \tag{7}$$

After the impact the small sphere moves through an angle θ_C and hence its velocity immediately after impact is

$$v_2 = \sqrt{2gR\left(1-\cos\theta_C\right)} \tag{9}$$

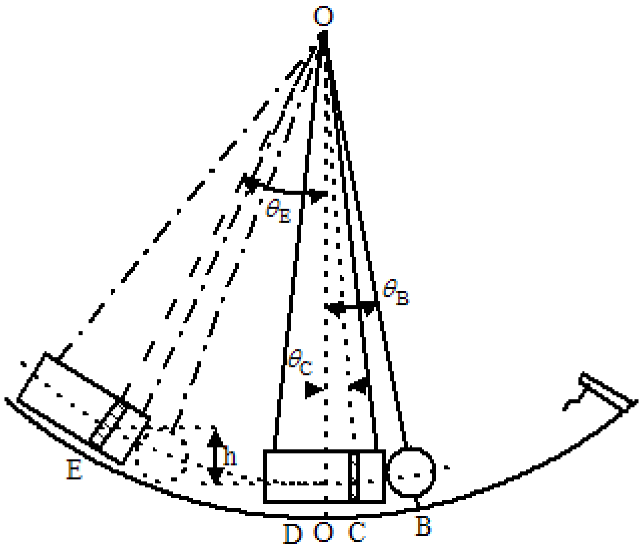

Fig. 5. Positions of cylinder and sphere after impact.

The larger sphere after impact attains a height of $R(1-\cos\theta_B)$ but it starts from a height of $R(1-\cos\theta_D)$, so that its velocity after impact is sufficient to raise it to a height of $R(\cos\theta_D - \cos\theta_B)$. Hence the velocity of the large sphere after impact is

$$v_1 = \sqrt{2gR(\cos\theta_D - \cos\theta_B)} \qquad (9)$$

Following the procedure outlined above, determine the various angles and calculate the velocities u and v_1, v_2. The masses M_1 and M_2 of the spheres may be supplied by the instructor. If not, they may be found by means of a suitable balance without detaching them from the cords.
Calculate the momentum before and after impact. State the percentage difference. Calculate the K.E. of the system before impact, $\left(\frac{1}{2} M_1 u^2\right)$, and the K.E. after impact, $\left(\frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2\right)$ and from this find the relative loss l in K.E. at impact. Calculate the coefficient of restitution e for the two spheres from Eq. (3).

QUESTIONS: 1. Using Eqs. (2), (3) and (4), show that the relative loss l in K.E. at impact is equal to $M_2(1-e^2)/M_1 + M_2$.

2. Show that for a perfectly elastic impact (e = 1) there is no loss in K.E. at impact.

3. Into what form of energy is that kinetic energy which is lost at impact transformed?