CENCO Physics — Selective Experiments In Physics

• C.J. Overbeck Northwestern University • R.R. Palmer Beloit College • Marsh W. White The Pennsylvania State University

INELASTIC IMPACT

OBJECT: To study the law of conservation of momentum for the case of two colliding objects which stick together after impact, and to measure the loss in kinetic energy at impact.

METHOD: Two objects of known masses are hung from the same point of support by cords of the same length so that both are capable of swinging as pendulums. One of the objects is pulled back through a measured angle and allowed to strike the other which is hanging vertically at rest. After collision both objects stick together and swing through an angle which is measured by means of a small rider moved along by the moving objects. From this angle and the length of the supporting cords the vertical heights to which the objects are raised may be calculated, and thus the velocity of the two objects after collision may be obtained. The velocity of the moving object before collision may be similarly calculated from the angle through which it falls. Knowing the velocities before and after collision and the masses of the objects, the momentum and kinetic energy before and after collision may be calculated.

THEORY: Suppose an object (Fig. 1) of mass M_1 and

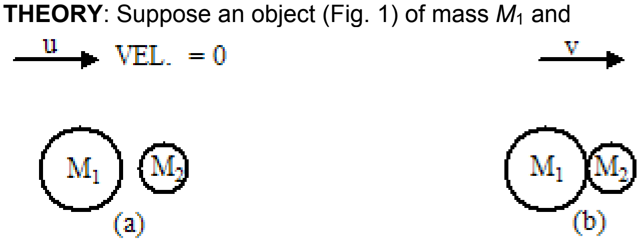

Fig. 1. Inelastic collision of two objects: (a) Velocities before impact; (b) Velocity after impact.

velocity u collides with another object of mass M_2 which is at rest. Let the impact be inelastic so that the two objects stick together. The common velocity with which they move immediately after impact is v. The change in momentum of mass M_1 is $M_1(v - u)$ and of the mass M_2 is $M_2(v - 0)$. If the time of the duration of the impact is Δt, then the rate of change of momentum of mass M_1 is $M_1(v - u)/\Delta t$ which, by Newton's second law of motion, is equal to the force exerted on object 1 by object 2.

Similarly the force exerted on object 2 by object 1 is equal to $M_2(v - 0)/\Delta t$. By Newton's third law of motion the force exerted by object 2 on object 1 is equal and opposite to the force exerted by object 1 on object 2, or

$$M_1(v-u)/\Delta t = -M_2(v-0)/\Delta t$$

Hence

$$M_1 u = (M_1 + M_2)v \tag{1}$$

Thus the *momentum of the system before impact is equal to the momentum of the system after impact*. This is the law of *conservation of momentum.*

Since in an inelastic impact the objects are permanently deformed by the impact, some work is done in deforming them. Thus the kinetic energy of the system is less after impact than before impact. The relative loss l of kinetic energy at impact is defined as

$$l = \frac{\text{Total K.E. before impact} - \text{Total K.E. after impact}}{\text{Total K.E. before impact}}$$

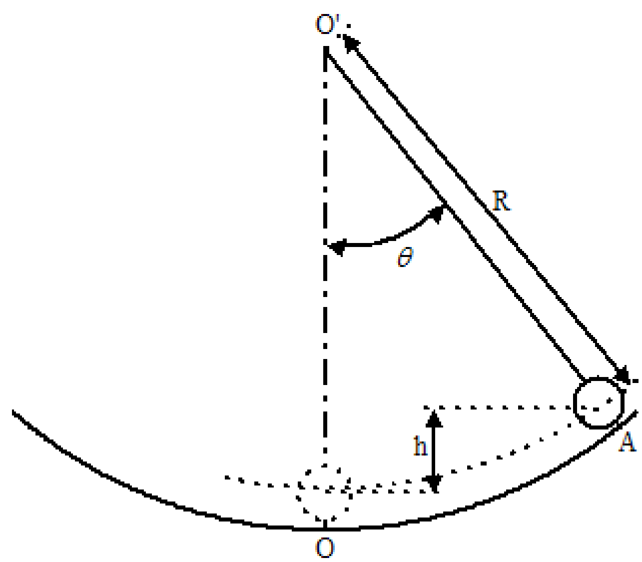

Fig. 2. Determination of velocity of ball from vertical height through which it falls.

For the objects M_1 and M_2 the relative loss l of kinetic energy at impact is given by

$$l = \frac{\frac{1}{2}M_1u^2 - \frac{1}{2}(M_1 + M_2)v^2}{\frac{1}{2}M_1u^2} \tag{2}$$

Substituting for v in terms of u from Eq. (1)

$$l = \frac{M_2}{M_1 + M_2} \quad (3)$$

The objects are hung from the same line of support by cords of the same length, and from the angles through which they swing their respective velocities are determined. The length of the pendulum R is measured from the point of support to the center of mass of the object. Suppose the object falls from the point A to the point O through an angle θ (Fig. 2) where the line OO' is vertical. Then the vertical height through which the center of mass falls is

$$h = R(1 - \cos\theta) \quad (4)$$

The velocity v acquired by the object in falling through the angle θ is given by $v^2 = 2gh$, since the P .E. at A equals the K.E. at O or $Mgh = 1/2Mv^2$. Thus

$$v = \sqrt{2gR(1 - \cos\theta)} \quad (5)$$

APPARATUS: The apparatus consists essentially of a metal

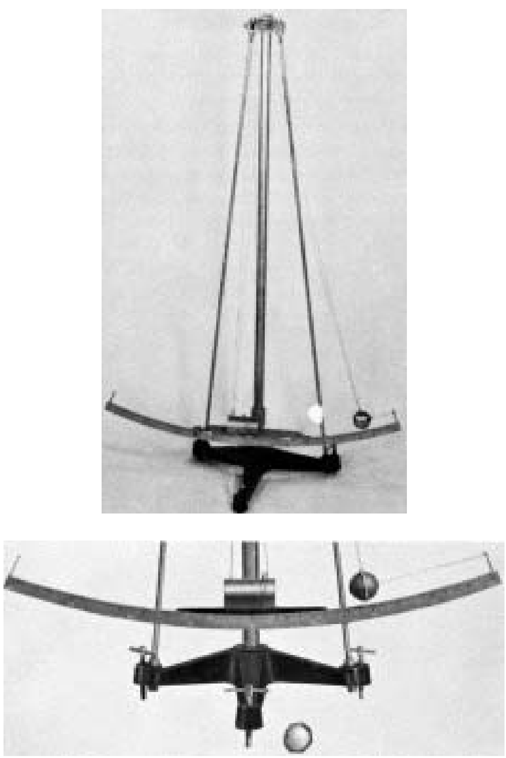

Fig. 3. Impact Apparatus

cylinder and a steel sphere, both suspended as pendulums of the same length from the same line of support (Fig. 3). The cylinder and sphere are hung symmetrically over a graduated arc of a circle whose center is the axis of suspension. On their under sides the cylinder and sphere are provided with indices which can move the riders along the graduated scale. In this manner positions of the center of mass of the cylinder and sphere, and of the sphere alone, may be determined.
A meter stick and a small quantity of wax are essential accessories.

PROCEDURE: Attach four long cords to the cylinder and two to the sphere. Connect each of the upper ends of the cords to one of the suspension screws at the top of the apparatus as shown in Fig. 3. Adjust the lengths of the cords until the cylinder and sphere hang slightly above the graduated scale so that the indices can move the riders along the scale. Level the apparatus by means of the three screws on the base until the cylinder and sphere each hang symmetrically over the graduated scale. When properly leveled, the index on the sphere (suspended alone) should hang directly over the zero of the scale. The index on the cylinder should now be adjusted to zero *when the cylinder and sphere hang together*. When all adjustments have been correctly made, the cylinder and sphere swing symmetrically over the graduated scale without sidewise motion after impact. The impact should be made inelastic by placing a small piece of beeswax or plasticine on the cylinder at the point where the impact takes place.
Displace the sphere about 20° to the position A in Fig. 4, using a piece of thread attached to the sphere and to the

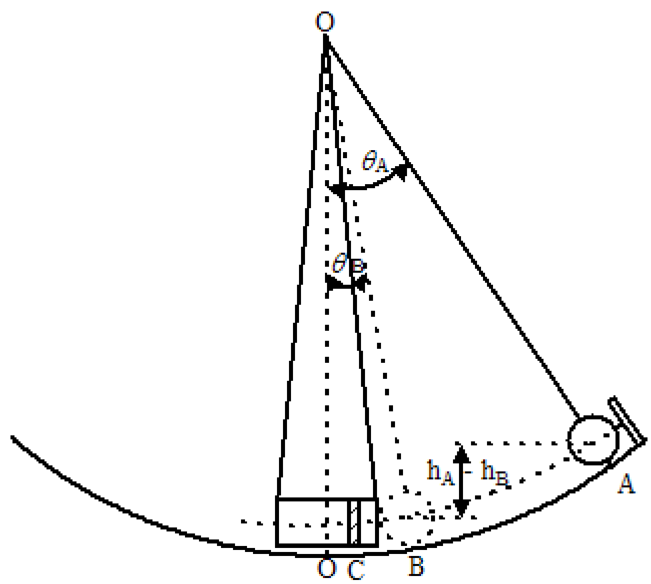

Fig. 4. Positions of cylinder and sphere before and at impact.

vertical post on the scale. Burn the thread near the sphere and allow the latter to strike the cylinder which is hanging freely. To make the impact quite inelastic it may be desirable to warm the wax slightly just before the ball is released. Note the position E (Fig. 5) of the index on the cylinder by means of the rider. Place the rider close to this point so that when final readings are taken the cylinder and sphere expend a negligible amount of energy in moving the rider.
Again attach a thread to the sphere and displace it to position A, noting its exact position with a rider which is placed to the right of the index. Burn the thread and determine accurately the angle θ. Repeat several times with

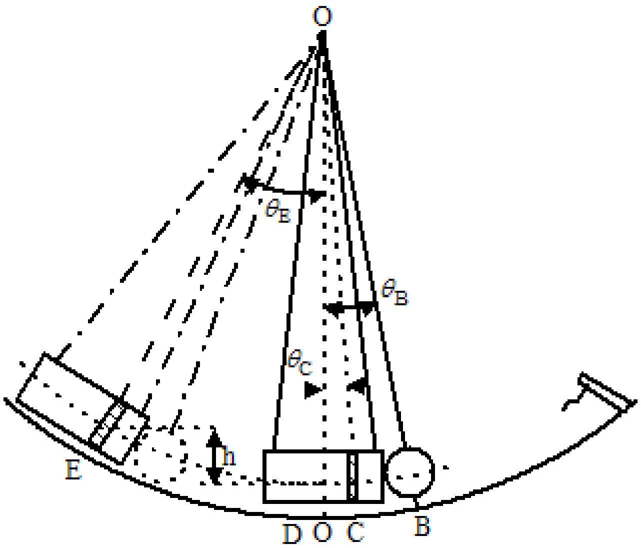

Fig. 5. Positions of cylinder and sphere after impact.

the same setting at A until consistent results are obtained. Measure the length R from the support to the center of mass of the sphere.

Bring the sphere carefully up to and just touching the cylinder and determine the angle θ_B by means of the rider. The position B gives the point of impact of sphere and cylinder.

The vertical height through which the sphere falls before striking the cylinder is given by

$$h_A - h_B = R(1-\cos\theta_A) - R(1-\cos\theta_B)$$
$$= R(\cos\theta_B - \cos\theta_A) \tag{6}$$

The velocity u of the sphere at the instant of impact is

$$u = \sqrt{2gR(\cos\theta_B - \cos\theta_A)} \tag{7}$$

With the cylinder hanging alone, observe the angle θ_c which is determined by the index on the cylinder. As a result of the impact, the center of mass of the cylinder and sphere is raised through the angle 6. - 6c. At the instant of impact the sphere is at position B. If the sphere were held at rest in position B and then released, the cylinder and ball would acquire sufficient momentum to move the index from C to D where OC = OD. Hence the actual height to which the center of mass of cylinder and sphere is raised due to their velocities immediately after impact is

$$h = R(1-\cos\theta_E) - R(1-\cos\theta_C) =$$
$$R(\cos\theta_C - \cos\theta_E) \tag{8}$$

The velocity of the cylinder and sphere immediately after impact is then

$$v = \sqrt{2gR(\cos\theta_C - \cos\theta_E)} \tag{9}$$

Following the procedure outlined above, determine the various angles and calculate the velocities u and v. The masses M_1 of the ball and M_2 of the cylinder may be supplied by the instructor. If not, they may be found with a suitable balance without detaching them from the cords. Calculate the momentum before and after impact. State the percentage difference. Calculate the K.E. of the system before impact, $\frac{1}{2}M_1u^2$, and the K.E. after impact, $\frac{1}{2}(M_1 + M_2)v^2$, and from this the relative loss l in K.E. at impact. Calculate l from Eq. (3). What is the percentage difference?

QUESTIONS: 1. Into what form of energy is the kinetic energy which is lost transformed?

2. When a pendulum is set vibrating, it ultimately comes to rest. What are the various causes of its coming to rest?

3. Check Eq. (9) for the velocity of the system after impact by considering the energies involved. Write down the K.E. and the P.E. of the system immediately after impact and the P.E. of the system when at its maximum position E, Fig. 5.