Selective Experiments In Physics

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INELASTIC IMPACT

OBJECT: To study the law of conservation of momentum for the case of two colliding objects which stick together after impact, and to measure the loss in kinetic energy at impact.

METHOD: Two objects of known masses are hung from the same point of support by cords of the same length so that both are capable of swinging as pendulums. One of the objects is pulled back through a measured angle and allowed to strike the other which is hanging vertically at rest. After collision both objects stick together and swing through an angle which is measured by means of a small rider moved along by the moving objects. From this angle and the length of the supporting cords the vertical heights to which the objects are raised may be calculated, and thus the velocity of the two objects after collision may be obtained. The velocity of the moving object before collision may be similarly calculated from the angle through which it falls. Knowing the velocities before and after collision and the masses of the objects, the momentum and kinetic energy before and after collision may be calculated.

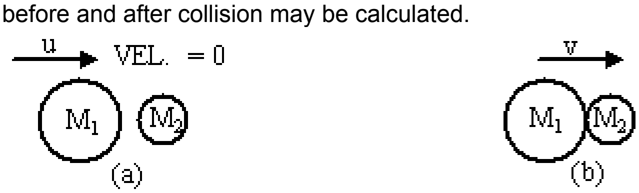

Fig. 1. Inelastic collision of two objects: (a) Velocities before impact; (b) Velocity after impact.

THEORY: Suppose an object (Fig. 1) of mass M_1 and velocity u collides with another object of mass M_2 which is at rest. Let the impact be inelastic so that the two objects stick together. The common velocity with which they move immediately after impact is v. The change in momentum of mass M_1 is $M_1(v - u)$ and of the mass M_2 is $M_2(v - 0)$. If the time of the duration of the impact is Δt, then the rate of change of momentum of mass M_1 is $M_1(v - u)/\Delta t$ which, by Newton's second law of motion, is equal to the force exerted on object 1 by object 2. Similarly the force exerted on object 2 by object 1 is equal to $M_2(v - 0)/\Delta t$. By Newton's third law of motion the force exerted by object 2 on object 1 is equal and opposite to the force exerted by object 1 on object 2, or

$$M_1(v-u)/\Delta t = -M_2(v-0)/\Delta t$$

Hence

$$M_1 u = (M_1 + M_2)v \tag{1}$$

Thus the *momentum of the system before impact is equal to the momentum of the system after impact*. This is the law of *conservation of momentum.*

Since in an inelastic impact the objects are permanently deformed by the impact, some work is done in deforming them. Thus the kinetic energy of the system is less after impact than before impact. The relative loss *l* of kinetic energy at impact is defined as

$$L = \frac{\text{Total K.E. before impact} - \text{Total K.E. after impact}}{\text{Total K.E. before impact}}$$

For the objects M_1 and M_2 the relative loss *l* of kinetic energy at impact is given by

$$l = \frac{\tfrac{1}{2}M_1u^2 - \tfrac{1}{2}(M_1 + M_2)v^2}{\tfrac{1}{2}M_1u^2} \tag{2}$$

Substituting for *v* in terms of u from Eq. (1)

$$l = \frac{M_2}{M_1 + M_2} \tag{3}$$

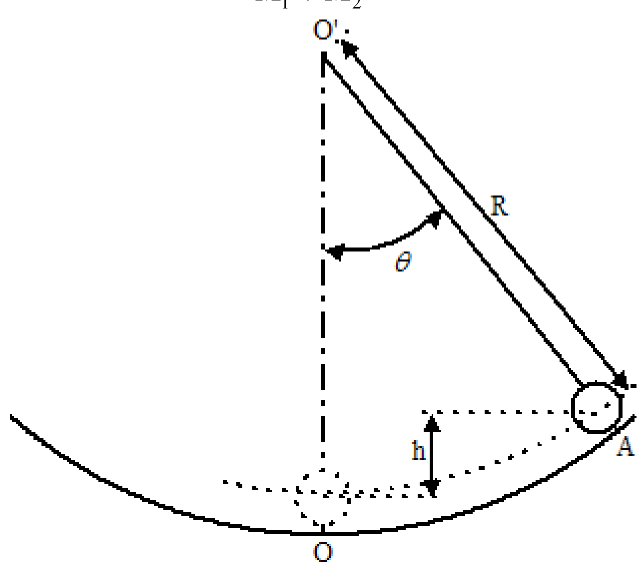

Fig. 2. Determination of velocity of ball from vertical height through which it falls.

The objects are hung from the same line of support by cords of the same length, and from the angles through which they swing their respective velocities are determined. The length of the pendulum *R* is measured from the point of support to the center of mass of the object. Suppose the object falls from the point A to the point O through an angle θ (Fig. 2) where the line OO' is vertical.

Then the vertical height through which the center of mass falls is

$$h = R(1 - \cos\theta) \qquad (4)$$

The velocity v acquired by the object in falling through the angle θ is given by $v^2 = 2gh$, since the P.E. at A equals the K.E. at O or $Mgh = 1/2Mv^2$. Thus

$$v = \sqrt{2gR(1 - \cos\theta)} \qquad (5)$$

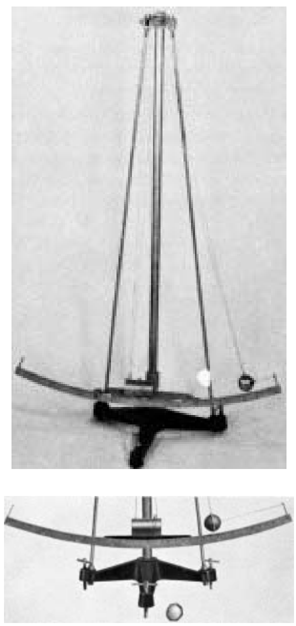

Fig. 3. Impact Apparatus

APPARATUS: The apparatus consists essentially of a steel sphere and a U-shaped block, both suspended as pendulums of the same length from the same line of support (Fig. 3). The sphere and the U-shaped block are hung symmetrically over a graduated arc of a circle whose center is the axis of suspension. On the under side of the sphere is an index which can move a light rider along the graduated scale. The U-shaped block is constructed so that when the moving sphere strikes it, the sphere is rigidly caught in the block in such a manner that the center of mass of the block and sphere together is at the same point as the center of mass of the U-shaped block alone.

A meter stick and a small quantity of wax are essential accessories.

PROCEDURE: Attach four long cords to the U-shaped block and two to the sphere. Connect each of the upper ends of the cords to one of the suspension screws at the top of the apparatus as shown in Fig. 3 in such a way that at impact the sphere is caught in the U-shaped block. The cavity on the inside of the U-shaped block should be filled with beeswax to make the impact inelastic. The amount of the wax should be adjusted until at impact the index on the sphere will be directly over the zero mark on the graduated scale. Adjust the length of the cords until the sphere and U-shaped block hang slightly above the graduated scale so that the index on the sphere can move the rider along the scale. Level the apparatus by means of the three screws on the base until the block and sphere hang symmetrically over the graduated scale. When all adjustments have been correctly made, the sphere and U-shaped block swing symmetrically over the graduated scale and have no sidewise motion after impact.

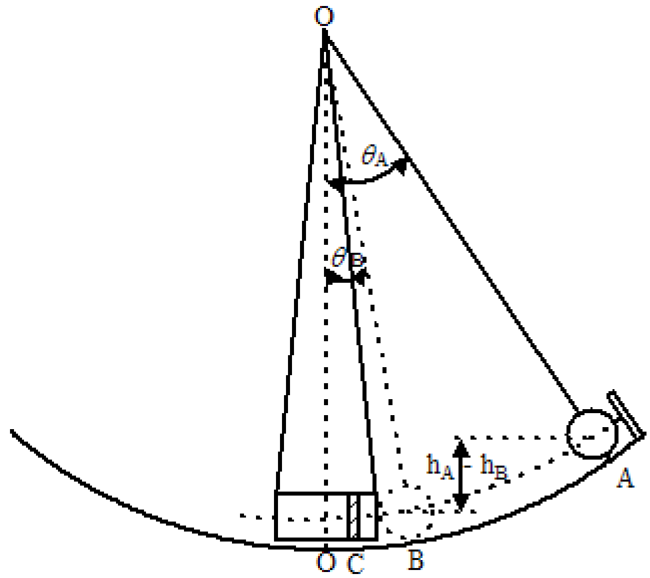

Fig. 4. Positions of cylinder and sphere before and at impact.

Displace the sphere about 20° to the position A in Fig. 4, using a piece of thread attached to the sphere and to the vertical post on the scale. Burn the thread near the sphere and allow it to strike the U-shaped block which is hanging vertically at rest. To make the impact quite inelastic it may be desirable to warm the wax slightly just before the ball is released. Note the position B of the sphere and block by means of the rider. Place the rider close to this position so that when final readings are taken the sphere and block expend a negligible amount of energy in moving the rider.

Again attach a thread to the sphere and displace it to position A, noting its exact position with a rider. Burn the thread and accurately determine the reading θ_B. Repeat several times with the same setting at A until consistent results are obtained. Measure the length *R* from the pendulum from the point of support to the center of mass of the sphere or U-shaped block. If time permits, repeat the observations with a different setting A of the sphere.

The masses M_1 of the sphere and M_2 of the U-shaped block may be supplied by the instructor. If not, they may be found with a suitable balance without detaching them from the cords. Calculate the velocities *u* and *v* before and after impact using Eq. (5) and the angles θ_A and θ_B. Calculate the momentum before and after impact and give the percentage difference. Calculate the kinetic energy of the system before and after impact and from this obtain the relative loss *I* in K.E. at impact. From Eq. (3) find *I* and give the percentage difference in the two values of *I* so obtained.

QUESTIONS: 1. Into what form of energy is the kinetic energy which is lost transformed?

2. When a pendulum is set vibrating, it ultimately comes to rest. What are the various causes of its coming to rest?

3. In order to increase the accuracy of the experiment would it be better to have very long or very short pendulum lengths.