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DENSITIES OF SOLIDS AND LIQUIDS USING A JOLLY BALANCE

OBJECT: To determine the densities of a solid and a liquid by using Archimedes' principle and a Jolly balance.

METHOD: A body is alternately weighed suspended in air and immersed in a liquid. The apparent loss in weight of the immersed body is known, by Archimedes' principle, to equal the weight of liquid displaced by the body. The apparent less in weight is measured by means of a spring. From these measurements the density and specific gravity of either the solid body or the liquid may be determined.

THEORY: The density of a substance is defined as its mass per unit volume, and in the metric system of units is measured in grams per cubic centimeter. The specific gravity of a solid or liquid is defined as the ratio of the density of the substance to the density of water. Since specific gravity is a numeric, it has zero dimensions.

A convenient method for determining densities or specific gravities is one which uses the principle of Archimedes. This principle states that when a body displaces a fluid there is exerted on the body a vertical upward force equal to the weight of fluid displaced. This force is called buoyancy; and it acts at the center of gravity of the displaced fluid.

Proof of Archimedes' Principle: Suppose aright circular cylinder of height h is immersed in a liquid of density d. The bottom of the cylinder is at a depth h_2, and the top at a depth h_1, below the surface of the liquid (Fig. 1). It is apparent that

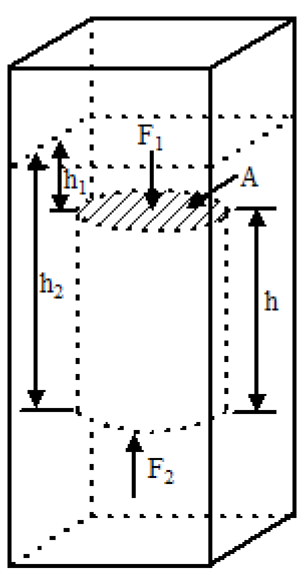

Fig. 1. Vertical forces acting on a right circular cylinder immersed in a liquid of density d.

$$h = (h_2 - h_1) \tag{1}$$

The pressure at a point in the liquid is the sum of that part of it which arises from the head of liquid above the point and that part which arises from the atmosphere. Thus $p_1 = h_1 gd$ + atmospheric pressure. Likewise, $p_2 = h_2 gd$ + atmospheric pressure. The total downward force on the upper surface of the cylinder is $F_1 = P_1 A$, where A is the area of cross section of the cylinder. Similarly the total upward force on the bottom of the cylinder is $F_2 = P_2 A$. Since the pressure in a fluid acts at right angles to any surface in contact with it, the pressure on the sides of the cylinder is everywhere horizontal and has no vertical component. Hence the total upward force on the cylinder is

$$F_2 - F_1 = p_2 A - p_1 A = (h_2 - h_1)Agd = hAgd \tag{2}$$

But $hA = V$, the volume of the cylinder, so that

$$F_2 - F_1 = Vgd \tag{3}$$

Thus the force of buoyancy equals the weight of fluid displaced. While this proof of Archimedes' principle is directly applicable to a simple geometrical object, the principle is true for an object of any shape which displaces any fluid, whether liquid or not.

Application of Archimedes' Principle: Suppose a body has a density D and a mass M. The volume of the body is $V = M/D$ and its weight in air is Mg. The apparent weight of the body when immersed in a liquid of density d is, by Archimedes' principle, $M_L g = Mg - Vdg$. Thus $M_L = M - Vd$ or $M_L = M - Md/D$.

Since M_L and M may be measured by means of a balance, it follows that if the density d of the liquid- say water- is know the, the density D of the body may be calculated from

$$D = \frac{Md}{M - M_L} \tag{4}$$

If the body whose density is to be measured is less dense than the liquid, it is necessary to fasten the body to a sinker so that the two together sink in the liquid. Let $M_1 g$ be the weight in air of the body, say a block of wood, whose density D_l is less than that of the liquid whose density is d. Suppose the body of mass M, density D, referred to above, is such that when fastened to the block of wood the two together sink in the liquid. Let the weight of the two together when completely immersed in the liquid be M_L'g. Then from Fig. 2 it is readily seen that

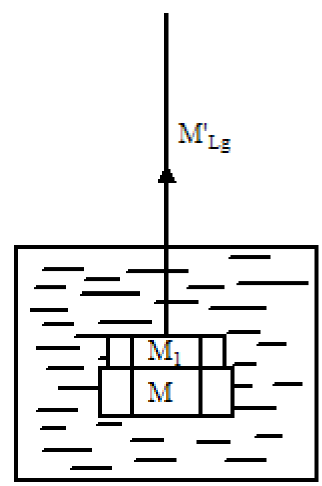

Fig. 2. Block of wood M_1 immersed in liquid, density d, by sinker M.

$$M_L{}'g = Mg + M_1 g - \frac{Mgd}{D} - \frac{M_1 gd}{D_1}$$

or

$$M_L{}' = M_L + M_1 - \frac{M_1 d}{D_1} \tag{5}$$

Hence

$$D_1 = \frac{M_1 d}{M_L + M_1 - M_L{}'} \tag{6}$$

The relative densities of liquids may be obtained by finding the weight of a sinker in the various liquids. Suppose that when the sinker is immersed in a liquid of density d_l its apparent loss in weight is $M_1 g$ and when immersed in a liquid of density d_2 the apparent loss in weight is $M_2 g$. By Archimedes' principle $M_1 g = Vd_1 g$ and $M_2 g = Vd_2 g$ where V is the volume of the sinker. Hence

$$V = M_1/d_1 = M_2/d_2$$

or

$$M_1/M_2 = d_1/d_2 \tag{7}$$

The masses or weights of the various bodies are measured in terms of the extension of a spring. When a properly constructed spring is stretched by an applied force, it is found that the elongation of the spring is proportional to the stretching force so long as the elastic limit of the spring is not exceeded. This is Hooke's law for a spring.

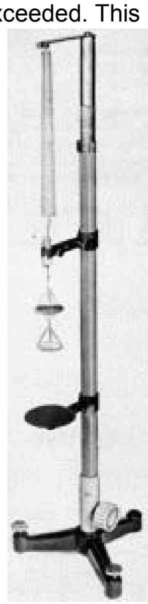

Fig. 3. Jolly Balance

If a force F_1 applied to the spring produces an elongation of x_1, then double the force produces double the elongation. In general,

$$F = Cx \tag{8}$$

where C is called the force constant of the spring and is numerically equal to the force required to produce unit elongation. The forces in this experiment are the weights hung on the lower end of the spring so that the extension of the spring is proportional to the weights. Since weights of bodies are proportional to their respective masses, it follows that for Eqs. (4), (6) and (7) the various masses are proportional to the respective extensions which are produced.

APPARATUS: The apparatus consists of a sensitive helical spring attached at its upper end to a movable calibrated pillar and carrying an index mark and scale pan at its lower end as shown in Fig. 3. The upper end may be moved by a screw at the base of the pillar and the position read by a scale and vernier. The index marks for indicating the position of the lower end of the spring are three horizontal lines marked on a small aluminum cylinder which hangs freely within a glass tube on which there is etched a horizontal line. Stops are provided at either end of the cylinder so that its motion is limited. The assembly, which consists of the spring, the standard on which it is mounted and the scale pans, is called a Jolly balance. In addition to it, the experiment requires a beaker and a bottle of some liquid, such as alcohol, .the density of which is to be measured. For solid objects whose densities are to be measured, a small brass block and a small block of paraffined wood will serve.

PROCEDURE:

Adjustments: The masses of the objects whose densities are to be determined depend on the load capacity of the spring used. With a suitable spring firmly clamped at its upper end, level the instrument by means of the leveling screws on the base so that the spring hangs parallel to the pillar. Bring the zero point of the scale on the pillar into coincidence with the zero point of the vernier. With two scale pans underneath one another at the lower end of the spring, move the clamp and glass tube until the etched line on the glass cylinder is in the same plane as the central index mark on the small aluminum cylinder.

Experimental:

Part A: Place a beaker partially filled with water on the stand attached to the pillar and adjust the position of the stand until the lower scale pan is under the water so that only a single strand of wire cuts the water surface. Place the heavier body whose density is to be found on the upper scale pan, and find the extension of the spring by raising its upper end until the central mark on the small cylinder is in the same plane as the mark etched on the glass. Read and record the scale and vernier reading. Then place the body on the lower scale pan under the water and again find the extension of the spring. Be careful to see that no air bubbles are present under the body.

Part B: Remove the heavy body from the lower scale pan. Place the piece of wood in the upper pan and determine the resulting extension of the spring. Attach the piece of wood to the heavy body by a fine thread whose mass is negligible and place both in the lower scale pan under the water. Find the extension of the spring. It is assumed here that the two bodies together will not float.

Part C: Replace the water in the beaker by alcohol or other liquid provided. Place the heavy body on the lower pan and completely submerge it in the liquid. Determine the extension of the spring.

Calculations: Find the density of the heavy body from the data of Part A using Eq. (4). The masses or weights of the various bodies are proportional to the respective extensions of the spring which they produce. Using Eq. (6) find the density of the block of wood. Find the densities of the liquids used in Part C making use of Eq. (7).

QUESTIONS: 1. Give the specific gravities of the substances whose densities have been measured.

2. Show that density measured in pounds per cubic foot is equal to specific gravity multiplied by 62.4lb./cu ft.

3. Why is it not necessary to calibrate the spring in this experiment?